

## Question 1

- a) The two rates are not directly comparable, because they are both Stated Annual Rates and have different compounding periods. In order to compare the loans, we need to calculate the Effective Annual Rate for the two Loans.

We will choose the lowest of the two rates. So, for the first loan we have that

$$(1 + EAR_1) = \left(1 + \frac{0.06}{2}\right)^2 \Leftrightarrow EAR_1 = 0.0609 = 6.09\%$$

For the second loan we have that

$$(1 + EAR_2) = \left(1 + \frac{0.05}{12}\right)^{12} \Leftrightarrow EAR_2 = 0.0512 = 5.12\%$$

The Loan with the lowest rate is the second loan.

The Loan will have 240 payments and a monthly interest rate of  $5\%/12=0.416\%$ . To calculate the monthly payments we need to solve the following expression for C

$$\$5,000,000 = CA_{0.416\%}^{240} \Leftrightarrow C = \$32,997.79$$

- b) The amount of money you owe on your Loan is the Present Value of the payments still to be made. Ten years from now, there will be still 120 monthly payments of \$32,997.79 to be made. So, to liquidate the loan, you need to make a payment of

$$P = \$32,997.79A_{0.416\%}^{120} = \$3,111,076$$

## Question 2

- a) The first dividend that is relevant is the dividend coming next year. Since  $2/3$  of the Earnings are re-invested in the firm and earn a 9% rate of return, the Earnings are growing at a rate of  $9\%*2/3=6\%$ .

So, the first relevant Earnings is  $\$3.00*1.06=\$3.18$ . Since only one third is paid as dividends, the next dividend to be paid is \$1.06. The Earnings and, therefore, the dividends will grow at a rate of 6%.

So, the Price of a share of stock is

$$P = \frac{\$1.06}{.16 - .06} = \$10.6$$

- b) The Present Value of the Growth Opportunities is the Present Value of the investments made by the firm in re-investing the Earnings. The easiest way to calculate the value is to compare the current price of the stock with the price it would have if it paid as dividends all the Earnings

In that case, the Earnings, and Dividends, will be constant and equal to \$3.18. It is not \$3.00 (the previous Earnings) because the firm ALREADY retained 2/3 of them by the time of this analysis. So, compared to the previous Earnings, they will also grow, for this year only, 6%. Therefore, the price per share would be

$$P = \frac{\$3.18}{0.16} = \$19.875$$

The Present Value of Growth Opportunities is  $\$10.6 - \$19.875 = -\$9.275$ .

- c) If they reduce the percentage of Earnings re-invested, two things will happen. The Dividends will be bigger, but will grow at a lower rate. The new rate will be  $9\% * 1/3 = 3\%$  and the new dividend will be  $\$3.00 * 1.03 * (2/3) = \$2.06$ . The new price per share will be

$$P = \frac{\$2.06}{0.16 - 0.03} = \$15.85$$

The reason for the price increase is that the firm loses money when it re-invest the earnings, because the internal rate of return (9%) is lower than the rate of return the investors require (16%). Therefore, a reduction of the amount re-invested will increase the value of the firm.

The advice you should give is to stop re-investing the earnings in the firm, and pay all the Earnings as dividends.

### Question 3

- a) The prices of the three bonds are respectively

$$P1 = \frac{\$1,000}{(1.09)^{10}} = \$422.41$$

$$P2 = \$100A_{9\%}^{10} + \frac{\$1,000}{(1.09)^{10}} = \$1,064.18$$

$$P3 = \$80A_{9\%}^{15} + \frac{\$1,000}{(1.09)^{15}} = \$919.39$$

- b) First, you need to calculate the prices of the Bonds one year from now.

$$P1 = \frac{\$1,000}{(1.09)^9} = \$460.43$$

$$P2 = \$100A_{9\%}^9 + \frac{\$1,000}{(1.09)^9} = \$1,059.95$$

$$P3 = \$80A_{9\%}^{14} + \frac{\$1,000}{(1.09)^{14}} = \$922.14$$

Therefore, the rates of return are:

