A Signaling Theory of Limited Supply

by

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This paper analyzes the role of seller-induced shortage as a signal of quality. Unlike dissipative advertising, the cost of inducing shortage is different for different quality types. It is shown that under certain conditions, a high-quality monopoly firm that signals quality by inducing shortage makes more profit than using price alone or combined with dissipative advertising. This is because the forgone profit from the lost sales is always lower for the high-quality firm than for the low-quality firm. The result explains why high-quality firms may prefer to initially limit supply with a price weakly lower than that in the complete-information case.

Keywords: seller-induced shortage, quality signaling, perfect Bayesian equilibrium

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1 Introduction

In some industries limiting supply is a commonly used business strategy. Becker (1991) considers a model of consumer demand to explain the rationality of not raising prices, even with persistent excess demand, as done by some successful restaurants, plays, sporting events, and so on. Recognizing that consumption of a good is to some extent a social activity, Becker postulates that demand by a typical consumer is positively related to aggregate demand. He analyzes implications of such consumption externality for the pricing behavior of a monopoly firm. Depending on the strength of the responsiveness of the market demand to itself, Becker demonstrates that the “self-dependence” of market demand may result in disconti-
This discontinuity leads to the existence of excess demand while the firm is maximizing its profit.\(^1\)

There have been several other papers on the strategic uses of seller-induced shortage. In DeGraba (1995), shortage is purposely induced by a monopoly firm to promote buying frenzy. In his model, consumers learn their types (their values of the product) over time. By selling fewer units, uncertainties can be created for those consumers who choose to wait until they have learned their types, since there may not be enough supply. DeGraba shows that an equilibrium exists in which all consumers purchase the product early on while uninformed, although they prefer to purchase after becoming informed provided the product is available. This induced buying frenzy allows the monopolist to price higher and earn more profit. Bose (1997) considers rationing problems in restaurants. He shows that restaurants can use capacities to screen out less profitable customers, because serious customers who will spend more care less about waiting time.

The signaling role of seller-induced shortage is not analyzed in the aforementioned papers. In this paper, we consider a different rationale for the existence of shortage. The basic idea is that consumers can observe the price and shortages using proxies such as queuing or the time required to order in advance. For example, consumers often signed up on waiting lists for hard-to-get cars, but car makers were not rushing to speed up production. The demand for Mazda Miata in 1989 when first introduced far exceeded the quantity that Mazda was producing. Indeed, dealers were selling the car for 20% more than the sticker price. But within a month of its introduction, Mazda informed the dealers that they were not allowed to increase the price above sticker. As a result, dealers sold Miatas at a price below the price that would clear the market (The Wall Street Journal, 1989). Although inducing shortage does not convey information directly, consumers can observe a proxy of it, for example, by how early in advance a reservation must be made or by the actual length of a queue.\(^2\) It is possible to have an equilibrium, in which consumers rationally expect the firm to induce different amounts of shortage for different product quality types.

Beginning with Nelson (1970, 1974), the signaling role of advertising has received considerable attention in the literature. A basic idea is that advertising may be dissipative, in the sense that it is only a signal that the firm is able to spend a lot of money. But consumers can observe the total amount of money (or a proxy of it) that the firm is spending on advertising. It is possible to have an equilib-

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\(^1\) Karni and Levin (1994) develop an analytical framework by which the aforementioned dependence is derived endogenously from primitive data such as utility functions and the distribution of tastes. They establish the existence of a Stackelberg equilibrium with the leader incurring excess demand and the follower incurring excess capacity, while no Bertrand equilibrium exists in which one or both firms incur excess demand.

\(^2\) When excess demand is present, demand must be rationed at the price being charged and a queue is therefore created. To simplify the discussion, we assume, as in Becker (1991) that the method used to ration demand is costless, such as having to make reservations in advance, so that the money price is the full cost to consumers.
rium in which consumers rationally expect the firm to spend different amounts on advertising for different product quality types.

To contrast with results using a price-advertising combination as the signal of quality, we carry out our analysis for a familiar two-period model of a monopoly firm providing a new indivisible experience good of either high or low quality. The firm knows the quality; but the consumers do not have this knowledge at the time the product is introduced. The firm cannot vary the quality across the two periods. In addition, there is no word-of-mouth learning or independent sources of quality revelation such as consumer reports. Signaling occurs during the first period. Thus, the consumers can learn the quality of the product either in the first period by observing a signal (if any) that separates the high-quality product from the low-quality one, or at the beginning of the second period by consuming the good in the first period.

As already shown in Tirole (1988, p. 120), price and dissipative advertising are perfect substitutes for their roles as signals of quality for the two-period model with indivisibility we consider. Furthermore, for the case with homogeneous consumers, using price or a price-advertising combination to signal quality generates a negative result, i.e., pooling always dominates signaling. In comparison, we show in this paper that the high-quality monopoly firm can signal its quality by properly setting the price and inducing shortage. Furthermore, signaling quality via price-shortage combination resolves the aforementioned signaling dilemma; it results in a separating equilibrium that is more profitable than the pooling equilibrium. For the case with heterogeneous consumers, we show that when it is profitable for the high-quality firm to target high-taste consumers with complete information, it is more profitable to signal using a price-shortage combination than by a price-advertising combination or pooling. These results provide a rationale for why high-quality product monopoly firms may prefer to initially limit supply with or without lowering price.

Stock and Balachander (2005) were the first to address the signaling role of seller-induced scarcity. Their approach differs from ours in several ways. First, they require that a nonzero fraction of consumers be informed of the quality of the product before purchasing. Second, informed and uninformed consumers make purchasing decisions sequentially in their model, with the informed consumers allowed to book the product in advance (without scarcity constraint) and the uninformed consumers allowed to enter the market only at a later stage (with possible scarcity). Third, they do not consider repeated purchase. Their setting implies that a separating equilibrium using a price-advertising combination as the signal of quality never exists if quality provision is costly. In reality, a consumer’s knowledge about the quality of a new product does not necessarily grant her the priority to buy the product first when the product is introduced.

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3 The intuition is straightforward: whatever the amount of advertising the high-quality type firm chooses, the low-quality type can mimic at no cost, since the firm sells to each consumer only once.
In contrast, we investigate the signaling role of limit supplying under the classical monopoly-firm setting with repeated purchase. We establish the existence of a least-cost separating equilibrium with the firm supplying the high-quality product inducing shortage at a price smaller than its complete-information monopoly price. We then compare shortage with dissipative advertising as high-quality signals. We show that under certain conditions it is more profitable to signal high quality by inducing shortage than by dissipative advertising. The major reason is that the signaling cost in forgone profits to the high-quality firm from inducing shortage is always lower than that to the low-quality firm.

