Transboundary Environmental Problems with a Mobile Population: 
Is There a Need for Central Policy?

by

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Abstract
It is a widely held view that efficient environmental policies regulating transboundary pollution will be adopted only if there is interjurisdictional coordination. Efficient policies can be adopted as a result of interstate treaties or mandated by a central authority. However, if the policies of states are chosen to maximize the same function of own-citizen welfare, and if individuals migrate freely between states, constrained-efficient environmental regulatory policies are a non-cooperative equilibrium. The policies are constrained-efficient in equilibrium, the policy choices are the same as those found by maximizing the social welfare function subject to a policy feasibility constraint.

Keywords: Transboundary pollution, Population mobility, Federalism.

JEL classification: D6, H7, Q20, R23

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1 Introduction
Transboundary environmental problems are characterized by the environment in a region being affected directly by actions taken in one or more other regions\(^1\). Transboundary environmental problems have received a large attention in the literature; early contributions include OECD (1976) and d’Arge (1975). The most obvious type of transboundary environmental problem is emissions of some physical substance from one region having a negative impact on the environment in other regions. However, transboundary environmental problems may also be of a non-physical kind, such as e.g. a concern about worldwide biodiversity, see Barrett (1992). In this paper we shall interpret our variables as physical emissions causing environmental damage, but our results are equally valid for non-physical environmental problems.

We use the term “region” as a geographical area that is a jurisdiction with some degree of political autonomy. The typical example of this type of region is a country. However, the regions could also be e.g. states, provinces or counties within a country. The important thing is that the region has some autonomy over policy instruments affecting the emissions in the region.

A standard result from the literature is that without any type of environmental policy coordination or other forms of environmental agreements between regions, the outcome will be socially inefficient. The reason for this is that when a region designs its environmental policy, it takes into account the affect of its emissions only on its own environment. In a socially efficient outcome, the effect of emissions in one region on all regions will be taken into consideration. This result is based on models in which it is assumed that the population in each region is exogenously given. For regions that are geographically and culturally close to each other, such as e.g. the states in USA or the provinces of Canada, this clearly is an unrealistic assumption. In this paper we therefore explore the consequences of an alternative assumption of the populations in the regions. We consider the case of perfect population mobility across regions, implying that the same types of people get the same utility in all regions. Our main finding is that with perfectly mobile populations, we may get an efficient outcome even if there is no policy coordination or environmental agreement between the regions.

A result similar to ours was first shown by Wellisch (1994), who analyzed the provision of a public good that generates interregional benefit spillovers. He showed that with perfect population mobility, the non-cooperative equilibrium would be socially efficient if each region in addition to deciding its level of the public good could set a head tax for

\(^{1}\) The term “directly affected” excludes any indirect effects via prices, incomes etc. making actions in one region affect the environment in other regions.
the residents in the region and also give non-negative transfers to the residents of other regions. A similar result is shown by Silva (1997), where the public good is pollution abatement, and where the level of pollution also is affected by the choices made by consumers in each region. Silva only considers the special case of a unidirectional spillover, i.e. in his 2 region model region 2 is affected by the consumption choices and the level of abatement in region 1, but not vice versa. He shows that in this case the efficiency property derived by Wellish is valid (in a second best sense) also if interregional transfers are ruled out. This result is valid for the 2-region case with a unidirectional transboundary spillover also if there is no population mobility between regions, see e.g. Hoel (1999).

Our contribution differs from the above-mentioned articles partly in the way we model the environmental externality. In our model this externality is explicitly linked to the production process. More importantly, we derive the efficiency result for very general types of spillovers and for very general assumptions about what policy instruments are available to the regions. The results of Wellisch and Silva follow as special cases of the general result we derive. In addition to the generality with regard to policy instruments, we also generalize previous literature by allowing for heterogeneous populations in each region. Finally, we give an explicit treatment of land and produced capital in the production functions that are not addressed in the previous literature.

The paper is organized as follows. A simple model of a transboundary environmental problem is introduced in Section 2, where we also derive the conditions for efficiency. Section 3 demonstrates that if interregional transfers are ruled out, the non-cooperative outcome is efficient. This result is generalized in Sections 4 and 6. Sections 7 and 8 discuss the role played by region specific land and mobile produced capital in models of these types.

