Capital Market Efficiency: A Reinterpretation

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Abstract

Realized returns on stocks and bonds in a standard consumption-based asset pricing model can be represented by an additive combination of the representative investor’s percentage forecast errors for certain endogenous variables. As a result, predictability of realized excess returns can arise from only two sources: (1) agents being risk neutral and (2) the model’s fundamental driving variables exhibit persistent stochastic volatility. This is a general result that holds for any stochastic discount factor, any consumption/dividend process, and any characterization of bond coupon payments. Under risk aversion this result implies that excess price volatility, forecastable excess returns and time-varying conditional variances of driving processes are equivalent: either all are present or none are.

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1 Introduction

Capital markets are said to be efficient if (1) agents have rational expectations and (2) returns or excess returns on securities and portfolios are unforecastable. Early

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investigations by Fama [8] and others concluded in favor of capital market efficiency: returns appeared to be uncorrelated over time, for example, so that current returns or prices do not predict future returns. Other studies, however, concluded that empirical stock price volatility exceeded the level of volatility consistent with market efficiency so defined (Shiller [19], LeRoy and Porter [13], Cochrane [5]).

LeRoy and Steigerwald [14] suggested that the discrepancy between these results might be due to tests of market efficiency based on price volatility being more powerful than tests based on direct estimates of return forecastability. If so, the fact that price volatility tests appeared to provide stronger evidence against market efficiency than return forecastability tests implied no puzzle. However, LeRoy and Steigerwald did not provide a specific account of where this proposed difference in power comes from.

Cochrane [5] advanced the discussion by providing an example in which one can see the reason for the power difference: in his setting power can be computed explicitly, based on simulations. This rendered his argument that capital markets appear to be inefficient especially persuasive.

Cochrane’s result was based on the fact that high price volatility is essentially the same thing as return forecastability. The argument for this is easily summarized. Drawing on earlier work by Campbell and Shiller [4], Cochrane observed that an identity derived from a log-linearization of the definition of the rate of return implies that price volatility is attributable to two sources: forecastability of dividend growth rates and forecastability of returns. Empirically dividend growth is virtually unforecastable, implying that to a close approximation the identity just cited defines a direct relation between price volatility and return forecastability. Under this identity the observed high level of price volatility is equivalent to forecastability of a substantial component of return variation. This is consistent with the earlier conclusion of Shiller and LeRoy-Porter.

The identification just described of market efficiency with return unforecastability has always been problematic. It has long been known that financial market equilibrium implies return unforecastability under rational expectations in the special, and unrealistic, case of risk neutrality. The argument for return unforecastability under risk neutrality is simple and appealing: if agents value only expected returns, ignoring higher moments of return distributions, then equilibrium conditionally expected returns will be equal across securities (that is, they will not depend on the realizations of the conditioning variables) and, at least under the simplest specification of utility functions, will be constant over time.

Risk-averse agents, in contrast, will balance expected returns against the higher moments of return distributions. As a result, one would expect that changes over time in conditional variances, for example, will be associated with changes over time in conditional expected returns (LeRoy [12], for example). In other words, the assumption that agents are risk averse implies that there is no reason to expect that financial markets will be efficient: conditional expected returns will be forecastable to the extent that conditional volatility is forecastable.
The argument just outlined suggests that if conditional volatilities in financial markets are constant over time, then returns will be unforecastable whether or not agents are risk neutral. To the extent that agents are nearly risk neutral returns will be nearly unforecastable whether or not conditional volatilities are time-varying. If agents’ risk aversion is not negligible and if volatilities are time-varying, we expect that returns will have a forecastable component. In this paper we show that this proposition is correct under very general conditions. Empirically we see both strongly time-varying volatilities and forecastable returns; the result of this paper implies that neither is surprising given the other.

It follows that the definition given above of financial market efficiency is equivalent to an identification of efficiency with rational expectations and risk neutrality (which is unrealistic) or constant volatilities. Why the idea of market efficiency should be connected in this way with the constancy of conditional volatilities is far from clear; we return to this question in the conclusion.

2 Excess Returns and Percentage Forecast Errors

The framework for our analysis is a class of standard representative agent consumption-based asset pricing models. We do not need to specify the model fully; for example, preferences are not specified and the growth rate of the dividend/consumption process is not characterized. Our results thus apply to a very general set of models.

