Price and Shortage as Signals of Quality

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Abstract

This paper analyzes the role of seller-induced shortage as a signal of quality for a monopoly firm. Unlike dissipative advertising, the cost of inducing shortage is different for different quality types. It is shown that under certain conditions, a high-quality monopoly firm (of a new indivisible product) that signals quality by properly setting price and inducing shortage makes more profit than using price and dissipative advertising. The result provides a rationale for why high quality product monopolies may prefer to initially limit supply with or without lowering the price.

Keywords: Shortage; Quality signaling; Pooling equilibrium; Separating equilibrium (JEL C72, L15.)

1 Introduction

In some industries limiting supply is a commonly used business strategy. For instance, the high quality handbag producer Louis Vuitton does not advertise against a cheap handbag imitation producer, but frequently produces limited edition bags. As a recent example, the automaker Jaguar put only 50 copies of a special Neiman Marcus edition of the redesigned 2010 Jaguar 5.0-liter V8, making 470 hp XJ at a price of $105,000

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for order in October 2009. But, it only took four hours and four minutes to finish all bookings (*Autoweek*, October 20, 2009).

There have been several papers on the strategic uses of seller-induced shortage. In DeGraba (1995), shortage is purposely induced by a monopoly firm to promote *buying frenzy*. In his model, consumers learn their types (their values of the product) over time. By selling fewer units, uncertainties can be created for those consumers who choose to wait until they have learned their types, since there may not be enough supply. DeGraba shows that an equilibrium exists in which all consumers purchase the product early on while uninformed, although they prefer to purchase after becoming informed provided the product is available. This induced buying frenzy allows the monopolist to price higher and earn more profit. Bose (1996) considers rationing problems in restaurants. He shows that restaurants can use capacities to screen less profitable customers, because serious customers who will spend more care less about waiting time.

The signaling role for seller-induced shortage is not analyzed in the aforementioned papers. In this paper, we analyze this signaling role. The basic idea is that consumers can observe the price and shortages using proxies such as queuing or the time required to order in advance. They can then update their beliefs about the quality type. It is possible to have a separating equilibrium in which consumers rationally expect the seller to choose different price-shortage combinations for different quality types.

Beginning with Nelson (1970, 1974), the signaling role of advertising has received considerable attention in the literature. A basic idea is that advertising may be dissipative, in the sense that it is only a signal that the firm is able to spend a lot of money. But consumers can observe the total amount of money (or a proxy of it) that the firm is spending on advertising. It is possible to have an equilibrium in which consumers rationally expect the firm to spend different amounts on advertising for different product quality types.

To contrast with results using a price-advertising combination as the signal of quality, we carry out our analysis for a familiar two-period model of a monopoly firm pr-

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1 Becker (1991) recognizes that the consumption of a product has some dimension of social activity and postulates that demand by a typical consumer is positively related to aggregate demand. Depending on the strength of the market demand’s responsiveness to itself, Becker demonstrates that the “self dependence” of market demand may result in discontinuity. This discontinuity leads to the existence of shortage while the monopoly is maximizing its profit. Karni and Levin (1994) develop an analytical framework by which the aforementioned dependence is derived endogenously from primitive data such as utility functions and the distribution of tastes. The signaling role of shortage is not analyzed in these papers either.
viding a new indivisible experience good of either high or low quality. The firm knows the quality; but the consumers do not have this knowledge at the time the product is introduced. The firm cannot vary the quality across the two periods. In addition, there is no word-of-mouth learning or independent sources of quality revelation such as consumer reports. Signaling occurs during the first period. So, the consumers can learn the quality of the product either in the first period by observing a signal (if any) that separates the high-quality product from the low-quality one, or in the beginning of the second period by consuming the good in the first period.

As already shown in Tirole (1988, p. 120), price and dissipative advertising are perfect substitutes for their roles as signals of quality for the two-period model we consider. Furthermore, for the case with homogeneous consumers, using price or price-advertising combination to signal quality generates a paradoxical result. Namely, the separating equilibrium is less profitable than pooling equilibrium. In comparison, we show in this paper that the high-quality monopoly firm can signal its quality by properly setting the price and inducing shortage. Furthermore, signaling quality via price-shortage combination resolves the aforementioned signaling paradox; it results in a separating equilibrium which is more profitable than pooling equilibrium. For the case with heterogeneous consumers, we show that when it is profitable for the high-quality firm to target high-taste consumers with complete information, it is more profitable to signal using price-shortage combination than signaling by price-advertising combination or pooling. These results provide a rationale for why high quality product monopoly firms may prefer to initially limit supply with or without lowering price.

Stock and Balachander (2005) were the first to address the signaling role of seller-induced scarcity. Their approach differs from ours in several ways. First, they require that a non-zero fraction of consumers be informed of the quality of the product before purchasing. Second, informed and uninformed consumers make purchasing decisions sequentially in their model. Moreover, the informed consumers are allowed to book the product in advance (without scarcity constraint) and the followers (uninformed consumers) are then allowed to enter the market only at a later stage (with possible scarcity). Third, they do not consider repeated purchase. Their setting implies that a separating equilibrium using price-advertising combination as the signal of quality never exists if quality provision is costly. In reality, a consumer’s knowledge about the quality

\[^2\] As explained in Milgrom and Roberts (1986), this may correspond to the situation where the firm’s R&D effort has generated the product of a given quality that the firm must then decide how to introduce. As usual, we assume that the product of high-quality is more costly to produce.

\[^3\] The intuition is straightforward: whatever the amount of advertising the high-quality type firm
quality of a new product does not necessarily grant her the priority to buy the product first when the product is introduced.

The rest of the paper is organized as follows. Section 2 introduces the model and results for the simple homogeneous consumer case. Section 3 presents results for the heterogeneous consumer case. Section 4 concludes the paper. Proofs of results are organized in the Appendix.

2 Signaling Quality with Homogeneous Consumers

Consider a two-period market for a new indivisible experience product supplied by a monopoly firm. The product can be of either high (type $s_1$) or low quality level (type $s_0$) with $s_0 < s_1$. The quality is not observable to the consumers nor is it adjustable by the firm across time. The marginal cost of the firm is constant for either quality type and is denoted by $c_t$ for type $s_t$. As usual, assume $c_0 < c_1$.

