Economics 250A:

Practice Questions on Personnel Economics and Static Labor Demand

1. The optimal piece rate

A firm is hiring a worker to do a job for one period. Once hired, the worker’s total pay will be given by \( y = a + bq + cq^2 \), where \( q \) is the worker’s output, and \( a, b \) and \( c \) are parameters the worker takes as given. In other words, once hired, the worker’s total pay will be a quadratic function of the the output she produces.

The worker’s output depends on her effort, \( e \), via the production function \( q = e \). The worker’s utility function \( U(y, e) \) is given by \( U = y - e^2 \), and the firm’s profits are just \( \Pi = q - y \).

(a) Think of \( a, b \) and \( c \) as the terms of the contract that has been agreed to between the worker and the firm. Suppose the worker chooses \( e \) after being hired, treating \( a, b \) and \( c \) as given. For given \( a, b \) and \( c \), find an expression for the level of effort that maximizes the worker’s utility.

(b) Using your results from part (a) above (which tell you how the worker will behave for any given contract \( (a,b,c) \)), your job now is to characterize Pareto optimal contracts between the worker and firm. For concreteness, fix the worker’s level of utility at \( \bar{U} \). (Imagine, for example, that labor markets are perfectly competitive ex ante; thus the firm must offer the market level of expected utility, \( \bar{U} \), to attract any worker. Now choose the terms of the contract (i.e. the parameters \( a, b \) and \( c \)) to maximize expected profits, subject to the following two constraints: (i) \( E[U] \geq \bar{U} \); and (ii) the fact (described in part (a) above) that the worker chooses effort after the contract is signed to maximize his/her expected utility (this is sometimes called the “incentive-compatibility constraint”). Show that, in the Pareto-optimal contract, \( b \) and \( c \) can take on any value, as long as they satisfy the relationship \( b = 1 - c \). Show that the Pareto-optimal level of effort equals \( .5 \).

c) Now set \( \bar{U} = .15 \) and suppose \( c=0 \) (i.e. the firm chooses to use a linear contract). Find the equilibrium levels of effort \( (e) \), output \( (q) \), the intercept of the compensation function \( (a) \), utility and profits. Illustrate graphically how this contract induces the worker to select the Pareto-optimal effort level (put effort=output on the horizontal axis and income \( y \) on the vertical).

d) Keeping \( \bar{U} = .15 \), now suppose \( c = -1 \) (i.e. the firm chooses to implement a concave reward schedule). Repeat part (c), illustrating your answer in the same diagram.

e) Given your answers to (c) and (d), is there any reason why firms would ever want to implement a nonlinear piece rate scheme in this environment?

2. **Monopsony Labor Demand.** Starting with the basic one-factor labor demand model we covered in class, replace the fixed wage, \( w \), by an increasing function \( w(L) \); i.e. an upward sloping labor supply curve. Derive the FOC for a profit maximum and illustrate them graphically, showing that optimal \( L \) is below the level where the VMP curve \( pF(L) \) intersects the labor supply curve \( w(L) \). Note that, because the wage is now endogenous, it no longer makes sense to speak of a “labor demand curve” as the firm’s response to an exogenous increase in \( w \). However, show that in this environment the firm’s response to an exogenous increase in \( p \) still satisfies \( dL/dp > 0 \).

3. **Monopoly Labor Demand.** Return to the case of a fixed wage, \( w \), but now replace the fixed output price, \( p \), by a decreasing function \( p(Q) \); i.e. a downward-sloping product demand curve, where \( Q \) is the quantity of output produced. Note that, because the output price is now endogenous, it no longer makes sense to solve for \( dL/dp \). However, show that in this environment the firm’s labor demand curve is still downward-sloping, i.e. \( dL/dw < 0 \).

4. **The Workers’ Co-operative Model** Suppose a worker-owned firm has a fixed cost of \( C \), for example the cost of renting the necessary capital equipment. Suppose that, after paying its fixed costs, all its remaining revenues are distributed evenly among its workers. Further, suppose this co-op is run in such as way as to maximize the payment per worker. Supposing that each (actual or potential) worker represents one unit of \( L \), argue that this means the co-op chooses \( L \) to maximize:

\[
\frac{(pF(L) - C)}{L}
\]

Show that the FOC for an optimum can be written:

\[
pF'(L) = \frac{(pF(L) - C)}{L}
\]

Interpret this intuitively and illustrate the solution graphically.

Finally, totally differentiate the above FOC and demonstrate that:

\( dL/dC > 0; dL/dp < 0 \). In words, the coop expands employment when its capital costs rise, and reduces employment when the demand for its product rises. If you can, provide some intuition for these “counterintuitive” results.