Cartelization of Cross-Holdings

Cheng-Zhong Qin, Shengping Zhang and Dandan Zhu†

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Abstract

Cross-holdings between firms cause their profits to be mutually dependent. This paper analyzes a model of cross-holdings that calls for the firms to consider financial interests of the competitors in their profits instead of operating earnings. It is shown that cross-holdings induce tacit formation of cartels when they are irreducible. Consequently, the irreducibility of cross-holdings should be regarded as a condition, under which the firms attempt to bring about substantial lessening of competition.

Keywords: Antitrust; Cartel; Cross-holding; Financial interest

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1 Introduction

Cross-holding refers to a situation where a firm acquires equity shares in another firm. Cross-holdings are frequently observed in practice. For example, Northwest Airlines acquired a 14% equity position in Continental Airlines in 1998. Microsoft acquired approximately 7% of the non-voting stock of Apple in 1997 and in 1999, it held a 10% stake in Inprise/Borland Corp. The former is a historic rival of Microsoft in PC market and the latter is a major competitor in the software applications market.1 The

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† Contact: Cheng-Zhong Qin, Department of Economics, University of California, Santa Barbara, CA 93106-9210, qin@econ.ucsb.edu; Shengping Zhang, Guanghua School of Management, Peking University; zsp@gsm.pku.edu.cn; Dandan Zhu, Department of Economics, University of California, Santa Barbara, CA 93106-9210, zhu@econ.ucsb.edu.

Japanese and the U.S. automobile industries are two of many that feature complex webs of cross-holdings (see Alley, 1997).

We consider cross-holdings which are passive, in the sense that the acquiring firm is entitled to a share of the acquired firm’s profit but not decision making. That is, the acquiring firm is entitled to financial interests in the acquired firm’s profits only. Partial ownership interests have been often examined in the context of Section 7 of the Clayton Act. According to the statutory language, Section 7 of the Clayton Act “shall not apply to persons purchasing such stock [1] solely for the investment and [2] not using the same by voting or otherwise to bring about, or in attempting to bring about, the substantial lessening of competition.” (see O’Brien and Salop, 2000). Due to the lack of decision right, passive cross-holdings are solely for the purposes of investment, because firms continue to be independent and compete with each other. However, it is not clear whether by engaging in cross-holdings firms attempt to bring about substantial lessening of competition.

Reynolds and Snapp (1986) were the first to analyze competitive implications of cross-holdings. They assumed that firms consider competitors’ financial interests in their operating earnings (profits from ordinary production). Firms’ profits under this approach are formulated by diving their earnings in proportion to equity shares. As pointed out in Reynolds and Snapp (p. 144), this formulation of firms’ profits is appropriate if owners are managers, or if managers view equity holdings by competitors as different from those by external equity holders (private investors or firms outside the industry). For an oligopoly with a homogenous product, Reynolds and Snapp showed that the Cournot equilibrium of the output market will become less competitive, with aggregate output level falling toward the monopoly level as cross-holdings increase. Cross-holdings do not imply complete mergers unless they are large enough according to these results.²

Flath (1991) adopted a different approach under which firms’ profits are given by their operating earnings plus financial interests in rivals’ profits. Firms under this approach do not consider competitors’ financial interests. As a result, total profits of the firms exceed total industry’s operating earnings, which implies double counting. Competitive implications of cross-holdings under this approach are qualitatively similar

²Alley (1997) applied the Reynolds and Snapp’s (1986) approach to analyze the effect of cross-holdings on the degree of competition in the US and Japanese automobile industries. Malheg (1992) considered the collusive effect of cross-holdings in a repeated Cournot duopoly also using this approach. Cross-holdings weakens output market competition following a breakdown of a collusive scheme. Contrasting to the Reynolds and Sanpp’s (1986) results on competitive implications of cross-holdings, Malheg showed that cross-holdings have an ambiguous effect on collusion.
to those in Reynolds and Snapp (1986).