There is a continuum of consumers with each demanding at most one unit of the product in each period. Let $\theta$ denote the common value of quality for a consumer. If he consumes the product of quality $s_k$ at price $p$, he obtains net utility

$$\theta s_k - p.$$  \hspace{1cm} (1)

Let $\delta \in (0, 1]$ be the common discount factor and let $\mu$ be the probability with which each consumer believes that the monopolist’s product has high quality prior to observing any signal or consuming the product. Since $s_1 > s_0$ and $c_0 < c_1$, the low-quality firm has incentives to mislead consumers into believing that it is of the high-quality type, provided that it is not too costly to do so.\footnote{Since technology is not adjustable, we can identify the type of the firm with the quality type of its product.}

There is no communication between the consumers. Those who purchase the product in the first period obtain information about the quality and can make their second period purchase decisions conditional on that information. However, the consumers who do not purchase the product in period 1 will base their period 2 decisions upon beliefs updated by signals they observe.
2.1 Signal Quality by Price and Advertising

Suppose that the high-quality firm uses price \( p_1 \) and dissipative advertising with expenditure \( A \), i.e., a price-advertising combination \((p_1, A)\), to signal its quality in period 1. As usual, in any separating equilibrium, the low-quality firm chooses its complete information monopoly price \( \theta_s_0 \) and does not advertise because it cannot mislead consumers unless it mimics the high-quality firm’s choice. The following incentive-compatibility constraints characterize all separating equilibria:

\[
p_1 - A - c_1 \geq \theta_s_0 - c_1 \tag{2}
\]

and

\[
p_1 - A - c_0 \leq \theta_s_0 - c_0. \tag{3}
\]

Note that (2) and (3) imply \( p_1 - A = \theta_s_0 \), which in turn implies that the first period price and dissipative advertising are perfect substitutes. Thus, the net price, \( p_1 - A \), of the high-quality firm is the same as the low-quality firm’s price. In contrast, the common period 1 price for both types of the firm in the pooling equilibrium is given by:

\[
p_1^\mu = \mu \theta s_1 + (1 - \mu) \theta s_0 > \theta s_0.
\]

Thus, the high-quality firm’s total profit across the two periods in the pooling equilibrium is equal to \(^5\)

\[
p_1^\mu - c_1 + \delta (\theta s_1 - c_1). \tag{4}
\]

Since \( s_0 < s_1 \), it follows that the pooling equilibrium is more profitable than the separating equilibrium with price and dissipative advertising as the signal of high quality (see also Tirole, 1988, p.120).

2.2 Signaling Quality by Inducing Shortage

Suppose that the high-quality firm limits its supply so that only a fraction \( \alpha \in (0, 1) \) of consumers can get the product during the first period. With the limited supply, the high-quality firm may reveal its type to the consumers by properly setting the price, so as to leave it undesirable for the low-quality firm to mimic. In the separating equilibrium, the low-quality firm does not induce any shortage. Consequently, all

\(^5\)Notice that in the pooling equilibrium, consumers learn the quality in the second period by purchasing the product in the first period. Thus both types can charge their complete-information monopoly prices.
separating equilibria are equally profitable for the low-quality firm, which is not true for the high-quality firm. We are interested in the most profitable separating equilibrium for the high-quality firm, for the reason that it satisfies equilibrium refinements, such as the Cho-Kreps intuitive criterion. Such equilibria are known as “least-cost” separating equilibria (e.g., Bagwell, 2007, p. 1801).

A price-shortage combination \((p^*_1, \alpha^*)\), with the high-quality firm’s period 1 price \(p^*_1\) and shortage \((1 - \alpha^*)\), is a least-cost separating equilibrium if and only if it solves

\[
\max_{0 \leq \alpha \leq 1, \ p_1 \leq \theta s_1} \alpha (p_1 - c_1) + \delta (\theta s_1 - c_1)
\]

subject to

\[
\alpha (p_1 - c_1) \geq \theta s_0 - c_1,
\]

and

\[
\alpha(p_1 - c_0) + \delta \max\{\theta s_0 - c_0, (1 - \alpha)(\theta s_1 - c_0)\} \leq (1 + \delta)(\theta s_0 - c_0).
\]

Condition (6) is equivalent to the incentive constraint

\[
\alpha(p_1 - c_1) + \delta(\theta s_1 - c_1) \geq \theta s_0 - c_1 + \delta(\theta s_1 - c_1).
\]

This constraint guarantees that the high-quality firm does not have any incentive to mimic the low-quality firm’s period 1 choice \((\theta s_0, 0)\). Condition (7) is the incentive constraint that makes it undesirable for the low-quality firm to mimic the high-quality firm’s period 1 choice \((p_1, \alpha)\) and then in period 2, either to supply the entire population of consumers at price \(\theta s_0\), or to supply only \((1 - \alpha)\) fraction who did not consume the product in period 1 by continuing to mimic the high-quality firm’s period 2 choice of charging price \(\theta s_1\).

**Lemma 1** Let \((p_1, \alpha)\) be the period 1 choice of the high-quality firm in a least-cost separating equilibrium. Then, constraint (7) is binding at \((p_1, \alpha)\).

Intuitively, when (7) is non-binding, the high-quality firm can slightly reduce the shortage without changing its price or violating (6). Keeping price constant, the high-quality firm’s profit increases as shortage reduces. It follows that the separating equilibrium cannot be the least-cost unless (7) is binding. We now apply Lemma 1 to

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6When mimicking in period 2, the low-quality firm can only mislead consumers who did not buy in period 1.
show that there is a unique solution for problem (5) with positive shortage under the following conditions.

\[ A1 : \quad \delta(\theta s_1 - c_0) > c_1 - c_0, \quad \theta(s_1 - s_0) > \theta s_0 - c_0, \quad \theta s_k > c_k, \quad k = 0, 1. \]

The first inequality requires that the discounting factor be large enough relative to the ratio of the cost differential \(c_1 - c_0\) to the profit the low-quality firm gets when it is perceived as the high-quality firm. With the second inequality, the consumers’ value differential due to quality difference exceeds the per-unit profit the low-quality firm gets under complete information. The third inequality is self-explanatory.