2 A simple model of transboundary pollution.
To formalize the analysis of a transboundary environmental problem, consider J regions with emissions \((e_1, \ldots, e_J)\). For each region \(j\) there is a variable \(z_j\) which measures environmental quality. This variable depends on emissions from all the \(J\) regions, and is defined so that it is declining in all \(e_i\). Denoting \(e=(e_1,\ldots,e_J)\) as the vector of emissions from all regions, we thus have \(z_j=z_j(e)\) where all partial derivatives \(z_{ji}\) are non-positive. The general description includes several special cases. One such special case is the case of only local environmental damage. For this case all the partial derivatives \(z_{ji}\) are zero for \(i \neq j\). Another special case is the one of a purely unidirectional environmental problem, like Silva (1996) assumes. An example of such a problem could be a river running through several regions which all pollute the river (for a recent discussion, see e.g. Rogers (1997)). Clearly, a country that is further downstream cannot pollute an upstream country. For this case, the partial derivatives \(z_{ji}\) are zero for \(i<j\) if we number countries so
that the region index is higher the further downstream the region is. Finally, climate change and depletion of the ozone layer are examples of environmental problems for which it is only the sum of emissions from all countries that matters for the environment. For this special case it is thus only the sum $\sum e_i$ that enters as an argument in the functions $z_j(e)$. Notice that even if the physical measure $\sum e_i$ of total emissions is the same for all regions, the countries may differ in the way these physical emissions affect the regions. A change in climate caused by an increase in total emissions of greenhouse gases could e.g. affect different regions differently.

Production in region $j$ is higher the population is, as it is assumed that labor input is proportional to (or at least increasing with) population in the region, denoted by $n_j$. It is also increasing in the emission level $e_j$ for the interesting sizes of emission levels. Production is denoted by $f_j(n_j,e_j)$, and we assume that per capita production $f_j/n_j$ is strictly declining in $n_j$ for any $e_j$ (this assumption is discussed further in Section 7).

In the simplest version there is a homogeneous population: everyone is equally productive and all share the same preferences. In this case it is natural to assume that income and consumption is divided equally among all residents of region $j$. Moreover, everyone is assumed to have the same utility function $u$ depending on their consumption and on the environmental quality of the region they live in. (These two assumptions are discussed further in Sections 6 and 8.) Denoting the per capita consumption in region $j$ by $c_j$, we thus have

$$U_j = u(c_j, z_j(e))$$  \hspace{1cm} (1)

While the level of welfare $U_j$ may be specific, the population homogeneity implies homogeneous preferences. Thus, there is no regional subscript for the function $u( )$. In most analyses of transboundary pollution, the distribution of the population is assumed exogenous, i.e. all $n_j$ are assumed exogenous. With this assumption an efficient allocation of emissions is found by maximizing a function

$$W = \sum_j \alpha_j U_j$$  \hspace{1cm} (2)

subject to (1) and the constraint

$$\sum_i n_i c_i \leq \sum_i f_i(n_i, e_i)$$  \hspace{1cm} (3)

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2 These examples are examples of global environmental problems. Obviously, our model is not suitable for such problems, since we assume perfect population mobility between all regions involved.
An allocation of emissions solving this maximization problem for an arbitrary vector of positive $\alpha_j$’s is a Pareto optimal allocation. It is well known, and straightforward to show that all Pareto optimal allocations must satisfy

$$f_{j_0}(n_j, e_j) = \sum_{i=1}^{n} n_i \frac{u_z(c_i, z_i)}{u_c(c_i, z_i)} (-z_{ij}(e))$$

(4)

If there is population mobility, we obviously cannot assume that the allocation of the population is exogenous. Instead, assume that all $n_j$’s are choice variables in the maximization problem above, and solve the maximization problem under the additional constraint

$$\sum_i n_i = N$$

(5)

where $N$ is the total population in the regions (assumed given). The Pareto optimal outcomes must also in this case satisfy (4). In addition, all Pareto optimal allocations of the population must satisfy

$$f_{i_0}(n_1, e_1) - c_1 = \ldots = f_{j_0}(n_j, e_j) - c_j$$

(6)

This is a well-known condition for the optimality of population distribution. It says that the marginal surplus gained by an individual moving into a state (the difference between his contribution at the margin -- his marginal product -- and what he consumes) should be equalized across all states.