The rest of the paper is organized as follows. Section 2 introduces the model and results for the simple homogeneous consumer case. Section 3 presents results for the heterogeneous consumer case. Section 4 concludes the paper. Proofs of some results are relegated to the appendix.

2 Signaling Quality with Homogeneous Consumers

To be tractable, we confine analysis to a static model of a monopoly firm providing a new product of either high or low quality. The firm knows the quality, but the consumers do not have this knowledge at the time the product is introduced. Further, the firm cannot vary the quality. The static model captures the introductory phase of the product under the following conditions on information transmission. The product’s life cycle can be decomposed into two phases: the introductory phase and the mature phase. Signaling occurs during the introductory phase. All consumers know the quality in the mature phase (e.g., Tirole, 1988, p. 118). Under these conditions, the firm will choose its complete-information monopoly price and will not induce shortage in the mature phase.

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4 This means that after the quality type of the product has been revealed in the introductory phase, the high-quality firm will raise the price to its complete-information monopoly price in the mature phase.

5 A result in Bagwell (2007) states that in a setting such as ours, but with advertising instead of seller-induced excess demand as the signal of high quality, dissipative advertising is not used and price is distorted upward for the high-quality type in the least-cost separating equilibrium. Fluet and Garella (2002) show that dissipative advertising may be used to signal high quality in a static duopoly model with one high-quality firm and one low-quality firm. A key assumption they make is that while consumers do not know which firm is the high-quality firm, the firms know each other’s quality types. Thus, one firm’s advertising choice may provide information about the other firm’s quality type. This enlarges the set of signals of high quality. They show that if the quality difference is not too large, then the high-quality firm advertises in any separating equilibrium.

6 As explained in Milgrom and Roberts (1986), this may correspond to the situation where the firm’s R&D effort has generated the product of a given quality that the firm must then decide how to introduce. As usual, we assume that the product of high quality is more costly to produce.

7 This is a natural consequence of models without first-phase shortages, i.e., all consumers will consume in the introductory phase, and they learn the true quality immediately.
Formally, consider a two-period market for a new indivisible experience product supplied by a monopoly firm. The product can be of either high (type $s_1$) or low quality level (type $s_0$) with $0 < s_0 < s_1$. The quality is not observable to the consumers, nor is it adjustable by the firm across time. The marginal cost of the firm is constant for either quality type and is denoted by $c_t$ for type $s_t$. As usual, assume $0 < c_0 < c_1$.

There is a continuum of consumers, with each demanding at most one unit of the product in each period. Let $\theta > 0$ denote the common value of quality for a consumer. If he consumes the product of quality $s_t$ at price $p$, he obtains net utility

$$\theta s_t - p.$$  

Let $\delta \in (0, 1]$ be the common discount factor, and let $\mu$ be the probability with which each consumer believes that the monopolist’s product has high quality prior to observing any signal or consuming the product. Since $s_1 > s_0$ and $c_0 < c_1$, the low-quality firm has incentives to mislead consumers into believing that it is of the high-quality type, provided that it is not too costly to do so.\(^8\)

### 2.1 Signaling Quality by Price and Advertising

Suppose that the high-quality firm uses price $p_1$ and dissipative advertising with expenditure $A$, i.e., a price–advertising combination $(p_1, A)$, to signal its quality in period 1. As usual, in any separating equilibrium with sequential rationality, the low-quality firm chooses its complete-information monopoly price $\theta s_0$ and does not advertise, because it cannot mislead consumers unless it mimics the high-quality firm’s choice. The following incentive-compatibility constraints characterize all separating equilibria:

\begin{align*}
(1) & \quad p_1 - A - c_1 \geq \theta s_0 - c_1 \\
(2) & \quad p_1 - A - c_0 \leq \theta s_0 - c_0.
\end{align*}

Note that (1) and (2) imply $p_1 - A = \theta s_0$, which in turn implies that the first-period price and dissipative advertising are perfect substitutes. Thus, the net price, $p_1 - A$, of the high-quality firm is the same as the low-quality firm’s price. In contrast, the common first-period price for both types of the firm in the pooling equilibrium is given by

$$p_1^p = \mu \theta s_1 + (1 - \mu) \theta s_0 > \theta s_0.$$  

\(^8\) Since technology is not adjustable, we can identify the type of the firm with the quality type of its product.
Thus, the high-quality firm’s total profit across the two periods in the pooling equilibrium is equal to

\[ p_1^u - c_1 + \delta s_1 - c_1. \]

Since \( s_0 < s_1 \), it follows that the pooling equilibrium is more profitable than the separating equilibrium with price and dissipative advertising as the signal of high quality (see also Tirole, 1988, p. 120).

### 2.2 Signaling Quality by Inducing Shortage

Suppose that the high-quality firm limits its supply so that only a fraction \( \alpha \in (0, 1) \) of consumers can get the product during the first period. With the limited supply, the high-quality firm may reveal its type to the consumers by properly setting the price, so as to leave it undesirable for the low-quality firm to mimic. In the separating equilibrium, the low-quality firm does not induce any shortage.

By introducing shortages, those who purchase the product in the first period obtain information about the quality and can make their second-period purchase decisions conditional on that information. However, the consumers who do not purchase the product in period 1 will base their period 2 decisions upon beliefs updated by signals they observe. This is guaranteed by the assumption that there is no communication between the consumers.

One of the most frequently applied equilibrium concepts for signaling games is perfect Bayesian equilibrium (PBE for short). This equilibrium concept involves a belief system for the uninformed, which in our setting is a collection \( \mu \) of probability distributions \( \mu(\cdot | p, z) \) over types 1 and 0 conditional on each price–shortage pair \( (p, \alpha) \) the firm may choose. Here, \( \mu(t | p, \alpha) \) is the probability with which consumers believe that the good has quality \( t \) upon observing the price–shortage pair \( (p, \alpha) \).

The PBE is often used to characterize the separating equilibrium. We will apply PBE for equilibrium refinement. We are interested in the most profitable separating equilibrium for the high-quality firm. Such equilibria are known as least-cost separating equilibria (e.g., Bagwell, 2007, p. 1801).