**Proposition 1** Assume \(A1\). Then, when the high-quality firm signals its quality type by price-shortage combination, there is a unique least-cost separating equilibrium in which

\[ \alpha^* = \frac{\theta(s_1 - s_0)}{\theta s_1 - c_0} \]

and

\[ p_1^* = c_0 + \frac{(\theta s_1 - c_0)(\theta s_0 - c_0)}{\theta(s_1 - s_0)}. \]

Note that \(p_1^* < \theta s_1\) if and only if \((\theta s_0 - c_0)/\theta(s_1 - s_0) < 1\), which is guaranteed by the second inequality in \(A1\). It follows that in the least-cost separating equilibrium, the high-quality firm charges a period 1 price which is below its complete information monopoly price. Note also that the high-quality firm’s total profit across the two periods in the least-cost separating equilibrium is:

\[ \alpha^*(p_1^* - c_1) + \delta(\theta s_1 - c_1) = \theta s_0 - c_0 + \delta(\theta s_1 - c_1) - \frac{\theta(s_1 - s_0)}{\theta s_1 - c_0}(c_1 - c_0). \quad (8) \]

By (4) and (8), the high-quality firm is better off signaling its quality type via the price-shortage combination in Proposition 1 than pooling if and only if \(p_1^* - c_1 < \alpha^*(p_1^* - c_1)\). This is equivalent to

\[ \mu < \frac{(\theta s_0 - c_0)(c_1 - c_0)}{\theta(s_1 - s_0)(\theta s_1 - c_0)}. \quad (9) \]

For later reference, we summarize this result in the following proposition whose proof will be omitted.

**Proposition 2** Assume \(A1\). Then, signaling quality by price-shortage combination is more profitable than pooling and hence more profitable than signaling quality by price-advertising combination if \(\mu\) satisfies (9).
Proposition 2 establishes a sharp comparison between the signaling roles of price-advertising and price-shortage combinations. The key reason behind the result is that the cost in foregone profits to the high-quality firm from inducing shortage is always lower than that of the low-quality firm.

3 Signaling with Heterogeneous Consumers

We now consider an extension of the model in Section 2 by allowing for heterogeneous consumers. We follow Wolinsky (1983), Chan and Leland (1982), Cooper and Ross (1984, 1985), Farrell (1980), and Tirole (1988), among others to consider unit-demand differentiated consumers. For simplicity, we assume that there are two types of consumers in terms of their tastes for quality. Specifically, the value of the product with quality level $s_k$ is $\theta_1 s_k$ for type 1 consumers and $\theta_0 s_k$ for type 0 consumers, where $\theta_t$ represents type $t$ consumers’ taste for quality. Assume $\theta_1 > \theta_0$. The proportion of type 1 consumers is denoted by $q_1$.

As with homogeneous consumers only, price and advertising are substitutes for the high-quality firm. But, unlike in the homogeneous case, we show that under certain conditions on the parameters of the model, the high-quality firm is better off signaling its quality type via price than not signaling at all. That is, the quality signaling paradox with homogeneous consumers is eliminated with the presence of heterogeneous consumers.

In the rest of this section, we characterize the least-cost separating equilibria when the high-quality firm signals its quality via price-advertising or price-shortage combination. We compare them in Section 4. First, due to consumer heterogeneity, we need the following assumptions.

\[ A1': \quad \delta(\theta_1 s_1 - c_0) > c_1 - c_0, \quad \theta_k s_k > c_k, \quad k = 0, 1. \]

\[ A2: \quad \frac{\theta_0 s_1 - c_1}{\theta_1 s_1 - c_1} < \frac{\theta_0 s_0 - c_0}{\theta_1 s_1 - c_0} < q_1 < \min \left\{ \frac{\theta_0 s_0 - c_0}{\theta_1 s_1 - c_0}, 1 - \frac{\theta_0 (s_1 - s_0)}{c_1 - c_0} + \frac{\theta_0 s_0 - c_0}{\theta_1 s_1 - c_0} \right\}. \]

The first two inequalities in $A2$ guarantee that when information is complete, the proportion of the high-taste consumers is large enough for the high-quality firm to target high-taste consumers only. Furthermore, if the low-quality firm is perceived by high-taste consumers as the high-quality firm when it charges period 1 price $\theta_1 s_1$, then the low-quality firm is more profitable than it can be with its complete information.
monopoly price $\theta_0s_0$ and targeting all consumers. These conditions make signaling non-trivial. In contrast, the last inequality in A2 with the first component on the right hand side implies that the proportion of the high-taste consumers is small enough, so that it is more profitable for the low-quality firm to target all consumers when information is complete. The first inequality together with $q_1(\theta_1s_1 - c_1) \leq 2(\theta_0s_0 - c_0)$ also implies that the high-quality firm targets high-taste consumers with period 1 price equal to $\theta_1s_1$ in least-cost separating equilibrium. On the other hand, the last inequality with the second component on the right hand together with $q_1(\theta_1s_1 - c_1) > 2(\theta_0s_0 - c_0)$ implies that the high-quality firm targets high-taste consumers with period 1 price strictly below $\theta_1s_1$ in least-cost separating equilibrium.

3.1 Signaling Quality by Price and Advertising

Let $D_h(p_1)$ denote the complete information aggregate demand for the high-quality product in period 1. Then, $D_h(p_1) = 1$ if $q_1 \leq \theta_0s_1$ and $D_h(p_1) = q_1$ if $q_1 > \theta_0s_1$. When the high-quality firm uses price-advertising combination to signal its quality, the incentive compatibility constraints for a separating equilibrium become

$$D_h(p_1)(p_1 - c_1) - A \geq \theta_0s_0 - c_1,$$  \hspace{1cm} (10)

and

$$D_h(p_1)(p_1 - c_0) - A \leq (\theta_0s_0 - c_0).$$  \hspace{1cm} (11)

Conditions (10) and (11) guarantee that neither type has any incentive to mimic the other type’s choice. Note also that these constraints are consistent, in the sense that there are prices and advertising expenses that simultaneously satisfy both of them.\(^7\)

In a least-cost separating equilibrium, the high-quality firm’s price solves

$$\max_{(p_1, A): A \geq 0, p_1 \leq \theta_1s_1} D_h(p_1)(p_1 - c_1) - A + \delta (\theta_1s_1 - c_1)$$

subject to constraints (10) and (11).\(^8\)

The following proposition characterizes the least-cost separating equilibria.