Generally, there is a continuum of Pareto optimal outcomes, i.e. outcomes satisfying (3)-(6). These outcomes differ in the distribution of utility levels across regions. However, in this paper we assume that there is perfect population mobility, and that migration therefore eliminates any potential differences in utility levels between regions. We thus have the following condition:

$$U_1 = U_2 = \ldots = U_j$$

(7)

Together with (3) – (6), this condition selects a particular Pareto optimal outcome. We call this outcome the first-best socially efficient outcome.

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3 See e.g. Markusen, (1975) or Hoel (1999).
The first-best optimum given by (3) – (7) gives a particular distribution of consumption per capita across regions. Generally, this distribution will differ from the distribution of per capita incomes (equal to \( f_j(n_j, e_j)/n_j \)). In other words, interregional transfers will generally be necessary in order to achieve the first-best optimum.

If for some reason interregional transfers are ruled out, it is generally not possible to achieve first-best efficiency. In this case the second-best constrained efficient allocation of emissions is found by maximizing the common utility subject to (1), (5), (7) and, instead of (3), the constraint

\[
n_i c_i \leq f_i(n_i, e_i) \quad \text{for all } i
\]

To see what the second-best efficiency condition for the allocation of emissions, consider the choice of the emission level \( e_j \), i.e. emissions from region \( j \). Clearly, at the optimum the derivative of \( U_j \) with respect to \( e_j \) must be equal to zero. Inserting (8) into (1) and differentiating gives

\[
\frac{dU_j}{de_j} = u(c_j, e_j) \left( \frac{\partial (f_j / n_j)}{\partial n_j} \frac{\partial n_j}{\partial e_j} + \frac{1}{n_j} \frac{\partial f_j}{\partial e_j} \right) + u_z(c_j, e_j) \frac{\partial z_j}{\partial e_j}
\]

where the term \( \frac{\partial n_j}{\partial e_i} \), together with \( \frac{\partial n_i}{\partial e_j} \) for all other \( i \), is determined by (5) and (7).

Setting the derivative in (9) equal to zero and rearranging gives us

\[
\frac{1}{n_j} \frac{f_j}{e_j}(n_j, e_j) = \frac{u_z(c_j, z_j)}{u(c_j, z_j)} (-z_{ji}(e)) - \frac{\partial (f_j / n_j)}{\partial n_j} \frac{\partial n_j}{\partial e_j}
\]

Together with the equations (1), (5), (7) and (8), this equation gives us the allocation of emissions that is socially efficient given the constraint that interregional transfers are ruled out.

3. The non-cooperative equilibrium
When there is no cooperation among regions, each region chooses its policies in order to maximize the utility level of its own residents, taking the policies of other regions as given. Consider region \( j \). Assume for a moment that population in this region is given, as is usually assumed in the literature of interregional environmental problems. Assume also that transfers between regions are ruled out. Inserting (8) into (1) and maximizing with
respect to $e_j$ gives the optimal emission level in region $j$. It is straightforward to see that the optimality condition in this case is

$$
\frac{1}{n_j} f_{je}(n_j, e_j) = u_z(c_j, z_j) (z_{ij}(e)) \quad (11)
$$

Comparing this with the condition (4) for first-best efficiency, we see that the conditions do not coincide. While each region only takes the effect of its emissions on its own residents into consideration when designing its optimal policy, the socially efficient allocation of emissions takes into consideration the effect of each region’s emissions also on the residents of all other regions. This difference is what makes some kind of cooperation across regions necessary.

When populations are mobile across regions, the optimal emission levels from each region’s point of view are no longer given by (11). To find its optimal emission level, region $j$ must take into consideration how its emission level affects the size of its population, and thereby the average consumption in the region. The same maximization as above, but taking account of this effect on population size, gives us equation (10) which was the condition for a constrained social optimum. The term $\partial n_j/\partial e_j$ is determined just as it was in the condition for the constrained social optimum, i.e. by (5) and (7).