In this paper, we analyze an alternative profit formulation that calls for firms to consider competitors’ financial interests as in Reynolds and Snapp (1986), but with financial interests in profits instead of operating earnings. Thus, firms under our approach divide profits in proportion to equity shares. Aside from the difference between financial interests in operating earnings and financial interests in profits, the appropriateness of this formulation can be justified by similar characteristics of the firms or behavioral conditions on managers as Reynolds and Snapp considered for their profit formulation. As with Flath’s (1991) approach, having financial interests in profits implies that firms consider indirect as well as direct financial interests. Total profits of the firms equal to total industry’s operating earnings under our approach. Furthermore, competitive implications of cross-holdings significantly differ from those found in the literature.

We show that, under the condition of “irreducibility” of cross-holdings, firms are each induced to guide their individual choices by the maximization of total operating earnings. That is, a cartel between the firms is tacitly induced. More generally, when cross-holdings can be grouped into “irreducible blocks”, firms are tacitly induced to partition themselves into disjoint cartels. These results do not depend on whether firms use price or quantity to compete in output market. An implication of these results is that the irreducibility of cross-holdings between firms within a group should be taken as a condition, under which member firms attempt to bring about substantial lessening of competition. The irreducibility of cross-holdings means that each firm has either direct or indirect equity holding in the profits of each of the other firms. On the other hand, an irreducible block is a group of firms such that cross-holdings between them are irreducible and separable from the rest of the firms.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 presents the results and Section 4 concludes.

2 Model

We consider an industry with $n$ firms. Firm $i$’s choice set is denoted by $X_i$ (e.g., the set of prices or quantities). Set $X = X_1 \times X_2 \times \cdots \times X_n$. The profits of firm $i$ resulting from a choice profile $x = (x_1, x_2, \ldots, x_n) \in X$ are called the operating earnings of firm $i$ and are denoted by $\pi^o_i(x)$. Set $\pi^o(x) = (\pi^o_1(x), \cdots, \pi^o_n(x))$.

By normalization, we can take the total number of equity shares within each firm
to be one. Denote by \( a_i = (a_{i1}, a_{i2}, \cdots, a_{in}) \) the row vector of equity shares \( a_{ij} \) held by firm \( i \) in firm \( j \) for \( i \neq j \) and

\[
a_{ii} = 1 - \sum_{j \neq i} a_{ji}.
\]

(1)

Let \( A \) denote the cross-holding matrix with \( a_i \) as the \( i \)-th row. As a non-harmful assumption, we assume \( a_{ii} > 0 \) for all \( i \). With (1), the transpose of \( A \) is a \( stochastic \) matrix or \( A \) is \( column stochastic \). We say that cross-holdings between member firms of a group \( S \) are \( separable \) if

\[
a_{ij} = a_{ji} = 0, \ i \in S, \ j \notin S.
\]

(2)

We use \( \pi(x) = (\pi_1(x), \cdots, \pi_n(x)) \) to denote profit allocation resulting from the operating earnings allocation \( \pi^\circ(x) \) and cross-holding matrix \( A \). Let \( A_S \) denote the sub-matrix obtained from \( A \) by removing the rows and columns that correspond to firms not in \( S \), and let \( \pi_S(x) = (\pi_i(x))_{i \in S} \) denote the sub-vector of \( \pi(x) \). If cross-holdings between firms in group \( S \) are separable, we require

\[
A_S \pi'_S(x) = \pi'_S(x).
\]

(3)

The right-hand-side of equation (3) is the column vector of profits that firms in group \( S \) end up getting at choice profile \( x \) and cross-holding matrix \( A \). In comparison, the left-hand-side is the column vector of profits which these firms will get when cross-holdings between them are characterized by matrix \( A \), and their profits are divided in proportion to equity shares. If follows that formulating profits according to (3) requires that firms consider financial interests of the competitors in their profits. Next, as a natural condition, we also require that the resulting profit allocation be feasible, in the sense that the allocation cannot involve profit sharing by member firms of a separable group in non-member firms nor conversely. This means that the allocation of profits must also satisfy:

\[
\sum_{i \in S} \pi_i(x) = \sum_{i \in S} \pi^\circ_i(x)
\]