A price–shortage combination \( (p^* \alpha^*) \), with the high-quality firm’s period 1 price \( p^* \) and shortage \( 1 - \alpha^* \), is a least-cost separating equilibrium if and only if it solves

\[
\max_{0 \leq \alpha \leq 1, \beta \leq \beta_1} \alpha(p_1 - c_1) + \delta(\beta s_1 - c_1)
\]

subject to

\[
\alpha(p_1 - c_1) \geq \beta s_0 - c_1
\]

and

\[
\alpha(p_1 - c_1) + \delta \max\{\beta s_0 - c_0, (1 - \alpha)(\beta s_1 - c_0)\} \leq (1 + \delta)(\beta s_0 - c_0).
\]

Notice that in the pooling equilibrium, consumers learn the quality in the second period by purchasing the product in the first period. Thus both types can charge their complete-information monopoly prices.
The condition (5) is equivalent to the incentive constraint
\[
\alpha(p_1 - c_1) + \delta(\theta s_1 - c_1) \geq \theta s_0 - c_1 + \delta(\theta s_1 - c_1).
\]

This constraint guarantees that the high-quality firm does not have any incentive to mimic the low-quality firm’s period 1 choice \((\theta s_0, 0)\). The condition (6) is the incentive constraint that makes it undesirable for the low-quality firm to mimic the high-quality firm’s period 1 choice \((p_1, \alpha)\) and then, in period 2, either to supply the entire population of consumers at price \(\theta s_0\), or to supply only the fraction \(1 - \alpha\) who did not consume the product in period 1 by continuing to mimic the high-quality firm’s period 2 choice of charging price \(\theta s_1\).

**Lemma 1** Let \((p_1, \alpha)\) be the period 1 choice of the high-quality firm in a least-cost separating equilibrium. Then the constraint (6) is binding at \((p_1, \alpha)\).

Intuitively, when (6) is nonbinding, the high-quality firm can slightly reduce the shortage without changing its price or violating (5). Keeping price constant, the high-quality firm’s profit increases as shortage reduces. It follows that the separating equilibrium cannot be the least-cost unless (6) is binding. We now apply Lemma 1 to show that there is a unique solution for the problem (4) with positive shortage under the following conditions:

\[
\begin{align*}
\delta(\theta s_1 - c_1) &> c_1 - c_0, \\
\theta(s_1 - s_0) &> \theta s_0 - c_0, \\
\theta s_k &> c_k, \quad k = 0, 1.
\end{align*}
\]

The first inequality requires that the discounting factor be large enough relative to the ratio of the cost differential \(c_1 - c_0\) to the profit the low-quality firm gets when it is perceived as the high-quality firm. With the second inequality, the consumers’ value differential due to quality difference exceeds the per-unit profit the low-quality firm gets under complete information. The third inequality is self-explanatory.

**Proposition 1** Assume (C1). Then, when the high-quality firm signals its quality type by a price–shortage combination, there is a unique least-cost separating equilibrium in which

\[
\alpha^* = \frac{\delta(\theta s_1 - s_0)}{\theta s_1 - c_0} \quad \text{and} \quad p_1^* = c_0 + \frac{(\theta s_1 - c_0)(\theta s_0 - c_0)}{\theta s_1 - s_0}.
\]

Note that \(p_1^* < \theta s_1\) if and only if \((\theta s_0 - c_0)/\theta s_1 - s_0) < 1\), which is guaranteed by the second inequality in (C1). It follows that in the least-cost separating equilibrium, the high-quality firm charges a period 1 price that is below its complete-information monopoly price. Note also that the high-quality firm’s total profit across the two periods in the least-cost separating equilibrium is

\[
\alpha^*(p_1^* - c_1) + \delta(\theta s_1 - c_1) = \theta s_0 - c_0 + \delta(\theta s_1 - c_1) - \frac{\theta(s_1 - s_0)}{\theta s_1 - c_0}(c_1 - c_0).
\]

\(\text{When mimicking in period 2, the low-quality firm can only mislead consumers who did not buy in period 1.}\)
By (3) and (7), the high-quality firm is better off signaling its quality type via the price–shortage combination in Proposition 1 than pooling if and only if
\[ p^h - c_1 < \alpha^*(p^* - c_1). \]
This is equivalent to
\[ \mu < \frac{(\theta s_0 - c_0)(c_1 - c_0)}{\theta (s_1 - s_0)(\theta s_1 - c_0)}. \]

For later reference, we summarize this result in the following proposition, whose proof will be omitted.

**Proposition 2** Assume (C1). Then, signaling quality by price–shortage combination is more profitable than pooling, and hence more profitable than signaling quality by price–advertising combination, if \( \mu \) satisfies (8).

Proposition 2 establishes a sharp comparison between the signaling roles of price–advertising and price–shortage combinations. The key reason behind the result is that the cost in forgone profits to the high-quality firm from inducing shortage is always lower than that of the low-quality firm.

3 Signaling with Heterogeneous Consumers and Indivisible Goods

We now consider an extension of the model in section 2 by allowing for heterogeneous consumers when the good is indivisible. We follow Wolinsky (1983), Chan and Leland (1982), Cooper and Ross (1984, 1985), Farrell (1980), and Tirole (1988), among others, to consider unit-demand differentiated consumers. For simplicity, we assume that there are two types of consumers in terms of their tastes for quality. Specifically, the value of the product with quality level \( s_k \) is \( \theta_k s_k \) for type \( k \) consumers and \( \theta_k s_k \) for type 0 consumers, where \( \theta_k \) represents type \( k \) consumers’ taste for quality. Assume \( \theta_1 > \theta_0 \). The proportion of type 1 consumers is denoted by \( q_1 \).

As in the case with homogeneous consumers only, price and advertising are substitutes for the high-quality firm. But, unlike in the homogeneous case, we show that under certain conditions on the parameters of the model, the high-quality firm is better off signaling its quality type via price than not signaling at all. That is, the Tirole-type quality-signaling dilemma with homogeneous consumers is eliminated in the presence of heterogeneous consumers.