It follows from the discussion above that if transfers between regions are ruled out, there is no reason to have an interregional environmental agreement or other type of coordination of environmental policies. With perfect population mobility, the non-cooperative equilibrium is (constrained) socially efficient. This result is contrary to the traditional result in the theory of transboundary environmental problems, where populations are assumed to be exogenously given. As mentioned in the Introduction, Silva (1996) has shown this result for the special case of a unidirectional pollution spillover. However, for this case the non-cooperative equilibrium is efficient also without population mobility, see e.g. Hoel (1999).

In the next sections we show that this important result is quite general, as long as there is perfect population mobility across the regions involved in the transboundary environmental problem we are considering.

4. The efficiency of the non-cooperative equilibrium: a generalization

It is convenient to consider the transboundary problem more formally as a game played between policy-autonomous regions. The strategies of the game are the regional environmental policies, and the payoffs are the utilities of the regional residents. It is
important to recognize, with respect to the previous model with free interstate migration, that market equilibrium imposes an important constraint on the payoffs. Namely the welfare of citizens in every region must be the same.

The game is described in the following way. The players are the regional governments, of which there are \( J \) (\( j = 1 \ldots J \)). The feasible strategies are the policy choices of the regional governments. In the model above, the most obvious example would be local contribution to environmental degradation \((e_j)\) as well as the local taxes and transfers to other regions. However, the reasoning of our analysis is applicable to larger, and more complex, sets of potential policies or strategies. Whatever the feasible strategies are, the payoffs are the utilities of the regional citizens \( U_j \). Market equilibrium restricts the inter-regional distribution of welfare to be such that the utilities are the same in all regions.

Formally, let \( S = S_1 \times S_2 \times \cdots \times S_J \) be the set of feasible strategies with \( S_j \) the set of feasible strategies the strategies of \( j \). The regional payoffs depend on the number of residents and the locally chosen policies. To illustrate, consider the case in which the policy vector \( s_j \) of region \( j \) consists of the choice of emissions \( e_j \), a head tax \( \tau_j \) and a vector of transfers to other countries per person in region \( j \), \( \tau_j1, \ldots, \tau_jJ \) (it is convenient, but not necessary to assume \( \tau_{jj} = 0 \)). A reasonable requirement of feasibility is that \( e_j \geq 0 \), all \( \tau_{ji} \geq 0 \) (region \( j \) cannot tax residents in other countries) and \( \sum \tau_{ji} = \tau_j \) (budget constraint). With these strategies, consumption of a person living in region \( j \) will be

\[
c_j = \frac{f_j(n_j,e_j) + \sum n_i \tau_{ij}}{n_j} - \tau_j
\]

Together with the migration equilibrium condition (7), the condition (12) for all \( j \) determines all population and consumption levels once the policy instruments are given. Given the strategy choices of all regional governments, utility levels thus follow from the preference function (1).

More generally, we have

\[
U_j = V_j(s)
\]

From the migration equilibrium condition (7) we know that we must equal utility in all regions, whatever the strategy vector \( s \) is. This means that the payoff functions \( V_j(s) \) must be the same for all regions, i.e.
Setting-up the transboundary pollution problem in this way yields a surprising conclusion: namely, central governmental mandate is unnecessary to induce the optimal policies:

**Theorem:** Let $s^* \in S$ be the vector of policy choices (e.g. environmental policy and tax/transfers) that maximizes $V(s)$ subject to feasibility constraints $s_j \in S_j$; then $s^*$ is the Nash Equilibrium of the policy game described above.

**Proof:** Suppose all regions but $j$ make the socially optimal choice, $s_{-j}^*$. Since the payoff to $j$, $V_j$, is identical to $V(s)$, region $j$ makes $V_j$ as large as possible by choosing $s_j^*$.

While the theorem focuses on the first best optima, for the transboundary pollution problem stated, the first best depends not only on the selection of the correct environmental policy, it also depends on the “correct” distribution of the population. In order to achieve that, the regions must be able to tax themselves and transfer to other regions. Both policy instruments might not be available. There may be laws in state prohibiting direct international monetary transfers, for instance. In other words, the set of policy instruments (feasible strategies) may be limited. The theorem is very hopeful in this regard. What it says, with respect to the limited feasible set of strategies, is that the “best” among the available constitute a Nash Equilibrium of the interregional game. In other words a “second-best” optimum is achievable non-cooperatively.

5. An Example

A simple example of two regions demonstrates the principle idea derived above. A common product is produced with local labor, $n_j$, and an environmentally degrading input, $e_j$. There is a total population of $n = 100$ ($n_1 + n_2 = 100$).