**Proposition 3** Assume A1’ and A2. Then, when the high-quality firm signals its quality type by price-advertising combination, the least-cost separating equilibria are

\(^7\)Let $p_1 > \theta_0s_1$. Condition (10) is equivalent to $p_1 \geq (\theta_0s_0 - c_1 + A)/q_1 + c_1$ and (11) is equivalent to $p_1 \leq (\theta_0s_0 - c_0 + A)/q_1 + c_0$. Thus, (10) and (11) are consistent if and only if $q_1(c_1 - c_0) \leq (c_1 - c_0)$ which is automatic.

\(^8\)
characterized by

\[ q_1 p_1^* - A^* = q_1 c_0 + \theta_0 s_0 - c_0, \quad \theta_0 s_1 < p_1^* \leq \theta_1 s_1, \quad A^* \geq 0. \]

Proposition 3 establishes the existence of a class of least-cost separating equilibria with price-advertising combination as the signal of high quality. The period 1 price of the high-quality firm increases with the advertising cost. Nonetheless, all of the equilibria yield the same profit for the high-quality firm as the one with zero advertising cost. This equivalence is due to the fact that price and advertising are perfect substitutes. The reason that signaling with price-advertising combination becomes more profitable than pooling is because the high-quality firm can charge a price to target high-taste consumers only, which leaves it less profitable for the low-quality firm to mimic.

The following example provides an illustration of Proposition 3. Both A1' and A2 are satisfied in this example.

**Example 1:** Let \( c_1 = .75, c_0 = 0, \theta_1 = 2, \theta_0 = 1, s_1 = 1, s_0 = .5, \delta = 1 \). Then, simple calculation shows that both A1' and A2 are satisfied whenever \( q_1 \in (.25, .5) \). With zero advertising cost, the least-cost separating equilibrium price of the high-quality firm monotonically decreases with respect to the proportion of high-taste consumers. This example is adapted from Exercise 2.7 in Tirole (1988, p. 121), where a separating equilibrium with price as the signal of quality is established, under the requirement that no one can purchase the product in period 2 unless he has purchased in period 1.

### 3.2 Signaling High Quality by Inducing Shortage

We now turn to the least-cost separating equilibrium in which the high-quality firm signals quality by properly setting the price and inducing shortage. Since a consumer cannot learn the quality of the product without consuming it, by mimicking the high-quality firm’s period 1 and period 2 choices, the low-quality firm can mislead those consumers who did not buy the product in period 1. Thus, the incentive compatibility constraints for a separating equilibrium become

\[ \alpha D_h(p_1) (p_1 - c_1) \geq \theta_0 s_0 - c_1 \] (13)
and
\[ \alpha D_h(p_1)(p_1 - c_0) + \delta \max \{\theta_0 s_0 - c_0, (1 - \alpha) q_1 (\theta_1 s_1 - c_0)\} \leq (1 + \delta) (\theta_0 s_0 - c_0). \]  

Constraint (13) implies that the high-quality firm does not have incentive to mimic the low-quality firm’s choice, while (14) ensures that the low-quality firm does not have incentive to mimic the choice of the high-quality firm.\(^8\)

In a least-cost separating equilibrium, the high-quality firm’s period 1 choice \((p_1, \alpha)\) solves
\[
\max_{0 \leq \alpha \leq 1, \ p_1 \leq \theta_1 s_1} \alpha D_h(p_1)(p_1 - c_1) + \delta q_1 (\theta_1 s_1 - c_1) \\
\text{subject to constraints (13) and (14).} 
\]

As with the previous case, the incentive compatibility constraint (14) is binding in any least-cost separating equilibrium. We summarize this result in the following lemma. Its proof is similar to the proof of Lemma 1. For this reason, the proof is omitted.

**Lemma 2** Let \((p_1, \alpha)\) be the high-quality firm’s period 1 choice in a least-cost separating equilibrium. Then, (14) must be binding at \((p_1, \alpha)\).

We now characterize the unique least-cost separating equilibrium separately for two mutually disjoint and jointly exhaustive subranges of \(A_2\).

**Proposition 4** Assume \(A_1'\) and \(A_2\). Then, when the high-quality firm signals its quality type by price-shortage combination, there exists a unique least-cost separating equilibrium with
\[
\tilde{\alpha}^* = 1 - \frac{\theta_0 s_0 - c_0}{q_1 (\theta_1 s_1 - c_0)}, \quad \tilde{p}_1^* = c_0 + \frac{(\theta_1 s_1 - c_0)(\theta_0 s_0 - c_0)}{q_1 (\theta_1 s_1 - c_0) - (\theta_0 s_0 - c_0)} \\
\text{if } q_1(\theta_1 s_1 - c_1) > 2(\theta_0 s_0 - c_0) \text{ and } \\
\hat{\alpha}^* = \frac{\theta_0 s_0 - c_0}{q_1 (\theta_1 s_1 - c_0)}, \quad \hat{p}_1^* = \theta_1 s_1 \\
\text{if } q_1(\theta_1 s_1 - c_1) \leq 2(\theta_0 s_0 - c_0). 
\]

When \(q_1(\theta_1 s_1 - c_1) > 2(\theta_0 s_0 - c_0)\), both the equilibrium price \(\hat{p}_1^*\) and shortage \((1 - \hat{\alpha}^*)\) decrease as the fraction \(q_1\) of the high-taste consumers increases. Simple