In the rest of this section, we characterize the least-cost separating equilibria when the high-quality firm signals its quality via a price–advertising or price–shortage combination. We compare them in section 4. First, due to consumer heterogeneity, we need the following assumptions:

(C1a) \[ \delta(\theta s_k - c_k) > c_1 - c_0, \quad \theta_k s_k > c_k, \quad k = 0, 1. \]
(C2) \[ \frac{\theta_k s_k - c_1}{\theta_k s_k - c_1} < \frac{\theta_k s_k - c_0}{\theta_k s_k - c_1} < q_1 < \min \left\{ \frac{\theta_k s_k - c_k}{\theta_k s_k - c_0}, 1 - \frac{\theta_k s_k - c_k}{\theta_k s_k - c_1} \right\}. \]
The first two inequalities in (C2) guarantee that when information is complete, the proportion of the high-taste consumers is large enough for the high-quality firm to target high-taste consumers only. Furthermore, if the low-quality firm is perceived by high-taste consumers as the high-quality firm when it charges period 1 price \( \theta_s s_1 \), then the low-quality firm is more profitable than it can be with its complete-information monopoly price \( \theta_s s_0 \) and targeting all consumers. These conditions make signaling nontrivial. In contrast, the last inequality in (C2) with the first component on the right-hand side implies that the proportion of the high-taste consumers is small enough so that it is more profitable for the low-quality firm to target all consumers when information is complete. The first inequality, together with \( q_1 (\theta_s s_1 - c_1) \leq 2(\theta_s s_0 - c_0) \), also implies that the high-quality firm targets high-taste consumers with period 1 price equal to \( \theta_s s_1 \) in least-cost separating equilibrium. On the other hand, the last inequality with the second component on the right, together with \( q_1 (\theta_s s_1 - c_1) > 2(\theta_s s_0 - c_0) \), implies that the high-quality firm targets high-taste consumers with period 1 price strictly below \( \theta_s s_1 \) in least-cost separating equilibrium.

3.1 Signaling Quality by Price and Advertising

Let \( D_h(p_1) \) denote the complete-information aggregate demand for the high-quality product in period 1. Then \( D_h(p_1) = 1 \) if \( p_1 \leq \theta_s s_1 \), and \( D_h(p_1) = q_1 \) if \( p_1 \in (\theta_s s_1, \theta_s s_1, \theta_s s_1) \]. When the high-quality firm uses a price–advertising combination to signal its quality, the incentive-compatibility constraints for a separating equilibrium become

(9) \( D_h(p_1) (p_1 - c_1) - A \geq \theta_s s_0 - c_1 \)

and

(10) \( D_h(p_1) (p_1 - c_0) - A \leq (\theta_s s_0 - c_0) \).

The conditions (9) and (10) guarantee that neither type has any incentive to mimic the other type’s choice. Note also that these constraints are consistent, in the sense that there are prices and advertising expenses that simultaneously satisfy both of them.\(^{11}\) In a least-cost separating equilibrium, the high-quality firm’s price solves

(11) \[ \max_{(p_1, q_1) \in \mathbb{R}^2 : p_1 \leq \theta_s s_1} D_h(p_1) (p_1 - c_1) - A + \delta (\theta_s s_1 - c_1) \]

subject to the constraints (9) and (10).

The following proposition characterizes the least-cost separating equilibria.

**Proposition 3** Assume (C1a) and (C2). Then, when the high-quality firm signals its quality type by a price–advertising combination, the least-cost separating equilibria are characterized by

\[ q_1 p_1^* - A^* = q_1 c_0 + \theta_s s_0 - c_0, \quad \theta_s s_1 < p_1^* \leq \theta_s s_1, \quad A^* \geq 0. \]

\(^{11}\) Let \( p_1 > \theta_s s_1 \). The condition (9) is equivalent to \( p_1 \geq (\theta_s s_0 - c_1 + A)/q_1 + c_1 \), and (10) is equivalent to \( p_1 \leq (\theta_s s_0 - c_0 + A)/q_1 + c_0 \). Thus, (9) and (10) are consistent if and only if \( q_1 (c_1 - c_0) \leq (c_1 - c_0) \), which is automatic.
Proposition 3 establishes the existence of a class of least-cost separating equilibria with price–advertising combination as the signal of high quality. The period 1 price of the high-quality firm increases with the advertising cost. Nonetheless, all of the equilibria yield the same profit for the high-quality firm as the one with zero advertising cost. This equivalence is due to the fact that price and advertising are perfect substitutes. The reason that signaling with price–advertising combination becomes more profitable than pooling is that the high-quality firm can charge a price to target high-taste consumers only, which leaves it less profitable for the low-quality firm to mimic.

The following example provides an illustration of Proposition 3. Both (C1a) and (C2) are satisfied in this example.

Example 1: Let $c_1 = 0.75$, $c_0 = 0$, $\theta_1 = 2$, $\theta_0 = 1$, $s_1 = 1$, $s_0 = 0.5$, $\delta = 1$. Then, simple calculation shows that both (C1a) and (C2) are satisfied whenever $q_1 \in (0.25, 0.5)$. With zero advertising cost, the least-cost separating-equilibrium price of the high-quality firm monotonically decreases with respect to the proportion of high-taste consumers. This example is adapted from Exercise 2.7 in Tirole (1988, p. 121), where a separating equilibrium with price as the signal of quality is established, under the requirement that no one can purchase the product in period 2 unless he has purchased in period 1.

3.2 Signaling High Quality by Inducing Shortage

We now turn to the least-cost separating equilibrium in which the high-quality firm signals quality by properly setting the price and inducing shortage. Since a consumer cannot learn the quality of the product without consuming it, by mimicking the high-quality firm’s period 1 and period 2 choices, the low-quality firm can mislead those consumers who did not buy the product in period 1. Thus, the incentive-compatibility constraints for a separating equilibrium become

$$\alpha D_s(p_1)(p_1 - c_1) \geq \theta s_0 - c_1$$
and
$$\alpha D_s(p_1)(p_1 - c_0) + \delta \max\{\theta s_0 - c_0, (1 - \alpha)q_1(\theta s_1 - c_0)\} \leq (1 + \delta)(\theta s_0 - c_0).$$

The constraint (12) implies that the high-quality firm does not have incentive to mimic the low-quality firm’s choice, while (13) ensures that the low-quality firm does not have incentive to mimic the choice of the high-quality firm.\(^\text{12}\)

In a least-cost separating equilibrium, the high-quality firm’s period 1 choice $(p_1, \alpha)$ solves

$$\max_{\theta s_0 \geq 1, p_1 \geq \theta s_1} \alpha D_s(p_1)(p_1 - c_1) + \delta q_1(\theta s_1 - c_1)$$
subject to the constraints (12) and (13).

\(^{12}\) Notice that due to (C2), the high-quality firm sets its period 2 price equal to $\theta s_1$ in separating equilibrium.
As in the previous case, the incentive-compatibility constraint (13) is binding in any least-cost separating equilibrium. We summarize this result in the following lemma. Its proof is similar to the proof of Lemma 1. For this reason, the proof is omitted.

**Lemma 2** Let \((p_1, \alpha)\) be the high-quality firm's period 1 choice in a least-cost separating equilibrium. Then, (13) must be binding at \((p_1, \alpha)\).

We now characterize the unique least-cost separating equilibrium separately for two mutually disjoint and jointly exhaustive subranges of \((C_2)\).