The production functions of the two regions are

$$f_1 = 150n_1 - 2 \frac{n_1^2}{e_1}, \quad f_2 = 140n_2 - \frac{n_2^2}{e_2}.$$  

Both production functions are homogeneous of degree one and strictly concave in the two inputs. The environmental damage functions are the same for both regions

$$z_1 = z_2 = -(e_1^2 + e_2^2).$$

Consumption of a resident in region 1 is

$$c_1 = 150 - 2 \frac{n_1}{e_1} - \tau_1 + \frac{n_2}{n_1} \tau_2.$$
and in region 2
\[ c_2 = 140 - \frac{n_2}{e_2} + \frac{n_1}{n_2} \tau_1 - \tau_2. \]

As previously defined the \( \tau \)'s are the per capita taxes and they are constrained to be non-negative.\(^4\) The expression for consumption builds in the balanced budget constraint that the amount transferred to one state exactly equals the amount collect from the other.

Individual preferences are assumed to be linear in consumption and environmental damage
\[ U_j = c_j + z_j. \]
With this form and the environmental damage function common to both regions, equal utility equilibrium condition, implies that consumption must be the same in both region in equilibrium. Thus, the necessary condition for an efficient population distribution, equation (6) is met if the regional marginal products of labor are the same.

The optimal policies, those that satisfy equations (4) and (6), along with the migration equilibrium condition (7), are, first, \( e_1 = 2.107 \) and \( e_2 = 2.888 \). The efficient population distribution is \( n_1 = 30.592 \) and \( n_2 = 69.708 \). This is an equilibrium distribution, given the desired environmental policies, if there is a tax of \( \tau_1 = 3.470 \) on every resident of region 1 with a subsequent transfer of 1.530 to every resident of region 2. The optimal tax for region 2 is \( \tau_2 = 0 \). These policies lead to a common consumption of 117.50 in both regions and a common utility of 104.71.

It is straightforward to show that these values also simultaneously satisfy the first order conditions for regional non-cooperative optimization. However, while each of the objective functions is concave, it is not obvious that concavity in the policy variables is maintained when the migration responses are included. In order to verify the coincidence of regional and social optimality, the welfare consequences of policy alternatives for each region were calculated on the assumption that the other region chose it efficient policy. The welfare of each region associated with its policies (conditional on the optimal policy of the other) if given in the following tables.

\(^4\) A negative tax would imply a tax on the other region. This is prohibited.
TABLE II
THE UTILITY OF REGION 2 GIVEN \( \tau_1 = 3.470 \) AND \( e_1 = 2.107 \)

<table>
<thead>
<tr>
<th>( e_2 )</th>
<th>( \tau_2 )</th>
<th>0.000</th>
<th>0.050</th>
<th>0.100</th>
<th>0.150</th>
<th>0.200</th>
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<tbody>
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<td>104.510</td>
<td>104.510</td>
<td>104.510</td>
<td>104.510</td>
<td>104.509</td>
<td></td>
</tr>
<tr>
<td>2.744</td>
<td>104.662</td>
<td>104.662</td>
<td>104.662</td>
<td>104.661</td>
<td>104.660</td>
<td></td>
</tr>
<tr>
<td>2.888</td>
<td>104.7148</td>
<td>104.7146</td>
<td>104.714</td>
<td>104.713</td>
<td>104.712</td>
<td></td>
</tr>
<tr>
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<td>104.484</td>
<td>104.482</td>
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</tr>
</tbody>
</table>

The confirmation of the coincidence of the Nash Equilibrium with the social optimum was done with a much finer division of policy option, the result of that examination would be much too extensive to present here. For presentation purposes, the example was computed for 5% deviations close to the optimal values. The utility levels, or payoffs, are presented with three significant decimal places, except in the case of the equilibrium value for region 2 where it is necessary to display four decimal places in order to indicate the equilibrium.

6. Non-homogeneous populations
The conclusion of this is surprising, but its application is somewhat limited. It requires an equality of welfare among all regions. This is achieved in the example with one type of individual and completely free mobility. Not surprisingly, there is a limit to how far the result can be generalized. It is not perfectly general, for it is easy to construct example, for instance, a case where there are locationally fixed factors, in which the result does not hold. However, it is possible to generalize the model to encompass the possibility of multiple types.