\(^8\)Notice that due to \(A_2\), the high-quality firm sets its period 2 price equal to \(\theta_1 s_1\) in separating equilibrium.
calculation shows \( \theta_0 s_1 < \tilde{p}_1^* < \theta_1 s_1 \). This means that the high-quality firm targets the high-stage consumers with a price below its complete-information monopoly price. In the homogeneous case, \( q_1 = 1 \) and \( \theta_1 = \theta_0 = \theta \). Thus, with homogeneity,

\[
\check{\alpha}^* = 1 - \frac{\theta s_0 - c_0}{q (\theta s_1 - c_0)} = \frac{\theta (s_1 - s_0)}{\theta s_1 - c_0} = \alpha^* \text{ and } \tilde{p}_1^* = p_1^*.
\]

When \( q_1(\theta_1 s_1 - c_1) \leq 2(\theta_0 s_0 - c_0) \), the proportion of high-taste consumers is not large enough for the high-quality firm to lower the price from its complete information monopoly price level and induce shortage at the same time. Proposition 4 shows that in that case, the complete information monopoly price together with a certain amount of shortage can signal high quality.

For the same parameter values of \( \theta_1, \theta_0, s_1, s_0, c_1, c_0, \delta \) in Example 1, \( q_1(\theta_1 s_1 - c_0) \leq 2(\theta_0 s_0 - c_0) \) for all \( q_1 \in (.25, .5) \). Under these parameter values, Proposition 4 shows that \((\tilde{p}_1^*, \check{\alpha}^*) \) with \( \tilde{p}_1^* = 2 \) and \( \check{\alpha}^* = q_1/4 \) is the least-cost separating equilibrium for each \( q_1 \in (.25, .5) \).

### 3.3 Signal Comparisons

Our results in the previous section show that under assumptions \( A1' \) and \( A2 \), both price-advertising and price-shortage combinations can be signals of high quality. Thus, as with the homogeneous case, it is natural to ask which signal is more profitable for the high-quality firm. The next proposition provides an answer.

**Proposition 5** Assume \( A1' \) and \( A2 \). Then, signaling quality by price-shortage combination is more profitable than using price-advertising combination.

In terms of the forgone profit, the high-quality firm has a cost advantage to induce shortage due to the cost differential in production. In contrast, dissipative advertising is equally costly for both types. The cost advantage associated with inducing shortage makes it more profitable to signal quality via price-shortage combination than using price-advertising combination, whenever the former is effective. We end this section with an illustration of Proposition 5.

**Example 2:** Consider the same parameter values of \( \theta_1, \theta_0, s_1, s_0, c_1, c_0, \delta \) as in Example 1. As noticed before, \( A1', A2 \), and \( q_1(\theta_1 s_1 - c_0) \leq 2(\theta_0 s_0 - c_0) \) are satisfied for all \( q_1 \in (.25, .5) \). By Proposition 3 and Proposition 4, with \( q_1 = 1/3 \):
• \( \hat{\alpha}^* = \frac{3}{4} \) and \( \hat{p}_1^* = 2 \) when signaling by price-shortage combination, implying that the high-quality firm’s period 1 profit in the least-cost separating equilibrium is

\[
\hat{\alpha}^* q_1 (\hat{p}_1^* - c_1) = \frac{5}{16}.
\]

• \( A^* = 0 \Rightarrow p_1^* = 1.5 \) when signaling by price-advertising combination, implying that the high-quality firm’s period 1 profit in the least-cost separating equilibrium is

\[
q_1 (p_1^* - c_1) = \frac{1}{4}.
\]

From the preceding equilibrium profits it follows that signaling quality by price-shortage combination is more profitable for the high-quality firm than signaling quality by price-advertising combination.

4 Conclusion

In this paper, we have analyzed the possibility for a monopoly firm of a new product to signal quality by properly setting the price and inducing shortage. For a stylized two-period model of a monopoly firm providing a new indivisible product with two possible unknown quality levels to homogeneous consumers, we have shown that for the high-quality firm, it is more profitable to signal quality using price-shortage combination than by pooling. Consequently, signaling via price-shortage combination eliminates the paradoxical result associated with signaling quality by price or price-advertising combination.

We have generalized the results to allow for heterogeneous consumers. Depending on the parameter values, the high-quality firm sets its price in period 1 below or equal to its complete information monopoly price. This enables the high-quality firm to extract as much consumer surplus as possible from high-taste consumers who purchase in period 1. Our results provide a rationale for a limited edition combined with a high or low introductory price for a high-quality new product supplied by a monopoly firm.

References


Appendix

Proof of Lemma 1: Let \((p_1, \alpha)\) be a price-shortage combination such that

\[
\alpha(p_1 - c_0) + \delta \max\{\theta s_0 - c_0, (1 - \alpha)(\theta s_1 - c_0)\} < (1 + \delta)(\theta s_0 - c_0)
\]  

(16)

Suppose first \(\theta s_0 - c_0 \geq (1 - \alpha)(\theta s_1 - c_0)\). In this case, if \(p_1 < \theta s_1\), then, there exists a price \(p'_1 > p_1\) such that \((p'_1, \alpha)\) also satisfies (16). Since \(\alpha(p'_1 - c_0) > \alpha(p_1 - c_0)\), \((p'_1, \alpha)\) satisfies (6) whenever \((p_1, \alpha)\) does. This shows that \((p_1, \alpha)\) cannot solve problem (5). If \(p_1 = \theta s_1\), then (16) reduces to \(\alpha(\theta s_1 - c_0) < \theta s_0 - c_0\). Hence, \(\alpha < 1\) because \(\theta s_1 - c_0 > \theta s_0 - c_0\). It follows that, by slightly increasing \(\alpha\) to \(\alpha' > \alpha\), we can guarantee that \((p_1, \alpha')\) also satisfies (16).\(^9\) Since \(\alpha'(p_1 - c_1) > \alpha(p_1 - c_1)\), \((p_1, \alpha)\) cannot solve problem (5).

Suppose now \(\theta s_0 - c_0 < (1 - \alpha)(\theta s_1 - c_0)\) which implies \(\alpha \neq 1\). Thus, as before, we can increase the maximum value of problem (5) by keeping price \(p_1\) while slightly increasing \(\alpha\) without violating (6) and (7).