**Proposition 4** Assume \((C_1a)\) and \((C_2)\). Then, when the high-quality firm signals its quality type by a price–shortage combination, there exists a unique least-cost separating equilibrium with

\[
\begin{align*}
\bar{\alpha}^* &= 1 - \frac{\theta_0 s_0 - c_0}{q_1(\theta_1 s_1 - c_0)}, \\
\bar{p}_1^* &= c_0 + \frac{(\theta_1 s_1 - c_0)(\theta_0 s_0 - c_0)}{q_1(\theta_1 s_1 - c_0) - (\theta_0 s_0 - c_0)}
\end{align*}
\]

if \(q_1(\theta_1 s_1 - c_1) > 2(\theta_0 s_0 - c_0)\), and

\[
\begin{align*}
\bar{\alpha}^* &= \frac{\theta_0 s_0 - c_0}{q_1(\theta_1 s_1 - c_0)}, \\
\bar{p}_1^* &= \theta_1 s_1
\end{align*}
\]

if \(q_1(\theta_1 s_1 - c_1) \leq 2(\theta_0 s_0 - c_0)\).

When \(q_1(\theta_1 s_1 - c_1) > 2(\theta_0 s_0 - c_0)\), both the equilibrium price \(\bar{p}_1^*\) and the shortage \(1 - \bar{\alpha}^*\) decrease as the fraction \(q_1\) of the high-taste consumers increases. Simple calculation shows \(\theta_0 s_1 < \bar{p}_1^* < \theta_1 s_1\). This means that the high-quality firm targets the high-stage consumers with a price below its complete-information monopoly price. In the homogeneous case, \(q_1 = 1\) and \(\theta_1 = \theta_0 = \theta\). Thus, with homogeneity,

\[
\begin{align*}
\bar{\alpha}^* &= 1 - \frac{\theta s_0 - c_0}{q(\theta s_1 - c_0)}, \\
\bar{p}_1^* &= \frac{\theta(s_1 - s_0)}{\theta_1 s_1 - c_0} = \alpha^* \quad \text{and} \quad \bar{p}_1^* = p_1^*.
\end{align*}
\]

When \(q_1(\theta_1 s_1 - c_1) \leq 2(\theta_0 s_0 - c_0)\), the proportion of high-taste consumers is not large enough for the high-quality firm to lower the price from its complete-information monopoly price level and induce shortage at the same time. Proposition 4 shows that in that case, the complete-information monopoly price together with a certain amount of shortage can signal high quality.

For the same parameter values of \(\theta_1, \theta_0, s_1, s_0, c_1, c_0, \delta\) in Example 1, \(q_1(\theta_1 s_1 - c_0) \leq 2(\theta_0 s_0 - c_0)\) for all \(q_1 \in (0.25, 0.5)\). Under these parameter values, Proposition 4 shows that \((\bar{p}_1^*, \bar{\alpha}^*)\) with \(\bar{p}_1^* = 2\) and \(\bar{\alpha}^* = 1/(4q_1)\) is the least-cost separating equilibrium for each \(q_1 \in (0.25, 0.5)\).

**3.3 Signal Comparisons**

Our results in the previous section show that under \((C_1a)\) and \((C_2)\), both price–advertising and price–shortage combinations can be signals of high quality. Thus, as with the homogeneous case, it is natural to ask which signal is more profitable for the high-quality firm. The next proposition provides an answer.
Proposition 5 Assume (C1a) and (C2). Then, signaling quality by price–shortage combination is more profitable than using price–advertising combination.

In terms of the forgone profit, the high-quality firm has a cost advantage to induce shortage, due to the cost differential in production. In contrast, dissipative advertising is equally costly for both types. The cost advantage associated with inducing shortage makes it more profitable to signal quality via a price–shortage combination than using price–advertising combination, whenever the former is effective. We end this section with an illustration of Proposition 5.

Example 2: Consider the same parameter values of \( \theta_1, \theta_0, s_1, s_0, c_1, c_0, \delta \) as in Example 1. As noticed before, (C1a), (C2), and \( q_1(\theta_1s_1-c_0) \leq 2(\theta_0s_0-c_0) \) are satisfied for all \( q_1 \in (0.25, 0.5) \). By Proposition 3 and Proposition 4, with \( q_1 = 1/3 \):

(i) \( \hat{\alpha}^* = 3/4 \) and \( \hat{\beta}^* = 2 \) when signaling by price–shortage combination, implying that the high-quality firm’s period 1 profit in the least-cost separating equilibrium is

\[
\hat{\alpha}^* q_1(\hat{\beta}^*_1 - c_1) = \frac{5}{16}.
\]

(ii) \( A^* = 0 \Rightarrow p^*_1 = 1.5 \) when signaling by price–advertising combination, implying that the high-quality firm’s period 1 profit in the least-cost separating equilibrium is

\[
q_1(p^*_1 - c_1) = \frac{1}{4}.
\]

From the preceding equilibrium profits it follows that signaling quality by price–shortage combination is more profitable for the high-quality firm than signaling quality by price–advertising combination.

4 Conclusion

In this paper, we have analyzed the possibility for a monopoly firm introducing a new product to signal quality by properly setting the price and inducing shortage. For a stylized two-period model of a monopoly firm providing a new indivisible product with two possible unknown quality levels to homogeneous consumers, we have shown that for the high-quality firm, it is more profitable to signal quality using price–shortage combination than by pooling. Consequently, signaling via price–shortage combination eliminates the negative result associated with signaling quality by price or price–advertising combination.

We have generalized the results to allow for heterogeneous consumers. Depending on the parameter values, the high-quality firm sets its price in the introductory phase (weakly) below its complete-information monopoly price, with a seller-induced shortage. With some additional conditions, seller-induced shortage is a more profitable signal of high quality than dissipative advertising. This is because
the signaling cost in terms of profits forgone by the high-quality firm by inducing shortage is always lower than that of the low-quality firm at any given quantity. Thus, seller-induced shortage is a better (less costly) strategy for the high-quality firm than dissipative advertising or high price alone per Milgrom and Roberts (1986). Furthermore, signaling high quality by inducing shortage also results in penetration pricing for the high-quality type. Our results provide a rationale for penetration pricing as well as for the presence of shortage in the introductory phase of a product provided by a monopoly firm.

Appendix

A.1 Proof of Lemma 1
Let \((p_1, \alpha)\) be a price–shortage combination such that

(A1) \(\alpha(p_1 - c_0) + \delta \max(\theta s_0 - c_0, (1 - \alpha)(\theta s_1 - c_0)) < (1 + \delta)(\theta s_0 - c_0)\).