For any traded asset and any specification of investor preferences, the first-order condition of the representative investor’s optimal consumption choice yields

\[ 1 = E_t \left[ M_{t+1} R_{t+1}^i \right], \]

where \( E_t \) is the mathematical expectation operator conditional on information available at time \( t \), \( M_{t+1} \) is the investor’s stochastic discount factor, and \( R_{t+1}^i \) is the gross holding period return on asset type \( i \) from time \( t \) to \( t + 1 \). With time-separable constant relative risk aversion (CRRA) preferences, we have \( M_{t+1} = \beta \left( c_{t+1}/c_t \right)^{-\alpha} \), where \( \beta \) is the subjective time discount factor, \( c_t \) is the investor’s consumption at \( t \), and \( \alpha \) is the risk aversion coefficient. With recursive preferences along the lines of Epstein and Zin [6], [7] and Weil [20] (IS THIS THE RIGHT CITATION?), we have \( M_{t+1} = \beta^\omega \left( c_{t+1}/c_t \right)^{-\omega/\sigma} \left( R_{t+1}^c \right)^{\omega-1} \), where \( R_{t+1}^c \) is the gross return on an asset that delivers a dividend of \( c_t \) each period, \( \sigma \) is the elasticity of intertemporal substitution, and \( \omega \equiv (1-\alpha)/(1-\sigma^{-1}) \). In the special case when \( \alpha = \sigma^{-1} \), we have \( \omega = 1 \) such that Epstein-Zin-Weil preferences coincide with CRRA preferences. With external habit formation preferences as in Campbell and Cochrane [3], we have \( M_{t+1} = \beta \left[ s_{t+1}c_{t+1}/(s_t c_t) \right]^{-\alpha} \), where \( s_t \equiv 1-x_t/c_t \) is surplus consumption ratio and \( x_t \) is the external habit level.

For a dividend-paying stock, we have \( R_{t+1}^s = (d_{t+1} + p_{t+1}^e)/p_t^s \), where \( R_{t+1}^s \) is the return on stock, \( p_t^s \) is the ex-dividend stock price at \( t \) and \( d_{t+1} \) is the dividend received
Turning to bonds, we assume that the coupon at time $t$ on a default-
free bond initiated at time $\tau$ is $\delta^{t-\tau}$, where $\tau \leq t$. This specification of geometrically
declining coupons allows parametrization of bonds of differing Macaulay duration
($\delta = 0$ represents a one-period bond, while $\delta = 1$ represents a perpetuity). Also,
bonds valued at the same date but initiated at different dates can be aggregated
using $p_{\tau,t}^b = \delta p_{\tau+1,t}^b$, where $p_{\tau,t}^b$ denotes the price at $t$ of a bond initiated at $\tau$ (in units
of the consumption good). The gross return $R_{t+1}$ from $t$ to $t+1$ on bonds initiated
at any date $\tau \leq t$ can be written as

$$R_{\tau,t+1}^b \equiv \frac{\delta^{t-\tau} + p_{\tau,t+1}^b}{p_{\tau,t}^b} = \frac{1 + \delta p_{\tau+1,t+1}^b}{p_{\tau,t}^b} \equiv R_{t+1}^b \equiv \frac{1 + \delta p_{t+1}^b}{p_t^b}, \tag{2}$$

where the third and fourth identities represent a notational simplification made pos-
sible by the fact that the first and second subscripts are the same in the third term.\(^1\)

For stocks, equation (1) can be written as

$$p_t^s/d_t = E_t \left[ M_{t+1} \left( \frac{d_{t+1}}{d_t} \right) \left( 1 + p_{t+1}^s/d_{t+1} \right) \right], \tag{3}$$

where $p_t^s/d_t$ is the price-dividend ratio (the inverse of the dividend yield) and $d_{t+1}/d_t$
is the gross growth rate of dividends. At this point it is convenient to define the
following nonlinear change of variables:

$$z_t^s \equiv M_t \left( \frac{d_{t+1}}{d_t} \right) \left( 1 + p_t^s/d_t \right), \tag{4}$$

where $z_t^s$ represents a composite variable that depends on the stochastic discount
factor, the growth rate of dividends, and the price-dividend ratio.\(^2\) The investor’s
first-order condition (3) becomes

$$p_t^s/d_t = E_t z_{t+1}^s, \tag{5}$$

which shows that the equilibrium price-dividend ratio is simply the investor’s rational
forecast of the composite variable $z_{t+1}^s$.

The gross stock return can now be written as

$$R_{t+1}^s = \frac{d_{t+1} + p_{t+1}^s}{p_t^s} = \left( \frac{1 + p_{t+1}^s/d_{t+1}}{p_t^s/d_t} \right) \frac{d_{t+1}}{d_t} \tag{6}$$

$$= \left( \frac{z_{t+1}^s}{E_t z_{t+1}^s} \right) \frac{1}{M_{t+1}}, \tag{7}$$

\(^1\)See, for example, Rudebusch and Swanson [17] and Lansing [10].