Proof of Proposition 1: Let \((p_1, \alpha)\) be a solution for problem (5). We break the rest of the proof into two cases.

Case 1: \(\alpha \geq \theta(s_1 - s_0)/(\theta s_1 - c_0)\).

In this case, we have \((\theta s_0 - c_0) \geq (1 - \alpha)(\theta s_1 - c_0)\).\(^{10}\) Thus, by Lemma 1, \(\alpha(p_1 - c_0) = (\theta s_0 - c_0)\), which implies \(\alpha p_1 = \alpha c_0 + (\theta s_0 - c_0)\). It follows that \(\alpha(p_1 - c_1) = \theta s_0 - c_0 - \alpha(c_1 - c_0)\) is decreasing in \(\alpha\).

Case 2: \(\alpha \leq \theta(s_1 - s_0)/(\theta s_1 - c_0)\).

\(^9\)Note that \(\theta s_0 - c_0 \geq (1 - \alpha)(\theta s_1 - c_0)\) implies \(\theta s_0 - c_0 > (1 - \alpha')(\theta s_1 - c_0)\) for all \(\alpha' > \alpha\).

\(^{10}\)Note that \(\theta s_0 - c_0 \geq (1 - \alpha)(\theta s_1 - c_0)\) implies if and only if \(\alpha(\theta s_1 - c_0) \geq \theta(s_1 - s_0)\) or equivalently \(\alpha \geq \theta(s_1 - s_0)/(\theta s_1 - c_0)\).
In this case, we have \((\theta s_0 - c_0) \leq (1 - \alpha)(\theta s_1 - c_0)\). By Lemma 1, \(\alpha(p_1 - c_0) + \delta(1 - \alpha)(\theta s_1 - c_0) = (1 + \delta)(\theta s_0 - c_0)\). It follows that

\[
\alpha(p_1 - c_1) = (1 + \delta)(\theta s_0 - c_0) - \delta(\theta s_1 - c_0) + \delta\alpha(\theta s_1 - c_0) - \alpha(c_1 - c_0).
\]

By A1, \(\alpha(p_1 - c_1)\) is increasing in \(\alpha\).

In summary, we have shown that \(\alpha(p_1 - c_1)\) is decreasing in \(\alpha\) when \(\alpha > \theta(s_1 - s_0)/(\theta s_1 - c_0)\) and increasing in \(\alpha\) when \(\alpha < \theta(s_1 - s_0)/(\theta s_1 - c_0)\). We can conclude that to solve (5) it must be

\[
\alpha = \frac{\theta(s_1 - s_0)}{\theta s_1 - c_0}.
\]

By the binding incentive constraint on the low-quality firm, it must also be

\[
p_1 = c_0 + \frac{\theta s_0 - c_0}{\alpha} = c_0 + \frac{(\theta s_0 - c_0)(\theta s_1 - c_0)}{\theta(s_1 - s_0)}.
\]

This establishes both the uniqueness and characterization of the solution for problem (5).

To show the existence, note first that by A1, the price-shortage combination \((p^*, \alpha^*)\) in the proposition satisfies 0 < \(\alpha^*\) < 1 and \(p_1^* < \theta s_1\). Thus, to complete the rest of the proof, it suffices to show that \((p_1^*, \alpha^*)\) also satisfies (6). To this end, note that

\[
\alpha^*(p_1^* - c_1) \geq (\theta s_0 - c_1) \text{ if and only if } \theta(s_1 - s_0) \leq \theta s_1 - c_0.
\]

Since \(\theta s_0 > c_0\), the above condition automatically follows from A1.

**Proof of Proposition 3:** Let \((p_1, A)\) satisfy (10), (11), and the condition \(p_1 > \theta_0 s_1\). Then, \(D_h(p_1) = q_1, q_1p_1 - A \geq q_1c_1 + \theta_0 s_0 - c_1\), and \(q_1p_1 - A \leq q_1c_1 + \theta_0 s_0 - c_0\). Notice also that the value of the objective function in (12) at \(p_1 > \theta_0 s_1\) is \(q_1p_1 - A - q_1c_1 + \delta(\theta_1 s_1 - c_1)\). It follows that for a price-advertising combination \((p_1, A)\) with \(\theta_0 s_1 < p_1 \leq \theta_1 s_1\) to solve (12), it must be \(^{11}\)

\[
q_1p_1 - A = q_1c_0 + \theta_0 s_0 - c_0.
\]

At such a combination, the first period profit of the high-quality firm is \(q_1(p_1 - c_1) - A = \theta_0 s_0 - c_0 - q_1(c_1 - c_0)\).

In contrast, for any combination \((p_1, A)\) satisfying (10), (11), and \(p_1 \leq \theta_0 s_1\), we have \(D_h(p) = 1, p_1 - A \geq \theta_0 s_0, \text{ and } p_1 - A \leq \theta_0 s_0\). It follows that \(p_1 - A = \theta_0 s_0\). The

\(^{11}\)Such a combination satisfies (10).
first period profit of the high-quality firm at such a combination is \((p_1 - c_1) - A = p_1 - A - c_1 = \theta_0 s_0 - c_1\). Notice that \(\theta_0 s_0 - c_0 - q_1 (c_1 - c_0) > \theta_0 s_0 - c_1\) if and only if \(q_1 < 1\) which is automatic. We can conclude that solutions of (12) are characterized by \(\theta_0 s_1 < p_1 \leq \theta_1 s_1\) and (17). Thus, the proof is completed if it can be shown that a combination \((p_1, A)\) satisfying (17) with \(\theta_0 s_1 < p_1 \leq \theta_1 s_1\) and \(A \geq 0\) exists. To this end, observe that the largest value of price \(p_1\) in the combinations satisfying those conditions is \(p_1 = c_0 + (\theta_0 s_0 - c_0)/q_1\). Since \(\theta_0 s_1 < c_0 + (\theta_0 s_0 - c_0)/q_1 \leq \theta_1 s_1\) if and only if

\[
\frac{\theta_0 s_0 - c_0}{\theta_1 s_1 - c_0} < q_1 < \frac{\theta_0 s_0 - c_0}{\theta_0 s_1 - c_0},
\]

it follows from A2 that (17) has a solution with \(\theta_0 s_1 < p_1 \leq \theta_1 s_1\) and \(A \geq 0\).