Begin by considering a partition of the population into \( T \) distinct types or classes. Membership in a class, labeled \( t = 1 \ldots T \), requires the same preferences and productivity as every other member of that class. In other words, there is homogeneity within a particular type. The welfare of an individual of class \( t \) in jurisdiction \( j \) is \( U^t_j \). Without
going through the details here, as before, the utility of every class will be a function of the strategies chosen by all regions

\[ U^t_j = u^t(c^t_j, z_j) = \Psi_j^t(s_j, s_{-j}) \quad \text{(15)} \]

The function \( \Psi_j^t \) is region specific in that the mapping of strategies into type welfare depends on the idiosyncratic aspects of each region. Regional differences in production technologies, for instance, affect the way regional policy determines local utility. Of course, with free migration, the utility levels are the same in all regions for all members of a particular partition, nonetheless, the mappings from strategies to utility may be region specific.

Suppose, further, that each region has the same social welfare function

\[ V_j = \Phi(U^1_j, U^2_j, \ldots, U^T_j) = \Phi(\Psi^1_j(s), \Psi^2_j(s), \ldots, \Psi^T_j(s)) = V_j(s) \quad \text{(16)} \]

Migration equilibrium requires that the utility of that type is the same no matter the residential region, \( U^t_j = U^t_k \quad \forall j, k \). Therefore if there is at least one type in every region, and every regions has the same welfare function then the feasible set of regional utilities are those for which the utility values are the same. Therefore, as in the case of the single homogeneous type

\[ V^*_j(s) = V(s) \quad \text{(17)} \]

Applying the same logic as before:

**Theorem:** If, in equilibrium, at least one of each population class resides in every region, the socially optimal policies are equilibrium strategies of the policy game played by the regions.

There is a general principle at work in these examples. The common welfare is naturally defined to be social welfare. If social, as well as regional, welfare, are mappings from set of feasible strategies to the reals

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5 For instance, if the regional social welfare function is of the Bergson-Samuelson type, then the weights are the same in every region: \( V_j = \sum_{t=1}^{T} \alpha^t U^t_j \)
\[ W : S \rightarrow \mathbb{R} \quad \text{and} \quad V_j : S \rightarrow \mathbb{R}, \]

and, if

\[ V_j(s) > V_j(s') \iff W(s) > W(s') \quad \forall s,s' \in S, \]

then \( s^* \) is a Nash Equilibrium (where \( s^* \in S \) as before is the vector of policy choices that maximizes \( W(s) \) subject to feasibility constraints \( s_j \in S_j \)).

A consequence of this is that if the policy instruments available to the regions are insufficient to produce a first best optimum, the non-cooperative equilibrium will be a second best. In other words, the non-cooperative equilibrium strategies are ones that maximize social welfare constrained by what are feasible strategies. An example of this is given previously. In order to produce a first best outcome, one that induces not only an optimal environmental policy, but also an efficient distribution of labor, regions must be able to tax themselves and make transfers to other regions in addition to being able to control their own emissions. If this is not a feasible policy alternative, the resulting equilibrium produces the highest social welfare possible without the tax transfer option. It is furthermore true of this that regions may have different feasible policies. One may be able to use command and control regulation while other may be prohibited from using those, but is able to employ pollution based taxes.

7. Properties of the production functions

To simplify the discussion of the present Section, we return to the case with only one type of persons. However, the discussion in this Section remains valid also for the case of several types discussed in the previous Section.

In the simple models of population mobility without any environmental variables, the only specified production factor is labor (or population), so that output in region \( j \) is \( f_j(n_j) \). With this specification, one must assume decreasing returns to labor in order to get equilibrium with positive populations in all regions. If there were constant returns to labor, the average productivity \( f_j/n_j \) would be independent of \( n_j \) and a socially efficient outcome as well as a non-cooperative equilibrium would have all of the population living in the region(s) with the highest average productivity. If \( f_j/n_j \) were equal in some or all regions, the allocation of the population across these regions would not be determined.

