Economics 242  
Problem Set #1 (Due Jan 29)  
Winter 2013

1. Consider a discretized first-prize sealed-bid auction for a single unit of an indivisible good with two risk-neutral buyers. Each buyer $i$ has three possible valuations of the good: $\theta^1_i = 1$, $\theta^2_i = 2$, and $\theta^3_i = 3$. Bids are restricted to be in \{.5, 1, 1.5, 2, 2.5, 3\}.

a) Suppose the common prior $p$ is given by

\[
\begin{array}{cccc}
\theta^1_i & \theta^2_i & \theta^3_i \\
\theta^1 & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\
\theta^2 & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\
\theta^3 & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\
\end{array}
\]

Model the auction as a Bayesian game and show that the bidding strategy $b^* = (b^*_1, b^*_2)$ where

\[
b^*_i(\theta^k_i) = \frac{1}{2}\theta^k_i, \ k = 1, 2, 3, i = 1, 2
\]

is a Bayesian-Nash equilibrium.

b) Now consider a type space ($\{T_i\}_i, p$), where the prior $p$ is given by

\[
\begin{array}{cccc}
t^1_i & t^2_i & t^3_i & t^4_i \\
t^1 & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & 0 \\
t^2 & \frac{1}{9} & \frac{1}{9} & \frac{27}{27} & \frac{1}{27} \\
t^3 & \frac{1}{9} & \frac{1}{27} & \frac{1}{27} & 0 \\
t^4 & 0 & \frac{1}{27} & 0 & \frac{27}{27} \\
\end{array}
\]

and $T_i = \{t^1_i, t^2_i, t^3_i, t^4_i\}$ with type $t^1_i$ having valuation $\theta^1_i$, type $t^2_i$ having value $\theta^2_i$, type $t^3_i$ and type $t^4_i$ both having valuation $\theta^3_i$. Show that $(b^{**}_1, b^{**}_2)$ where

\[
b^{**}_i(t^1_i) = \frac{1}{2}\theta^1_i, \ b^{**}_i(t^2_i) = \frac{1}{2}\theta^2_i, \ b^{**}_i(t^3_i) = 1, \ b^{**}_i(t^4_i) = 1.5
\]

is a Bayesian-Nash equilibrium.

c) Comment on the differences between the BNEs in parts a) and b).
2. Consider two agents each hold 50% of an asset. The payoff of agent $i$ for owning a fraction $\sigma_i \in [0, 1]$ of the asset and obtaining a monetary transfer $\tau_i$ is:

$$\theta_i \sigma_i + \tau_i.$$ 

The parameter $\theta_i$ is privately known to agent $i$ only. Assume $\theta_1$ and $\theta_2$ are independently and identically distributed in $[0, 1]$. Characterize a Bayesian incentive-compatible budget-balanced mechanism that allocates the ownership to the highest valuation agent.

3. A monopolist seller produces a product with a CRS technology at a per-unit cost equal to $c > 0$. The monopolist sells to a consumer whose utility function is not observable to the monopolist. A consumer of payoff type $\theta$ has a utility function

$$u(x, \theta) = \theta v(x) - \tau$$

where $x$ is a quantity of the product, $\tau$ total amount the consumer pays to the monopolist for quantity $x$, and $v$ is a function of $x$ only such that $v' > 0$ and $v'' < 0$. Assume $\theta$ is randomly distributed on $[\underline{\theta}, \bar{\theta}]$ with density function $\phi(\cdot) > 0$ and distribution function $\Phi(\cdot)$ such that

$$\theta - \frac{1 - \Phi(\theta)}{\phi(\theta)}$$

is non-decreasing in $\theta$. Characterize the optimal selling mechanism for the monopolist, assuming the consumer can always choose not to buy.