Suppose first \(\theta s_0 - c_0 \geq (1 - \alpha)(\theta s_1 - c_0)\). In this case, if \(p_1 < \theta s_1\), then there exists a price \(p'_1 > p_1\) such that \((p'_1, \alpha)\) also satisfies (A1). Since \(\alpha(p'_1 - c_0) > \alpha(p_1 - c_0)\), \((p'_1, \alpha)\) satisfies (5) whenever \((p_1, \alpha)\) does. This shows that \((p_1, \alpha)\) cannot solve the problem (4). If \(p_1 = \theta s_1\), then (A1) reduces to \(\alpha(\theta s_1 - c_0) < \theta s_0 - c_0\). Hence, \(\alpha < 1\) because \(\theta s_1 - c_0 > \theta s_0 - c_0\). It follows that, by slightly increasing \(\alpha\) to \(\alpha' > \alpha\), we can guarantee that \((p_1, \alpha')\) also satisfies (A1).\(^{13}\) Since \(\alpha'(p_1 - c_1) > \alpha(p_1 - c_1)\), \((p_1, \alpha)\) cannot solve the problem (4).

Suppose now \(\theta s_0 - c_0 < (1 - \alpha)(\theta s_1 - c_0)\), which implies \(\alpha \neq 1\). Thus, as before, we can increase the maximum value of the problem (4) by keeping the price \(p_1\) while slightly increasing \(\alpha\) without violating (5) and (6).

Q.E.D.

A.2 Proof of Proposition 1
Let \((p_1, \alpha)\) be a solution for the problem (4). We break the rest of the proof into two cases.

Case 1: \(\alpha \geq \theta(s_1 - s_0)/(\theta s_1 - c_0)\).

In this case, we have \(\theta s_0 - c_0 \geq (1 - \alpha)(\theta s_1 - c_0)\).\(^{14}\) Thus, by Lemma 1, \(\alpha(p_1 - c_0) = (\theta s_0 - c_0)\), which implies \(\alpha p_1 = \alpha c_0 + (\theta s_0 - c_0)\). It follows that \(\alpha(p_1 - c_1) = \theta s_0 - c_0 - \alpha(c_1 - c_0)\) is decreasing in \(\alpha\).

\(^{13}\) Note that \(\theta s_0 - c_0 \geq (1 - \alpha')(\theta s_1 - c_0)\) implies \(\theta s_0 - c_0 > (1 - \alpha')(\theta s_1 - c_0)\) for all \(\alpha' > \alpha\).

\(^{14}\) Note that \(\theta s_1 - c_0 \geq (1 - \alpha)(\theta s_1 - c_0)\) if and only if \(\alpha(\theta s_1 - c_0) \geq \theta(s_1 - s_0)\) or equivalently \(\alpha \geq \theta(s_1 - s_0)/(\theta s_1 - c_0)\).
Case 2: \( \alpha \leq \theta(s_1 - s_0)/(\theta s_1 - c_0) \).

In this case, we have \( \theta s_0 - c_0 \leq (1 - \alpha)(\theta s_1 - c_0) \). By Lemma 1, \( \alpha(p_1 - c_0) + \delta(1 - \alpha)(\theta s_1 - c_0) = (1 + \delta)(\theta s_0 - c_0) \). It follows that
\[
\alpha(p_1 - c_1) = (1 + \delta)(\theta s_0 - c_0) - \delta(\theta s_1 - c_0) + \delta\alpha(\theta s_1 - c_0) - \alpha(c_1 - c_0).
\]

By (C1), \( \alpha(p_1 - c_1) \) is increasing in \( \alpha \).

In summary, we have shown that \( \alpha(p_1 - c_1) \) is decreasing in \( \alpha \) when \( \alpha > \theta(s_1 - s_0)/(\theta s_1 - c_0) \) and increasing in \( \alpha \) when \( \alpha < \theta(s_1 - s_0)/(\theta s_1 - c_0) \). We can conclude that to solve (4) it must be that
\[
\alpha = \frac{\theta(s_1 - s_0)}{\theta s_1 - c_0}.
\]

By the binding incentive constraint on the low-quality firm, it must also be that
\[
p_1 = c_0 + \frac{\theta s_0 - c_0}{\alpha} = c_0 + \frac{(\theta s_0 - c_0)(\theta s_1 - c_0)}{\theta(s_1 - s_0)}.
\]

This establishes both the uniqueness and the characterization of the solution for the problem (4).

To show the existence, note first that by (C1), the price–shortage combination \((p^*, \alpha^*)\) in the proposition satisfies \( 0 < \alpha^* < 1 \) and \( p_1^* < \theta s_1 \). Thus, to complete the rest of the proof, it suffices to show that \((p^*, \alpha^*)\) also satisfies (5). To this end, note that
\[
\alpha^*(p^* - c_1) \geq (\theta s_0 - c_1) \quad \text{if and only if} \quad \theta(s_1 - s_0) \leq \theta s_1 - c_0.
\]

Since \( \theta s_0 > c_0 \), the above condition automatically follows from (C1). \( \quad Q.E.D. \)

A.3 Proof of Proposition 3

Let \((p_1, A)\) satisfy (9), (10), and the condition \( p_1 > \theta_0 s_1 \). Then, \( D_s(p_1) = q_1 \), \( q_1 p_1 - A \geq q_1 c_1 + \theta_0 s_0 - c_1 \), and \( q_1 p_1 - A \leq q_1 c_0 + \theta_0 s_0 - c_0 \). Notice also that the value of the objective function in (11) at \( p_1 > \theta_0 s_1 \) is \( q_1 p_1 - A = q_1 c_0 + \theta_0 s_0 - c_0 \). It follows that for a price–advertising combination \((p_1, A)\) with \( \theta_0 s_1 < p_1 \leq \theta_1 s_1 \), we have
\[
q_1 p_1 - A = q_1 c_0 + \theta_0 s_0 - c_0.
\]

At such a combination, the first-period profit of the high-quality firm is
\[
q_1(p_1 - c_1) - A = \theta_0 s_0 - c_0 - q_1(c_1 - c_0).
\]

In contrast, for any combination \((p_1, A)\) satisfying (9), (10), and \( p_1 \leq \theta_0 s_1 \), we have \( D_s(p) = 1 \), \( p_1 - A \geq \theta_0 s_0 \), and \( p_1 - A \leq \theta_0 s_0 \). It follows that \( p_1 - A = \theta_0 s_0 \). The first-period profit of the high-quality firm at such a combination is
\[
(p_1 - c_1) - A = p_1 - A - c_1 = \theta_0 s_0 - c_1. \]

