Consider the design of an efficient labor contract between a single risk averse worker and a firm. Let the worker’s utility function be given by the strictly concave function $U(C, L)$ where $C$ is consumption and $L$ is leisure. The worker does not have access to capital or insurance markets so his/her consumption in any state must equal his/her income. This worker is paired with a firm, whose profits in state $\theta$ are given by: $\theta (1 - L) - C$.

Let the probability density function for $\theta$ be given by the continuous function $g(\theta)$ on the interval $(\underline{\theta}, \overline{\theta})$. Suppose an optimal contract chooses the functions $C(\theta)$ and $L(\theta)$ to maximize the worker’s expected utility, subject to a constraint of zero expected profits.

(a) set up expressions for expected profits and for expected utility.

(b) set up the Lagrangian expression whose maximization defines the optimal contract

(c) derive the first-order conditions for an efficient contract

(d) prove that, in an optimal contract, sign $dC/d\theta = -\text{sign}(U_{CL})$. In other words, consumption will be higher in “better” states of nature only if consumption and leisure are substitutes (in other words, the marginal utility of consumption declines with the amount of leisure consumed). Provide some intuition for this result. If $U_{CL} > 0$, will workers be better or worse off when a “better” (i.e. higher- $\theta$) state is realized? Explain.