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\(^6\) Monderer and Shapley (1996) would label \( W( ) \) a 'potential function' and the resulting game a 'potential game'. In Shapiro and Petchey (1999) this condition implies that there is a "coincidence of collective and individual interests".
If labor is the only specified factor and there are decreasing returns to labor, we have implicitly assumed that there is a fixed factor in the background. It is natural to think of this factor as region specific land. In the simple models of population mobility without environmental variables, the mobility condition (7) implies that consumption per capita should be equalized across regions. It is not obvious how this should be interpreted when per capita income (before taxes and transfers) consists of wage income and rent from land ownership. With perfect mobility it is reasonable to assume after tax wages per person to be equalized across regions. However, there is no reason why land rents per capita should be equalized. This income component is after all an income one gets from the region in which one owns land, independent of the region in which one lives. The simplest way out of this problem is to assume that land rents in each region are taxed at a rate of 100 percent by the authorities of the region, and that the tax revenues are distributed as equal lump sum transfers to all residents in the region. (This is one way of interpreting Wellisch (1994), who assumes that “land in region I is owned only by residents of that region on an equal per capita basis”. Silva (1996) is not clear about how he treats income from land, although land is explicitly used in his production functions.)

In our model with environmental variables, there is no need to have a fixed factor (“land”) in the production functions. Even if the production functions $f_j(n_j,e_j)$ are homogenous of degree 1 and differ across regions, we may have an outcome satisfying (7) with positive populations in all regions, as illustrated in the example presented in Section 5. This is most obvious if the only policy instrument in each country is its emission level. In this case all $e_j$’s are fixed once the authorities have chosen their policies, implying that average productivities $f_j(n_j,e_j)/n_j$ are declining in $n_j$. If emission levels are determined through direct regulation, there is a residual or rent in the firms equal to $f_j(n_j,e_j)-f_j(n_j,e_j)n_j$. As in the case with land rents, we must assume that this residual is claimed by the authorities, who reimburse it to the residents in the region. An obvious interpretation is that the chosen emission level of the region is allocated to individual firms in the region through quotas that the firms buy from the regulator at their equilibrium price.

Instead of choosing a fixed level of emissions and allocating it to firms through direct regulation or auctioned quotas, the authorities of the regions could set emission taxes. In this case emissions would be endogenously determined along with populations. Moreover, there will be no residual or rent left in the firms in this case (when constant returns are assumed). However, in this case it is not obvious that there exists an equilibrium with positive populations in all regions. To see this, let firms in region $j$ face an emission tax $\theta_j$ and a wage rate $w_j$. Define $g_j(e_j/n_j)=f_j(1,e_j/n_j)$ and $\epsilon_j=e_j/n_j$. Profit maximization (i.e. cost minimization combined with zero profit) implies

$$g_j'(\epsilon_j)=\tau_j$$ (18)
and

\[ g_j(\varepsilon_j) - \varepsilon_j g'_j(\varepsilon_j) = w_j \]  

(19)

Given the emission tax \( \theta_j \), the emissions per capita \( \varepsilon_j \) and the wage rate \( w_j \) thus follow. The equilibrium condition (7) may in this case be written as

\[ u(g_1(\varepsilon_1), z_1(n_1\varepsilon_1,...,n_j\varepsilon_j)) = ... = u(g_j(\varepsilon_j), z_j(n_1\varepsilon_1,...,n_j\varepsilon_j)) \]  

(20)

For an equilibrium with positive populations to exist, it must be the case that a population move from region \( j \) to \( k \) must increase \( U_j - U_k \) when \( U_j < U_k \) initially. This will certainly hold if there is no regional spillover of emissions. In this case the environmental quality in each region depends only on emissions in the same region. If population moves from region \( j \) to \( k \) emissions will go down in \( j \) and up in \( k \), since emissions per capita are given in both regions. This increases \( U_j \) and reduces \( U_k \). However, with transboundary pollution it is not obvious that \( U_j - U_k \) will increase as people move from region \( j \) to \( k \). For instance, if all \( z_j \)'s depend only on the sum of emissions from all regions, such a migration will increase or reduce all \( z_j \)'s depending on whether \( \varepsilon_j \) is smaller or larger than \( \varepsilon_k \) (since the sum of emissions given by \( \sum_i n_i \varepsilon_i \)). Moreover, even given the change in total emissions, the change in \( U_j - U_k \) will depend on how much \( z_j \) and \( z_k \) change. Assume for instance that \( \varepsilon_k > \varepsilon_j \), so that total emissions increase when people move from \( j \) to \( k \). If \( z_j \) is only weakly affected by this increase in total emissions, while \( z_k \) declines sharply, \( U_j - U_k \) will increase.