Notice that \( \theta_0 s_0 - c_0 - q_1(c_1 - c_0) > \theta_0 s_0 - c_1 \) if and only if \( q_1 < 1 \),

\[\text{Such a combination satisfies (9).}\]
which is automatic. We can conclude that solutions of (11) are characterized by
\( \theta_{s_1} < p_i \leq \theta_{s_1} \) and (A2). Thus, the proof is completed if it can be shown that a
combination \((p_1, A)\) satisfying (A2) with \( \theta_{s_1} < p_i \leq \theta_{s_1} \) and \( A \geq 0 \) exists. To this
end, observe that the largest value of the price \( p_1 \) in the combinations satisfying those conditions is
\( p_1 = c_0 + \frac{(\theta_{s_0} - c_0)}{q_1} \). Since \( \theta_{s_1} < c_0 + \frac{(\theta_{s_0} - c_0)}{q_1} \) if and only if
\[
\frac{\theta_{s_0} - c_0}{\theta_{s_1} - c_0} \leq q_1 < \frac{\theta_{s_0} - c_0}{\theta_{s_1} - c_0},
\]
it follows from (C2) that (A2) has a solution with \( \theta_{s_1} < p_i \leq \theta_{s_1} \) and \( A \geq 0 \).

Q.E.D.

A.4 Proof of Proposition 4

Suppose first that \( q_1 (\theta_{s_1} - c_2) > 2(\theta_{s_0} - c_0) \). By Lemma 2, (13) is binding in any
least-cost separating equilibrium:

(A3) \[ \alpha D_0(p_1)(p_1 - c_0) + \delta \max \{ \theta_{s_0} - c_0, (1 - \alpha)q_1 (\theta_{s_1} - c_0) \} = (1 + \delta)(\theta_{s_0} - c_0). \]

Let \((p_1, \alpha)\) satisfy (A3) and \( p_1 > \theta_{s_1} \). Then, \( D_0(p_1) = q_1 \).

Case 1: \( \alpha \geq 1 - \frac{\theta_{s_1} - c_0}{q_1 (\theta_{s_1} - c_0)} \). In this case, \( \theta_{s_0} - c_0 \geq (1 - \alpha)q_1 (\theta_{s_1} - c_0) \),

Thus, by (A3), \( \alpha q_1 (p_1 - c_0) = \theta_{s_0} - c_0 \), which is equivalent to \( p_1 = c_0 + \frac{(\theta_{s_0} - c_0)}{\alpha q_1} \).

It follows that the high-quality firm’s period 1 profit \( \alpha q_1 (p_1 - c_1) = \theta_{s_0} - c_0 - \alpha q_1 (c_1 - c_0) \) is decreasing in \( \alpha \).

Case 2: \( \alpha \leq 1 - \frac{\theta_{s_1} - c_0}{q_1 (\theta_{s_1} - c_0)} \). In this case, \( \theta_{s_0} - c_0 \leq (1 - \alpha)q_1 (\theta_{s_1} - c_0) \).

Thus, by (A3), \( \alpha q_1 (p_1 - c_0) + \delta (1 - \alpha)q_1 (\theta_{s_1} - c_0) = (1 + \delta)(\theta_{s_0} - c_0) \). Consequently, the high-quality firm’s period 1 profit is

\[ \alpha q_1 (p_1 - c_1) = (1 + \delta)(\theta_{s_0} - c_0) - \delta q_1 (\theta_{s_1} - c_0) + \alpha q_1[\delta (\theta_{s_1} - c_0) - (c_1 - c_0)]. \]

By (C1a), \( \delta (\theta_{s_1} - c_0) > c_1 - c_0 \) and \( \theta_{s_1} > c_1 \). It follows that \( \alpha q_1 (p_1 - c_1) \) is increasing
in \( \alpha \).

Thus, case 1 and case 2 together with (A3) imply that in any least-cost separating
equilibrium, \( p_i > \theta_{s_1} \) implies

(A4) \[ \alpha = 1 - \frac{\theta_{s_0} - c_0}{q_1 (\theta_{s_1} - c_0)} \]

and

(A5) \[ p_1 = c_0 + \frac{(\theta_{s_1} - c_0)(\theta_{s_0} - c_0)}{q_1 (\theta_{s_1} - c_0) - (\theta_{s_0} - c_0)} \]

By (C2), the price \( p_i \) in (A5) satisfies \( \theta_{s_1} < p_i \leq \theta_{s_1} \) if and only if

\[ q_1 < \frac{\theta_{s_0} - c_0}{\theta_{s_1} - c_0} + \frac{\theta_{s_0} - c_0}{\theta_{s_1} - c_0}. \]
which is guaranteed by (C2). Since \( q_1(\theta_{s1} - c_0) > 2(\theta_{s0} - c_0) \), \( p_i \) in (A5) also satisfies \( p_i \leq \theta_{s1} \), implying that \( p_i \) is feasible. Thus, to show that \( (p_i, \alpha) \) in (A4) and (A5) also satisfies (12), note that \( \alpha q_1(p_i - c_1) \geq \theta_{s0} - c_1 \) if and only if

\[
q_1 \leq 1 + \frac{\theta_{s0} - c_0}{\theta_{s1} - c_0}
\]

which is clearly satisfied.

To complete the rest of the proof under the supposition that \( q_1(\theta_{s1} - c_0) > 2(\theta_{s0} - c_0) \), it only remains to show that no price below or equal to \( \theta_{s1} \) can be the high-quality firm’s period 1 price in least-cost equilibrium. To this end, note first that (A5) implies that \( \alpha q_1(p_i - c_1) \geq \theta_{s0} - c_1 \) if and only if

\[
q_1 < 1 - \frac{\theta_{s0} - c_0}{\theta_{s1} - c_0} + \frac{\theta_{s0} - c_0}{\theta_{s1} - c_0}.
\]

By (C2), the preceding condition is satisfied. Since the high-quality firm’s period 1 profit is no more than \( \theta_{s1} - c_1 \) with a period 1 price \( p_i \leq \theta_{s1} \), the high-quality firm’s period 1 price in least-cost equilibrium must exceed \( \theta_{s1} \).

Suppose now that \( q_1(\theta_{s1} - c_0) \leq 2(\theta_{s0} - c_0) \). Let \( (p_i, A) \) satisfy (A3) with \( p_i > \theta_{s1} \). Consider first \( \theta_{s0} - c_1 < (1 - \alpha)q_1(\theta_{s1} - c_0) \). This together with (A3) implies

(A6) \( \alpha < 1 - \frac{\theta_{s0} - c_0}{q_1(\theta_{s1} - c_0)} \), \( p_i(\alpha) = c_0 + \frac{(1 + \delta)(\theta_{s0} - c_0) - \delta(1 - \alpha)q_1(\theta_{s1} - c_0)}{\alpha q_1} \).