**Proof of Proposition 4:** Suppose first that \(q_1 (\theta_1 s_1 - c_0) > 2(\theta_0 s_0 - c_0)\). By Lemma 2, (14) is binding in any least-cost separating equilibrium:

\[
\alpha D_h(p_1) (p_1 - c_0) + \delta \max \{\theta_0 s_0 - c_0, (1 - \alpha) q_1 (\theta_1 s_1 - c_0)\} = (1 + \delta) (\theta_0 s_0 - c_0). \tag{18}
\]

Let \((p_1, \alpha)\) satisfy (18) and \(p_1 > \theta_0 s_1\). Then, \(D_h(p_1) = q_1\).

**Case 1:** \(\alpha \geq 1 - \frac{\theta_0 s_0 - c_0}{q_1 (\theta_1 s_1 - c_0)}\).

In this case, \(\theta_0 s_0 - c_0 \geq (1 - \alpha) q_1 (\theta_1 s_1 - c_0)\). Thus, by (18), \(\alpha q_1 (p_1 - c_0) = (\theta_0 s_0 - c_0)\) which is equivalent to \(p_1 = c_0 + (\theta_0 s_0 - c_0)/\alpha q_1\). It follows that the high-quality firm’s period 1 profit \(\alpha q_1 (p_1 - c_1) = \theta_0 s_0 - c_0 - \alpha q_1 (c_1 - c_0)\) is decreasing in \(\alpha\).

**Case 2:** \(\alpha \leq 1 - \frac{\theta_0 s_0 - c_0}{q_1 (\theta_1 s_1 - c_0)}\).

In this case, \(\theta_0 s_0 - c_0 \leq (1 - \alpha) (\theta_1 s_1 - c_0)\). Thus, by (18), \(\alpha q_1 (p_1 - c_0) + \delta (1 - \alpha) q_1 (\theta_1 s_1 - c_0) = (1 + \delta) (\theta_0 s_0 - c_0)\). Consequently, the high-quality firm’s period 1 profit is

\[
\alpha q_1 (p_1 - c_1) = (1 + \delta) (\theta_0 s_0 - c_0) - \delta q_1 (\theta_1 s_1 - c_0) + \alpha q_1 \left[\delta (\theta_1 s_1 - c_0) - (c_1 - c_0)\right].
\]

By A1’, \(\delta (\theta_1 s_1 - c_0) > (c_1 - c_1)\) and \(\theta_1 s_1 > c_1\). It follows that \(\alpha q_1 (p_1 - c_1)\) is increasing in \(\alpha\).

In summary, Case 1 and Case 2 together with (18) imply that in any least-cost separating equilibrium, \(p_1 > \theta_0 s_1\) implies

\[
\alpha = 1 - \frac{\theta_0 s_0 - c_0}{q_1 (\theta_1 s_1 - c_0)}, \tag{19}
\]

17
and

\[
p_1 = c_0 + \frac{(\theta_1 s_1 - c_0)(\theta_0 s_0 - c_0)}{q_1 (\theta_1 s_1 - c_0) - (\theta_0 s_0 - c_0)}.
\]  

(20)

By A2, price \( p_1 \) in (20) satisfies \( p_1 > \theta_0 s_1 \) if and only if

\[
q_1 < \frac{\theta_0 s_0 - c_0}{\theta_0 s_1 - c_0} + \frac{\theta_0 s_0 - c_0}{\theta_1 s_1 - c_0}
\]

which is guaranteed by A2. Since \( q_1(\theta_1 s_1 - c_0) > 2(\theta_0 s_0 - c_0) \), \( p_1 \) in (20) also satisfies \( p_1 \leq \theta_1 s_1 \) implying that \( p_1 \) is feasible. Thus, to show that \( (p_1, \alpha) \) in (19) and (20) also satisfies (13), note that \( \alpha q_1(p_1 - c_1) \geq \theta_0 s_0 - c_1 \) if and only if

\[
q_1 \leq 1 + \frac{\theta_0 s_0 - c_0}{\theta_1 s_1 - c_0}
\]

which is clearly satisfied.

To complete the rest of the proof under the supposition that \( q_1(\theta_1 s_1 - c_1) > 2(\theta_0 s_0 - c_0) \), it only remains to show that no price below or equal to \( \theta_0 s_1 \) can be the high-quality firm’s period 1 price in least-cost equilibrium. To this end, note first that (20) implies that \( \alpha q_1(p_1 - c_1) > (\theta_0 s_1 - c_1) \) if and only if

\[
q_1 < 1 - \frac{\theta_0 (s_1 - s_0)}{c_1 - c_0} + \frac{\theta_0 s_0 - c_0}{\theta_1 s_1 - c_0}.
\]

By A2, the preceding condition is satisfied. Since the high-quality firm’s period 1 profit is no more than \( \theta_0 s_1 - c_1 \) with a period 1 price \( p_1 \leq \theta_0 s_1 \), the high-quality firm’s period 1 price in least-cost equilibrium must exceed \( \theta_0 s_1 \).

Suppose now that \( q_1(\theta_1 s_1 - c_1) \leq 2(\theta_0 s_0 - c_0) \). Let \( (p_1, A) \) satisfy (18) with \( p_1 > \theta_0 s_1 \). Consider first \( \theta_0 s_0 - c_0 < (1 - \alpha)q_1(\theta_1 s_1 - c_0) \). This together with (18) implies

\[
\alpha < 1 - \frac{\theta_0 s_0 - c_0}{q_1(\theta_1 s_1 - c_0)}, \quad p_1(\alpha) = c_0 + \frac{(1 + \delta)(\theta_0 s_0 - c_0) - \delta(1 - \alpha)q_1(\theta_1 s_1 - c_0)}{\alpha q_1}.
\]

(21)

Note that

\[
\frac{dp_1(\alpha)}{d\alpha} = \frac{\delta q_1(\theta_1 s_1 - c_0) - (1 + \delta)(\theta_0 s_0 - c_0)}{\alpha^2 q_1}.
\]