(4)

if cross-holdings between firms in \( S \) are separable.\(^3\)

\(^3\)Firm \( i \) maximizes \( \sum_{j=1}^{n} a_{ij} \pi_j^\circ(x) \) in Reynolds and Snapp (1986) while it maximizes \( \pi_i(x) = \pi_i^\circ(x) + \sum_{j \neq i} a_{ij} \pi_j(x) \) in Flath (1991). The former restricts financial interests to be in operating earnings only while the latter covers financial interests in profits, but it implies \( \sum_{i=1}^{n} \pi_i(x) > \sum_{i=1}^{n} \pi_i^\circ(x) \).
3 Cartelization

As noted before, (1) implies that each cross-holding matrix $A$ is column stochastic. If condition (2) holds for group $S$, then the sub-matrix $A_S$ of cross-holdings between member firms of group $S$ is also column stochastic.

3.1 Cartel Formation

We say that cross-holdings between firms in group $S$ is irreducible if for any two firms in $S$, a family of intermediate member firms of group $S$ can be found such that when they are properly ordered, each firm holds equity share in the next firm according to the order. That is, each firm in group $S$ has either direct or indirect equity holding in each of the other firms within group $S$. Formally, cross-holdings between firms of group $S$, represented by $A_S$, are irreducible if any two member firms $i \neq j$ of $S$, there is a family $i_0, i_1, \ldots, i_m$ of $m + 1$ member firms for some finite integer $m$ such that $i = i_0$, $j = i_m$, and $a_{i_{k-1}i_k} > 0$ for $k = 1, 2, \ldots, m$ (see Seneta, 1981, p. 18).

Proposition 1 Assume cross-holdings between firms in group $S$ are irreducible and separable. (I) If $\pi_i^*(x) \geq 0$ for all $x \in X$ and all $i \in S$, then there exists a unique vector $\gamma_S = (\gamma_i)_{i \in S}$ such that

$$\gamma_i > 0, \ i \in S, \sum_{i \in S} \gamma_i = 1$$

and

$$\pi_i(x) = \gamma_i \sum_{i \in S} \pi_i^*(x), \ x \in X, \ i \in S$$

is the unique non-negative solution for (3) and (4). (II) If $\pi_i^*(\cdot)$ is continuous and $X_i$ is compact for all $i \in S$, then there exists a vector $\gamma_S = (\gamma_i)_{i \in S}$ such that (5) holds and $\pi_S(x)$ as determined by (6) is a solution for (3) and (4).

Proof. See Appendix. ■

Observe that only non-negative solutions for (3) and (4) are sensible when firms’ operating earnings are non-negative. Observe also that the Bertrand model satisfies the non-negativity assumption when firms marginal costs are constant and fixed costs are zero. The reason is that the choice set $X_i$ can be restricted, without loss of generality,
to contain prices not below firm \(i\)'s marginal cost. In contrast, the Cournot model satisfies the continuity and compactness assumption if demand is continuous and there is either a finite capacity for each firm or a finite total quantity resulting in price equal to 0.

By (5) and (6), \(\gamma\) can be regarded as a distribution of relative shares in total operating earnings among firms within group \(S\). Since \(\gamma\) is independent of the choice profile \(x\), (6) implies that the choices of the member firms of group \(S\) are each guided by the maximization of the group's total profits. Consequently, a cartel among them with \(\gamma\) as the distribution of their relative shares of operating earnings is induced. The difference between part (I) and part (II) is that the relative share distribution is unique in the former, while this uniqueness is not guaranteed in the latter.