Note that

\[
\frac{dp_i(\alpha)}{d\alpha} = \frac{\delta q_1(\theta_{s1} - c_0) - (1 + \delta)(\theta_{s0} - c_0)}{\alpha^2 q_1}.
\]

It follows that

\[
\frac{dp_i(\alpha)}{d\alpha} < 0 \quad \text{if and only if} \quad q_1 < \left( 1 + \frac{\delta}{\theta_{s1} - c_0} \right) \left( \frac{\theta_{s0} - c_0}{\theta_{s1} - c_0} \right).
\]

Thus, \( dp_i(\alpha)/d\alpha < 0 \) for all \( q_1 \) satisfying \( q_1(\theta_{s1} - c_0) \leq 2(\theta_{s0} - c_0) \).\(^{16}\) By (C2),

\[
\alpha = 1 - \frac{\theta_{s0} - c_0}{q_1(\theta_{s1} - c_0)} \Rightarrow p_i(\alpha) = c_0 + \frac{\theta_{s0} - c_0}{\alpha q_1} > \theta_{s1}.
\]

Thus, \( p_i(\alpha) \) in (A6) is not feasible in the sense that it exceeds \( \theta_{s1} \) for all \( 0 < \alpha < \frac{1 - (\theta_{s0} - c_0)}{q_1(\theta_{s1} - c_0)} \).

Consider now \( \theta_{s0} - c_0 \geq (1 - \alpha)q_1(\theta_{s1} - c_0) \). This together with (A3) implies

(A7) \( \alpha \geq 1 - \frac{\theta_{s0} - c_0}{q_1(\theta_{s1} - c_0)} \) and \( p_i(\alpha) = c_0 + \frac{\theta_{s0} - c_0}{\alpha q_1} \).

\(^{16}\) For \( \delta \in (0, 1) \), \((1 + \delta)/\delta > 2 \).
In this case, the high-quality firm’s total profit is decreasing in $\alpha$ over the range $1 - \frac{\theta_s s_0 - c_o}{q_1(\theta_s s_1 - c_o)} \leq \alpha < 1$, because

$$q_1[\alpha (p_1 - c_1) + \delta(\theta_s s_1 - c_1)]' = -q_1(c_1 - c_o) < 0.$$ 

On the other hand,

$$\alpha = 1 - \frac{\theta_s s_0 - c_o}{q_1(\theta_s s_1 - c_o)} \Rightarrow p_1 = c_o + \frac{\theta_s s_0 - c_o}{q_1 - \frac{\theta_s s_0 - c_o}{\theta_s s_1 - c_o}} > \theta_s s_1.$$ 

Thus, by (A7), the smallest $\alpha$ that results in a feasible price is

(A8) $\alpha = \frac{\theta_s s_0 - c_o}{q_1(\theta_s s_1 - c_o)} \Rightarrow p_1(\alpha) = \theta_s s_1.$

With $p_1$ and $\alpha$ in (A8), $(p_1, \alpha)$ satisfies the incentive constraint (12) for the high-quality firm if and only if

$$\frac{\theta_s s_0 - c_o}{\theta_s s_1 - c_o} > \frac{\theta_s s_0 - c_o}{\theta_s s_1 - c_o}.$$ 

The preceding inequality automatically holds because $c_1 > c_0$.\textsuperscript{17}

By (A8), $\alpha q_1(p_1 - c_1) > \theta_s s_1 - c_o$ if and only if

$$\frac{\theta_s s_1 - c_o}{\theta_s s_1 - c_o} < \frac{\theta_s s_0 - c_o}{\theta_s s_1 - c_o},$$

which is guaranteed by (C2). Since the high-quality firm’s period 1 profit is no more than $\theta_s s_1 - c_o$ with a period 1 price $p_1 \leq \theta_s s_1$, the preceding analysis shows that the high-quality firm’s period 1 price in least-cost equilibrium must exceed $\theta_s s_1$. Q.E.D.

### A.5 Proof of Proposition 5

Since the second-period profit of the high-quality firm is the same in separating equilibrium with either price–advertising or price–shortage combination as the signal, we only need to show that the high-quality firm’s first-period profit is higher with price–shortage combination as the signal.

By Proposition 3, the high-quality firm’s first-period separating-equilibrium profit with price–advertising combination as the signal is

(A9) $q_1(p_1^* - c_1) - A^* = \theta_s s_0 - c_o - q_1(c_1 - c_o).$

Suppose first $q_1(\theta_s s_1 - c_1) > 2(\theta_s s_0 - c_o)$. In this case, Proposition 4 implies that the high-quality firm’s first-period separating-equilibrium profit with price–shortage combination as the signal is

(A10) $\tilde{\alpha}^* q_1(\tilde{\alpha}^* s_1 - c_1) = \theta_s s_0 - c_o - q_1(c_1 - c_o) + \frac{(\theta_s s_0 - c_o)(c_1 - c_o)}{\theta_s s_1 - c_o}$.

\textsuperscript{17} Given two constants $a$ and $b$, the ratio $(a - x)/(b - x)$ is decreasing in $x$ if and only if $a < b$. 

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By (A9) and (A10),
\[ \hat{\alpha}^* q_1 (\hat{p}_1^* - c_1) > q_1 (p_1^* - c_1) - A^* \iff \frac{(\theta_0 s_0 - c_0)(c_1 - c_0)}{\theta_1 s_1 - c_0} > 0, \]
which holds under (C1a). Suppose now that \( q_1 (\theta_1 s_1 - c_1) \leq 2(\theta_0 s_0 - c_0) \). Proposition 4 implies that in this case the high-quality firm’s first-period separating-equilibrium profit with price–shortage combination as the signal becomes
\[ (A11) \quad \hat{\alpha}^* q_1 (\hat{p}_1^* - c_1) = \frac{(\theta_0 s_0 - c_0)(\theta_1 s_1 - c_1)}{\theta_1 s_1 - c_0}. \]
Using (A9) and (A11), \( \hat{\alpha}^* q_1 (\hat{p}_1^* - c_1) > q_1 (p_1^* - c_1) - A^* \) if and only if
\[ q_1 > \frac{\theta_0 s_0 - c_0}{\theta_1 s_1 - c_0}, \]
which is guaranteed by (C2). \( Q.E.D. \)

References