It follows that

\[
\frac{dp_1(\alpha)}{d\alpha} < 0 \text{ if and only if } q_1 < \left( \frac{1 + \delta}{\delta} \right) \left( \frac{\theta_0 s_0 - c_0}{\theta_1 s_1 - c_0} \right).
\]
Thus, \( dp_1(\alpha)/d\alpha < 0 \) for all \( q_1 \) satisfying \( q_1(\theta_1 s_1 - c_0) \leq 2(\theta_0 s_0 - c_0) \).\(^{12}\) By \( A2 \),

\[
\alpha = 1 - \frac{\theta_0 s_0 - c_0}{q_1(\theta_1 s_1 - c_0)} \Rightarrow p_1(\alpha) = c_0 + \frac{\theta_0 s_0 - c_0}{\alpha q_1} > \theta_1 s_1.
\]

Thus, \( p_1(\alpha) \) in (21) is not feasible in the sense that it exceeds \( \theta_1 s_1 \) for all \( 0 < \alpha < 1 - (\theta_0 s_0 - c_0)/q_1(\theta_1 s_1 - c_0) \).

Consider now \( \theta_0 s_0 - c_0 \geq (1 - \alpha)q_1(\theta_1 s_1 - c_0) \). This together with (18) implies

\[
\alpha \geq 1 - \frac{\theta_0 s_0 - c_0}{q_1(\theta_1 s_1 - c_0)} \text{ and } p_1(\alpha) = c_0 + \frac{(\theta_0 s_0 - c_0)}{\alpha q_1}. \quad (22)
\]

In this case, the high-quality firm's total profit is decreasing in \( \alpha \) over the range \( 1 - (\theta_0 s_0 - c_0)/q_1(\theta_1 s_1 - c_0) \) \( \leq \alpha < 1 \) because

\[
\left\{ q_1[\alpha(p_1 - c_1) + \delta(\theta_1 s_1 - c_1)] \right\}' = -q_1(c_1 - c_0) < 0.
\]

On the other hand,

\[
\alpha = 1 - \frac{\theta_0 s_0 - c_0}{q_1(\theta_1 s_1 - c_0)} \Rightarrow p_1 = c_0 + \frac{\theta_0 s_0 - c_0}{q_1 - \left( \frac{\theta_0 s_0 - c_0}{\theta_1 s_1 - c_0} \right)} > \theta_1 s_1.
\]

Thus, by (22), the smallest \( \alpha \) that results in a feasible price is

\[
\alpha = \frac{\theta_0 s_0 - c_0}{q_1(\theta_1 s_1 - c_0)} \Rightarrow p_1(\alpha) = \theta_1 s_1. \quad (23)
\]

With \( p_1 \) and \( \alpha \) in (23), \( (p_1, \alpha) \) satisfies the incentive constraint (13) for the high-quality firm if and only if

\[
\frac{\theta_0 s_0 - c_0}{\theta_1 s_1 - c_0} \geq \frac{\theta_0 s_0 - c_1}{\theta_1 s_1 - c_1}.
\]

The preceding inequality automatically holds because \( c_1 > c_0 \).\(^{13}\)

By (23), \( \alpha q_1(p_1 - c_1) > (\theta_0 s_1 - c_1) \) if and only if

\[
\frac{\theta_0 s_1 - c_1}{\theta_1 s_1 - c_1} < \frac{\theta_0 s_0 - c_0}{\theta_1 s_1 - c_0}
\]

which is guaranteed by \( A2 \). Since the high-quality firm's period 1 profit is no more than \( \theta_0 s_1 - c_1 \) with a period 1 price \( p_1 \leq \theta_0 s_1 \), the preceding analysis shows that the

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\(^{12}\)For \( \delta \in (0, 1), (1 + \delta)/\delta > 2. \)

\(^{13}\)Given two constants \( a \) and \( b \), the ratio \( (a - x)/(b - x) \) is decreasing in \( x \) if and only if \( a < b \).
Proof of Proposition 5: Since the second period profit of the high-quality firm is the same in separating equilibrium with either price-advertising or price-shortage combination as the signal, we only need to show that the high-quality firm’s first period profit is higher with price-shortage combination as the signal.

By Proposition 3, the high-quality firm’s first period separating equilibrium profit with price-advertising combination as the signal is

\[ q_1(p_1^* - c_1) - A^* = \theta_0s_0 - c_0 - q_1(c_1 - c_0). \]

(24)

Suppose first \( q_1(\theta_1s_1 - c_1) > 2(\theta_0s_0 - c_0) \). In this case, Proposition 4 implies that the high-quality firm’s first period separating equilibrium profit with price-shortage combination as the signal is

\[ \alpha^*_1 q_1(\hat{p}_1^* - c_1) = \theta_0s_0 - c_0 - q_1(c_1 - c_0) + \frac{(\theta_0s_0 - c_0)(c_1 - c_0)}{\theta_1s_1 - c_0}. \]

(25)

By (24) and (25),

\[ \alpha^*_1 q_1(\hat{p}_1^* - c_1) > q_1(p_1^* - c_1) - A^* \iff \frac{(\theta_0s_0 - c_0)(c_1 - c_0)}{\theta_1s_1 - c_0} > 0 \]

which holds under \( \textbf{A1}' \). Suppose now \( q_1(\theta_1s_1 - c_1) \leq 2(\theta_0s_0 - c_0) \). Proposition 4 implies that in this case the high-quality firm’s first period separating equilibrium profit with price-shortage combination as the signal becomes

\[ \alpha^*_1 q_1(\hat{p}_1^* - c_1) = \frac{(\theta_0s_0 - c_0)(\theta_1s_1 - c_1)}{\theta_1s_1 - c_0}. \]

(26)

Using (24) and (26), \( \alpha^*_1 q_1(\hat{p}_1^* - c_1) > q_1(p_1^* - c_1) - A^* \) if and only if

\[ q_1 > \frac{(\theta_0s_0 - c_0)}{\theta_1s_1 - c_0} \]

which is guaranteed by \( \textbf{A2} \).