\(^2\)This nonlinear change of variables technique is also employed by Lansing [9], [10] and Lansing
where we have eliminated $p_t^b/d_t$ using the first-order condition (5) and eliminated $p_{t+1}^b/d_{t+1} + 1$ using the definitional relationship (4) evaluated at time $t + 1$.

Starting again from equation (1) and proceeding in a similar fashion, the bond price is determined by the following first-order condition:

$$p_t^b = E_t (1 + \delta p_{t+1}^b) = E_t z_{t+1}^b,$$

where $z_t^b \equiv M_t (1 + \delta p_t^b)$. The gross bond return can be written as

$$R_{t+1}^b = \frac{1 + \delta p_{t+1}^b}{p_t^b} = \frac{z_{t+1}^b}{E_t z_{t+1}^b} \frac{1}{M_{t+1}}.$$  \hfill (9)

Notice that the above expression simplifies to $R_{t+1}^b = R_{t+1}^F = 1/(E_t M_{t+1})$ when $\delta = 0$.

Combining equations (7) and (9) yields the following ratio of the gross stock return to the gross bond return:

$$\frac{R_{t+1}^s}{R_{t+1}^b} = \frac{z_{t+1}^s}{E_t z_{t+1}^b} \frac{E_t z_{t+1}^b}{z_{t+1}^b}.$$  \hfill (10)

Taking logs of both sides of equation (10) yields the following compact expression for the excess stock return, i.e., the realized equity premium:

$$\log (R_{t+1}^s) - \log (R_{t+1}^b) = \log \left( \frac{z_{t+1}^s}{E_t z_{t+1}^b} \frac{E_t z_{t+1}^b}{z_{t+1}^b} \right) \log \left( \frac{z_{t+1}^s}{E_t z_{t+1}^b} \frac{E_t z_{t+1}^b}{z_{t+1}^b} \right),$$

where the second term simplifies to $-\log [M_{t+1}/(E_t M_{t+1})]$ when $\delta = 0$. For example, in the special case of CRRA utility and iid dividend growth, the realized equity premium with $\delta = 0$ is $\log (R_{t+1}^s/R_{t+1}^F) = \varepsilon_{t+1} + (\alpha - 0.5) \sigma_\varepsilon^2$, where $\varepsilon_{t+1}$ is the innovation to consumption/dividend growth and $\sigma_\varepsilon^2$ is the associated variance.\(^3\)

The first term on the right-hand side of eq. (11), $\log (z_{t+1}^s) - \log E_t (z_{t+1}^s)$, is almost equal to the forecast error for $\log (z_{t+1}^s)$. To obtain an exact expression for the forecast error for $\log (z_{t+1}^s)$ we need to interchange the log and expectations operators. To do so we assume that the $z$ terms are lognormally distributed. If a random variable $x_t$ is log-normal, then we have

$$\log (E_t x_{t+1}) = E_t [\log (x_{t+1})] + \frac{1}{2} \text{Var}_t [\log (x_{t+1})].$$  \hfill (12)

Starting from equation (11), we can make the assumption that the composite variables $z_{t+1}^s$ and $z_{t+1}^b$ are both conditionally log-normal. Making use of equation (12) to eliminate $\log (E_t z_{t+1}^s)$ and $\log (E_t z_{t+1}^b)$ yields the following alternative expression for

\(^3\)For the derivation, see Abel [1], p. 353.
the realized excess return:

\[
\log (R_{t+1}^s) - \log (R_{t+1}^b) = \left[\log \left( z_{t+1}^s \right) - E_t \log \left( z_{t+1}^s \right) \right] - \left[ E_t \log \left( z_{t+1}^b \right) \right] \\
- \frac{1}{2} \text{Var}_t \left[ \log \left( z_{t+1}^s \right) \right] + \frac{1}{2} \text{Var}_t \left[ \log \left( z_{t+1}^b \right) \right] 
\]

(13)

where \( z_{t+1}^b = M_{t+1} \) when \( \delta = 0 \).