A more general result can be established when it is possible to partition the cross-holding matrix into irreducible blocks. The proof is similar to the proof of Proposition 1. For this reason, the proof is omitted.

**Proposition 2** Assume firms can be partitioned into disjoint groups \(S_1, S_2, \ldots, S_l\) such that cross-holdings between firms in group \(S_h\) are irreducible and separable for all \(h = 1, 2, \ldots, l\). (I) If \(\pi_i^o(x) \geq 0\) for all \(x \in X\) and all \(i\), then there exists a unique vector \(\gamma \in \mathbb{R}^n\) such that for all \(h\), (i) \(\gamma_i > 0\) for \(i \in S_h\), (ii) \(\sum_{i \in S_h} \gamma_i = 1\), and (iii) \(\pi_i(x) = \gamma_i \sum_{j \in S_h} \pi_j^o(x)\) for \(x \in X\) and \(i \in S_h\) is the unique non-negative solution for (3) and (4) with \(S = S_h\). (II) If \(\pi_i^o(\cdot)\) is continuous and \(X_i\) is compact for all \(i\), then there exists a vector \(\gamma \in \mathbb{R}^n\) such that \(\gamma\) satisfies (i) and (ii) and \(\pi_{S_h}(x)\) as in (iii) is a solution for (3) and (4) with \(S = S_h\) for all \(h\).

A sufficient condition for the irreducibility of cross-holdings is for each firm to hold a positive amount of equity share in each of the other firms. However, such complete direct holdings are not necessary for the irreducibility of cross-holdings. As an example, suppose that each firm \(i\) holds 1\% of equity share in firm \(m + 1\) and zero percent in any other firm’s.\(^4\) That is, suppose that the cross-holding matrix is

\[
A = \begin{pmatrix}
\frac{99}{100} & \frac{1}{100} & 0 & \cdots & 0 \\
0 & \frac{99}{100} & \frac{1}{100} & \cdots & 0 \\
: & : & : & \cdots & : \\
\frac{1}{100} & 0 & 0 & \cdots & \frac{99}{100}
\end{pmatrix}.
\]

\(^4\)We take \(i + k\) to be \(i + k - n\) when \(i + k > n\).
Simple analysis shows that cross-holdings between the \( n \) firms as described by the preceding matrix \( A \) are irreducible.

4 Conclusion

Behavioral implications of cross-holdings depend on the formulation of firms’ profits. Two approaches to profit formulation have been proposed and analyzed in the literature. One approach assumes that firms consider competitors’ financial interests in operating earnings, while the other approach assumes that firms do not consider competitors’ financial interests. Under both approach, cross-holdings do not imply formation of cartels unless they are large enough.

We analyzed a different approach to the formulation of firms’ profits, under which firms divide profits instead of operating earnings in proportion to equity shares. Thus, firms under our approach consider competitors’ financial interests in their profits instead of operating earnings. Competitive implications under this approach significantly differ from those found in the literature. We showed that firms will be tacitly induced to form a cartel if cross-holdings between them are irreducible, even though they continue to be independent in their decision making and compete with one another. Accordingly, firms attempt to bring about substantial lessening of competition when cross-holdings are irreducible. Consequently, our results suggest that the irreducibility of cross-holdings should be regarded as a condition to raise antitrust concerns.

Appendix

Proof of Proposition 1: Since \( A_S \) is irreducible and \( a_{ii} > 0 \) for all \( i \in S \), \( A_S \) is also primitive (Seneta, 1981, Theorem 1.4). Consider first the case where \( \pi_i^0(x) \geq 0 \) for all \( x \in X \) and all \( i \in S \). In this case, part (I) holds automatically when \( \sum_{i \in S} \pi_i^0(x) = 0 \). Thus, assume \( \sum_{i \in S} \pi_i^0(x) > 0 \). Since \( A_S \) is stochastic, the Perron-Frobenius eigenvalue of \( A_S \) is 1 (Seneta, 1981, Corollary 1 on p. 8). By results for primitive non-negative matrices (Seneta, 1981, Theorems 1.1 and 1.6), there exists a unique vector \( \gamma_S = (\gamma_i)_{i \in S} \) such that

\[
A_S \gamma_S' = \gamma_S', \quad \sum_{i \in S} \gamma_i = 1, \text{ and } \gamma_i > 0, \ i \in S,
\]