It is clear for the discussion above that it is not obvious that there exists an equilibrium with positive populations in all regions. Whether or not such an equilibrium exists depends partly on the structure of the economy (given by the functions \( u, f_j \) and \( z_j \)), but also on what policy instruments are used.

8. Land and capital

In the previous Section, we argued that it was not necessary to include land or other fixed factors in our model, contrary to similar models without environmental variables. However, even if factors such as land and capital are not necessary for the logic of the model, these factors are important in the real world. In this Section we therefore show how they can be incorporated in our model.
Denote output in region \( j \) by \( x_j \). Instead of the simple production function presented in Section 2, let us now assume that

\[
x_j = f_j(n_j^1, \ldots, n_j^T, k_j, L_j, e_j)
\]

where \( n_j^t \) is labor (or population) of type \( t \) employed in region \( j \), \( k_j \) is capital used in region \( j \), \( L_j \) is region specific land, and \( e_j \) represents emissions in region \( j \). This production function is assumed to be homogeneous of degree one.

Land is exogenously given in each region. The sum of capital in all regions is given, but capital is assumed to be completely mobile across regions. The marginal productivity of capital is therefore equalized across regions, and we denote this common return on capital by \( q \). We also denote the rent per unit of land in region \( j \) by \( r_j \), and wages of type \( t \) labor by \( w_j^t \).

There are \( T \) types of persons in the world. The total number of people of type \( t \) is \( N_j^t \). These types may differ in their labor productivity, implying different wages. They also may differ in their ownership of land and capital. We assume that a person of type \( t \) owns \( \sigma_j^t \) \( L_j \) of the land in region \( j \), and \( \sigma_K^t \) \( K \) of the total capital stock (since the rate of return on capital is the same in all regions, it makes no difference where the capital owned by a particular person is used). These ownership parameters must satisfy

\[
\sum_t \sigma_j^t N_j^t = 1
\]

(22)

\[
\sum_t \sigma_K^t N_j^t = 1
\]

(23)

A person of type \( t \) choosing to live and work in region \( j \) will have the following consumption:

\[
c_j^t = w_j^t + \sum_i \sigma_i^t r_i L_i + \sigma^t q K + Y_j^t
\]

(24)

where \( Y_j^t \) are the net transfers this person receives (positive or negative). The utility index of this person is as usual given by

\[
U_j^t = u'(c_j^t, z_j(e))
\]

(25)
But this is the same specification as we gave in Section 6, see (15). The results from Section 6 are therefore valid for the model presented above, where incomes from capital and land go to the owners of this capital and land, independently of in which region they live.

**Conclusion**

We have shown that the efficient regulation of transboundary pollution is possible without explicit cooperative agreements or central mandates. What is required is that the policy options available to each state are adequate; that states choose policies to maximize the same function of own-citizen welfare and that individual are fully mobile between states. These conditions are unlikely to be met in general, but, even if the restrictions are unrealistic, the model does point to an important aspect of policy making.

The conditions set up an interrelationship between autonomous states that, in itself, can induce states to make policy choice consistent with overall welfare maximization while pursuing their own self-interested objectives. The migration equilibrium (equal utility) condition generates a coincidence of interests between the states. In equilibrium the welfare of one state is tied to the welfare of all others: the well being of one state cannot be improved unless the welfare of all states improve. It is not necessary for states to individually recognize this coincidence; they need only know the migration responses to their own environmental policy choice.

An interesting aspect of the analysis is that equilibrium policy choices may not be globally efficient. They may, instead, be second best efficient in the sense that the chosen policies are the best, given the limited set of policy options open to states. This suggests that central intervention might take the form of expanding the policy choices open to individual states rather than direct regulatory control. The results, however, do not fully mitigate the desirability of more active central intervention.

Central government may have an important role beyond simply expanding the feasible set of state policies. Although efficient policies are an equilibrium, the equilibrium may not be unique. Some form of central coordination can be a mechanism for insuring the best equilibrium is, in fact, the one achieved.
References