The next step is to take expectations conditional on \( t \) in eq. (13). There results

\[
E_t[\log (R_{t+1}^s) - \log (R_{t+1}^b)] = \frac{1}{2} \left\{ -\text{Var}_t \left[ \log \left( z_{t+1}^s \right) \right] + \text{Var}_t \left[ \log \left( z_{t+1}^b \right) \right] \right\} 
\]

(14)

Here the left-hand side gives the forecastable component of the log excess return one period ahead, and the right-hand side shows that this depends on the conditional variances of \( z_{t+1}^s \) and \( z_{t+1}^b \). Finally, take the unconditional variance in eq. (14):

\[
\text{Var}_t \left[ E_t[\log (R_{t+1}^s) - \log (R_{t+1}^b)] \right] = \frac{1}{2} \text{Var}_t \left[ -\text{Var}_t \left[ \log \left( z_{t+1}^s \right) \right] + \text{Var}_t \left[ \log \left( z_{t+1}^b \right) \right] \right]. 
\]

(15)

The left-hand side gives a measure of the predictable variation in excess returns. It shows that if the conditional variances of \( \log z^s \) and \( \log z^b \) are constant (although not necessarily equal to zero), then excess returns on stock are unpredictable.

### 3 An Example

That time-varying volatilities imply forecastable excess returns is intuitive: it makes sense that when asset prices are unusually volatile investors expect that prices will continue to be volatile in the future. In this case investors, being risk averse, will trade assets so as to generate an unusually high risk premium. In periods of calm, in contrast, risk premia will be near zero. The point that autocorrelated volatilities are likely to imply forecastable excess returns when investors are risk averse has been stated by several authors (for example, Attanasio [2], Ludvigson [16]). Our analysis strengthens the result as usually presented: if agents are strictly risk averse, time-varying volatilities are both necessary and sufficient for return forecastability, at least in a large class of models. This result applies only when investors are strictly risk averse, and therefore is consistent with the well-known fact that if investors are risk neutral excess returns are unforecastable whether or not volatilities are autocorrelated, at least under standard conditions (Samuelson [18]).

Before engaging in general discussion of the result just reported we describe an example of a fully specified model—presented elsewhere—in which agents are strictly risk averse, but conditional variances of shocks are not time-varying. In that model we can compute an explicit expression for future excess returns. As required by the result of this paper, in that setting excess returns do not have a forecastable component.
In (Lansing and LeRoy [11]) the representative agent has strictly positive and constant relative risk aversion. As in the analysis of Lucas [15], the dividend on stock equals the representative agent’s consumption, so that asset prices are such as to induce the representative agent to consume his endowment. The log dividend growth rate—equivalently, the log consumption growth rate—is generated as a first-order autoregression, with autocorrelation parameter \( \rho \). In the special case \( \rho = 0 \) log dividends follow a random walk, but we are interested in the equilibrium for general \( \rho \), so that log dividend growth contains a forecastable component.

In this model the error in the expression for the log dividend growth rate is assumed to have a constant variance, so the model just described fits the environment required for our result for any value of \( \rho \). We derive an expression for the log excess return on equity and verify that this variable is unforecastable. Incidentally, it is worth noting that in the model under discussion the return on equity, as distinguished from the excess return on equity, has a forecastable component in the case \( \rho \neq 0 \). This is so because \( \rho \neq 0 \) implies that the risk-free return is autocorrelated, and this autocorrelation induces a nonzero serial dependence in equity returns.

4 Interpretation: Rational Expectations

It is worth noting the role of the assumption of rational expectations—that the subjective probability distributions of the agents being modeled coincide with the relevant objective distributions, so that agents can be represented as fully understanding the environment generating the equilibrium—in the derivation presented above. Rational expectations, of course, is much weaker than complete information, however defined: agents optimize subject to probability distributions conditional on the variables that they can observe, and there is no presumption that the variables they can observe are informative about those that they cannot. Under rational expectations the analyst is free to specify which variables are and are not observable at any date by the agents being modeled.

On the simplest interpretation of the model here the derivation of the link between forecastability of excess returns and time-varying volatilities involves only the representative agent’s subjective distributions. Thus on this interpretation the question of how subjective probability distributions are related to the associated objective distributions does not come up. Accordingly, the variance of the forecastable component of excess returns is appropriately interpreted as the representative agent’s subjective variance.

Interpreting that variance as an objective variance, as in any empirical implementation, requires going to a broader setting: unless one is willing to introduce an explicit distinction between objective and subjective distributions and provide a rederivation of the results in that setting, one is led to assume that the two coincide. On this interpretation we include rational expectations in the definition of market efficiency.
5 Conclusion

The results presented above take the topic of financial market efficiency a long way from what is found in the early literature. Our result is that if one assumes that agents have rational expectations and are strictly risk averse, it follows that associating market efficiency with unforecastable excess returns implies that an efficient market is one in which conditional volatilities are constant over time. This is an unexpected outcome: it is not easy to see any connection between the intuitive idea of market efficiency as presented in the early literature—however that idea is made precise—and the assumption of constant volatilities. Perhaps the conclusion is that we need to give further thought to why exactly we are interested in the question of whether future excess returns have forecastable components.

References