For Bond1

$$\text{Current Yield} = 0\%,$$

$$\text{Capital Gains Yield} = \frac{\$460.43 - \$422.41}{\$422.41} = 9\%,$$

$$\text{Total Return} = 9\%$$

For Bond2

$$\text{Current Yield} = \frac{\$100}{\$1,064.18} = 9.4\%,$$

$$\text{Capital Gains Yield} = \frac{\$1,059.95 - \$1,064.18}{\$1,064.18} = -0.4\%,$$

$$\text{Total Return} = 9\%$$

For Bond3

$$\text{Current Yield} = \frac{\$80}{\$919.39} = 8.7\%,$$

$$\text{Capital Gains Yield} = \frac{\$922.14 - \$919.39}{\$919.39} = 0.3\%,$$

$$\text{Total Return} = 9\%$$

- c) If the yield to maturity of the Bonds drops to 7%, the prices of the Bonds one year from now will change.

$$P1 = \frac{\$1,000}{(1.07)^9} = \$543.93$$

$$P2 = \$100A_{7\%}^9 + \frac{\$1,000}{(1.07)^9} = \$1,195.46$$

$$P3 = \$80A_{7\%}^{14} + \frac{\$1,000}{(1.07)^{14}} = \$1,087.46$$

Therefore, the rates of return are:

For Bond1

$$\text{Current Yield} = 0\%,$$

$$\text{Capital Gains Yield} = \frac{\$543.93 - \$422.41}{\$422.41} = 28.77\%,$$

$$\text{Total Return} = 28.77\%$$

For Bond2

$$\text{Current Yield} = \frac{\$100}{\$1,064.18} = 9.4\%,$$

$$\text{Capital Gains Yield} = \frac{\$1,195.46 - \$1,064.18}{\$1,064.18} = 12.34\%,$$

$$\text{Total Return} = 21.74\%$$

For Bond3

$$\text{Current Yield} = \frac{\$80}{\$1,087.46} = 7.4\%,$$

$$\text{Capital Gains Yield} = \frac{\$1,087.46 - \$919.39}{\$919.39} = 18.28\%,$$

$$\text{Total Return} = 25.68\%$$

- d) What matters to you is to pay the least taxes while at the same time obtain \$2million to pay the loan you took.

The first thing you need to consider is that you would pay the Current Income tax, even if you wouldn't sell the Coupon bearing Bonds. This is because you would always receive the Coupons and therefore, pay the tax.

So, the only tax that is incremental for this analysis is the Capital Gains tax.

If that is the case, the Bonds you should sell to re-pay the Loan is the 10% coupon Bond. Because the price is dropping, you will actually get a tax break if you sell them. The one you want the least to sell is the zero coupon Bond. In this Bond, the variation is the largest, which means you will pay more taxes and therefore need to sell more Bonds to get the \$2million you need.

In the zero coupon Bonds you are making a gain of \$38.02 per bond. This means you are paying in taxes \$11.406 for every bond you sell. Therefore, you are only able to use \$449.024 per bond to re-pay the loan. This means, you need to sell 4,454 Bonds and pay \$50,803.52 in taxes.

For the other Bond, you make a gain of \$2.75 per Bond. So, you pay \$0.825 in taxes per Bond sold. So, the amount you use per Bond is \$921.315. This means selling 2,171 Bonds and pay \$1,790.918 in taxes.

#### Question 4

This is the table with the Cash Flows

Description	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
Investment	<b>-300,000</b>					<b>100,000</b>
(-) Tax on Gain						<b>35,000</b>
(-) Revenues		400,000	500,000	500,000	400,000	300,000
(-) Variable Costs		240,000	300,000	300,000	240,000	180,000
(-) Fixed Costs		36,000	36,000	36,000	36,000	36,000
(-) Depreciation		99,000	132,000	45,000	24,000	0
(=) EBIT		25,000	32,000	119,000	100,000	84,000
(-) Taxes		8,750	11,200	41,650	35,000	29,400
(=) Net Income		<b>16,250</b>	<b>20,800</b>	<b>77,350</b>	<b>65,000</b>	<b>54,600</b>
Add Depreciation		<b>99,000</b>	<b>132,000</b>	<b>45,000</b>	<b>24,000</b>	0
-Var. NWC	<b>-9,000</b>	0	0	0	0	<b>9,000</b>
(=) Total Cash Flow	<b>-309,000</b>	<b>115,250</b>	<b>152,800</b>	<b>122,350</b>	<b>89,000</b>	<b>128,600</b>
PV Cash Flow	<b>-309,000</b>	<b>96,042</b>	<b>106,111</b>	<b>70,804</b>	<b>42,921</b>	<b>51,681</b>
NPV	<b>58,559</b>					

You will make a Bid of \$58,559 to operate the concession stand

The Profitability Index will be  $\frac{367,559}{309,000} = 1.19$

The Payback Period is 2.33 years

The Discounted Payback is 3.84 years