(A1)

\[\text{The positive vector } \gamma_S \text{ is the normalized Perron-Frobenius eigenvector of matrix } A_S.\]
\[ A_S \eta'_S = \eta'_S, \quad \sum_{i \in S} \eta_i = 1, \quad \text{and} \quad \eta_i \geq 0 \Rightarrow \eta_S = \gamma_S. \quad (A2) \]

Since \( \sum_{i \in S} \pi_i^0(x) > 0 \), (4) implies \( \sum_{i \in S} \pi_i(x) > 0 \). Thus, if \( \pi_i(s) \geq 0 \) for all \( i \in S \), then it follows from (3), (A1) and (A2) that

\[
\left( \frac{\pi_i(x)}{\sum_{j \in S} \pi_j(x)} \right)_{i \in S} = \gamma_S.
\]

By (4), the preceding equation is equivalent to

\[
\pi_i(x) = \gamma_i \sum_{i \in S} \pi_i^0(x), \quad i \in S.
\]

Since \( \gamma \) is unique, \( \pi_S(x) \) given in the preceding equation is the unique non-negative solution for (3) and (4). This completes the proof of part (I).

Consider now the case where \( \pi_i^0(x) \) is continuous and \( X_i \) is compact for all \( i \in S \). In this case, let \( \bar{\pi}_i^0 \) be the minimum of \( \pi_i^0(x) \) over \( X \) and let

\[
\tilde{\pi}_i^0(x) = \pi_i^0(x) - \bar{\pi}_i^0. \quad (A3)
\]

Then, \( \tilde{\pi}_i^0(x) \geq 0 \) for all \( x \in X \) and all \( i \in S \). Fixing \( x \in X \), by replacing \( \pi_i^0(x) \) with \( \tilde{\pi}_i^0(x) \) as the operating earnings, it follows from the preceding proof of part (I) that there exists a positive vector \( \gamma_S = (\gamma_i)_{i \in S} \) such that it satisfies (A1), (A2), and

\[
\tilde{\pi}_i(x) = \gamma_i \sum_{i \in S} \tilde{\pi}_i^0(x), \quad i \in S \quad (A4)
\]

satisfies

\[
A_S \tilde{\pi}_S(x) = \tilde{\pi}'_S(x) \quad (A5)
\]

and

\[
\sum_{i \in S} \tilde{\pi}_i(x) = \sum_{i \in S} \tilde{\pi}_i^0(x). \quad (A6)
\]

Next, set

\[
\pi_i(x) = \tilde{\pi}_i(x) + \gamma_i \sum_{i \in S} \tilde{\pi}_i^0, \quad i \in S. \quad (A7)
\]
Then, by \((A1)\) and \((A3) - (A7)\),

\[
A_S \pi'_S(x) = A_S \tilde{\pi}'_S(x) + \left( \sum_{i \in S} \pi^0_i \right) A_S \gamma'_S = \tilde{\pi}'_S(x) + \left( \sum_{i \in S} \pi^0_i \right) \gamma'_S = \pi'_S(x)
\]

\[
\sum_{i \in S} \pi_i(x) = \sum_{i \in S} \tilde{\pi}_i(x) + \left( \sum_{i \in S} \gamma_i \right) \left( \sum_{i \in S} \pi^0_i \right) = \sum_{i \in S} \pi^0_i(x)
\]

and

\[
\pi_i(x) = \gamma_i \sum_{i \in S} \pi^0_i(x).
\]

This shows that \(\pi_S(x)\) given in \((A7)\) solves \((3)\) and \((4)\). 

References


