Excess Volatility

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It has been known for many years that stock prices frequently undergo big changes that do not coincide with commensurate changes in prospective corporate earnings or dividends or in variables that can readily be connected to discount factors, such as interest rates (see Cutler, Poterba and Summers [2], for example). The best-known episode occurred on October 19, 1987, when stock prices dropped around the world—22 per cent in the United States—in the complete absence of news about fundamentals. Such events appear to conflict with finance theory: if stock prices equal the present value of future expected dividend streams, changes in prices should be attributable to news about dividends or discount factors. However, it is difficult to draw reliable conclusions about price volatility from individual episodes, if only because there is no obvious way to evaluate statistical significance.

The question is whether stock price volatility systematically exceeds that justified by fundamentals, not whether it appears to do so in individual episodes. That question was first addressed by Shiller [20] and LeRoy and Porter [15]. These papers, written independently and approximately contemporaneously, derived the existence of bounds on the volatility of prices and returns that are implied by the present-value relation. The found excess volatility. However, the papers made this point using different analytical methods. Shiller observed that if stock prices equal the expectation of summed discounted dividends, then stock price volatility should be bounded above by the volatility of what he called ex-post rational stock prices, defined as actual summed discounted dividends. Ex-post rational prices p_t^*, he pointed out, obey the relation

$$p_t^* = \beta(p_{t+1}^* + d_{t+1}), \tag{1}$$

where β is a discount factor (assumed constant). From (1), a time series for p_t^* can be constructed by a backward recursion, at least given an initial condition. Shiller constructed graphs of p_t^* and p_t and argued from visual inspection of these graphs that the former was much smoother than the latter, proving that volatility is excessive. Since Shiller did not specify the model assumed to generate the data, he had no way to evaluate statistical significance.

LeRoy-Porter, in contrast, adopted a model-based analytical procedure. They specified that dividends, stock prices and any auxiliary variables that serve as predictors for future dividends are generated by a linear vector autoregression (to use a term that was not yet then in vogue). They proved that a certain function of the variance of stock price and the variance of stock payoffs can be derived from the parameters describing the bivariate autoregression for dividends and prices. The reason both price volatility and dividend volatility enter the expression is that if the auxiliary variables are accurate predictors of future dividend innovations, then price volatility will be high but payoff volatility will be low. The opposite will be the case if the auxiliary variables do not give accurate predictions of future dividends.

Using this result, LeRoy-Porter constructed a joint test of price volatility and payoff volatility from a bivariate model for dividends and prices. It was unnecessary to estimate the forecasting power of the auxiliary variables, or even to specify them. LeRoy-Porter conducted this test and reported a confidence interval based on the asymptotic distribution of the coefficients of the bivariate process for dividends and prices. They found excess volatility, but it appeared to be of borderline significance. See LeRoy [13] for a fuller, but still brief, summary of the variance-bounds tests in the context of the efficient capital markets literature.

Both Shiller's and LeRoy-Porter's procedures had econometric problems (for a survey of these problems, see Gilles and LeRoy [5]). These problems were serious enough to invalidate the results, in the opinion of some. Early discussion focused on Shiller's paper. Kleidon [7], [8] and Flavin [4] pointed out that while the present-value model implied that the unconditional variance of p_t^* exceeded that of p_t, one would expect the variance of p_t^* conditional on its neighbors p_{t+j}^* to be lower than the corresponding conditional variance of p_t. This is so because p_t^* is much more highly autocorrelated than p_t, as is evident from the absence of an error term in (1). In visually evaluating the volatility of p_t^* and p_t it is not easy to distinguish unconditional from conditional volatility. This is a major drawback of Shiller's procedure. Kleidon computed simulations of p_t^* and p_t in which the present-value model was true by construction, and argued that they looked much the same as the actual data.

LeRoy-Porter's procedure had the drawback that the assumed linear process for dividends and prices implies that these are stationary in levels. That being so, some sort of trend correction to remove the upward trend in both variables must be imposed, and LeRoy-Porter did so. Trend-correction algorithms can easily distort the time-series properties of variables, and that may have happened in this case. It is possible that LeRoy-Porter's finding that excess price and return volatility is only marginally significant statistically reflects difficulties with the trend correction.

Analysts questioned the interpretation of the variance-bounds tests along other lines. They pointed out that the volatility implications of the present-value relation were just repackaged versions of the fundamental implication of the present-value relation (plus the assumption of rational expectations) that stock returns should be serially uncorrelated. If so, why do direct tests of the return orthogonality implication of the present-value relation tend to accept the null, whereas the variance bounds tests appear to reject it? Part of the resolution of this apparent contradiction is that the evidence on return uncorrelatedness began to look less favorable to the null hypothesis in the late 1980s (see Campbell and Shiller [1], for example). Another possibility, discussed by LeRoy and Steigerwald [16], is that the volatility tests are more powerful than the return nonautocorrelatedness tests under whatever alternative hypothesis generated the data.

Discussion of these issues ended fairly suddenly in the mid-1990s. This happened mostly because of a growing realization that the hypothesis being tested required an assumption of risk neutrality: in general, stock prices equal the discounted value of expected dividends only if agents are risk neutral (when the expectation is taken under the natural probabilities). If agents are risk averse there is no reason to presume that either the orthogonality implications or the volatility implications of the present-value relation will be satisfied. This dependence on risk-neutrality had not been brought out clearly in the major papers developing the orthogonality implications of market efficiency. For example, Samuelson's otherwise superb paper [19] developing the connection between martingale models and the present-value relation glossed over this point. Similarly, Fama's classic 1970 survey of the efficient capital markets literature [3] observed that "market efficiency" (meaning, presumably, rational expectations) could only be tested jointly with a model specifying how returns are generated. Despite this, Fama did not go on to observe that the returns model that underlies conventional market efficiency tests took no account of risk aversion. Shortly it became clear that the nonautocorrelatedness of returns would not occur in general if agents are risk averse (LeRoy [11], [12], Lucas [17]).

Initially the dependence of the variance-bounds tests on risk neutrality appeared to be a somewhat abstract point. However, arguments were shortly presented that this might not be so. LaCivita and LeRoy [10], using a two-state version of Lucas's [17] tree model, showed that allowing for risk aversion could be expected to increase the predicted volatility of stock prices. Risk averse agents will try to smooth consumption across time by transferring consumption from low-marginal-utility states to high-marginal-utility states. However, in an exchange economy they cannot do so in the aggregate. The representative agent must consume the aggregate endowment in equilibrium, so prices must counteract preferences. If stock prices are very high when the marginal utility of consumption is low, and vice-versa, agents must buy financial assets when they are expensive and sell them when they are cheap if they are to transfer claims on consumption as desired. This price pattern decreases their desire to do so. If price volatility is high enough, this effect will induce agents to consume the endowment. A related argument was presented by Grossman and Shiller [6].

The excess volatility debate shifted with the arrival of Mehra and Prescott's well-known paper [18] on the equity premium puzzle. This paper was relevant to the excess volatility question for several reasons. The Mehra-Prescott paper followed

LaCivita-LeRoy in specializing Lucas's tree model to two states, but modified the states so that they described the growth rate of the aggregate endowment rather than its level, as in Lucas and LaCivita-LeRoy. This leads to a tractable model when the representative agent has homothetic utility, as with power utility. A major advantage of the Mehra-Prescott specification is that when consumption growth rates rather than levels are stationary there is no need for trend correction.

Mehra-Prescott imposed drastic simplifications so as to obtain a tractable model. For example, their model did not distinguish corporate earnings, dividends and aggregate consumption, despite the fact that these variables behave differently. Some analysts expected that Mehra-Prescott's finding that the equity premium is excessive would be reversed when these simplifications were reversed, but that has turned out not to be the case (see, for example, Kocherlakota [9]).

LeRoy and Parke [14] observed that the Mehra-Prescott framework can be adapted to the investigation of volatility by imposing the assumption that consumption growth rates are independently distributed. This is a special case of the Markov distribution that Mehra-Prescott specified. In that case the ratio of equity value to consumption follows a stationary process. The volatility of that variable depends on how much information agents have about future consumption beyond that contained in present consumption. In the simplest case, when agents have no such information, price is a constant markup of consumption, implying that stock returns have the same volatility as the consumption growth rate. This is true regardless of the degree of risk aversion, implying that the implications of risk aversion for volatility are very different in stationary-growth-rate models than in the stationary-levels models discussed above. This prediction is rejected by the US data: the standard deviation of annual consumption growth is about 2%, whereas that of annual stock returns is on the order of 20%. In contrast to the equity premium puzzle, which can in principle be resolved with sufficiently high risk aversion, the prediction that equity returns should have standard deviations around 2% holds for any level of risk aversion.

Even if one accepts that consumption is a geometric random walk, assuming that agents have no information variables for future consumption other than current consumption is unacceptable. If agents do have such information, equity prices will not be a constant markup of consumption. However, LeRoy-Parke showed that in that case the variances of the price-consumption ratio and the return on stock obey a relation similar to that obtained by LeRoy-Porter. They found that the resulting joint test on the volatility of the price consumption ratio and the volatility of stock returns results in excess volatility for either variable or both.

Most analysts believe that no single convincing explanation has been provided for the volatility of equity prices. The conclusion that appears to follow from the equity premium and price volatility puzzles is that, for whatever reason, prices of financial assets do not behave as the theory of consumption-based asset pricing predicts.

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