Economics 250A: Practice Problems 1
(Static Labor Supply)

I. Use DIAGRAMS only to demonstrate/illustrate your answers to the following questions:

1. Freda’s effective hourly wage rate has just fallen because the government has raised the Social Security (FICA) payroll tax rate. Freda responds to this by working more hours. It follows that leisure must be a normal good to Freda. (True/False/Explain?).

2. Suppose income is an inferior good to Herman. Does it follow that an increase in wages must reduce Herman’s labor supply?

3. The government has decided it wants to make Wolfgang better off. It is deciding whether to give him a lump sum grant or to subsidize his hourly wages. Show, in a diagram, that:

(a) If both grants make Wolfgang equally well off, the demogrant will reduce Wolfgang’s labor supply by more than the wage subsidy.

(b) Under the demogrant, Wolfgang can be made just as well off as under the wage subsidy at a lower total cost to the government.

4. Until recently, recipients of Social Security in the United States faced an earnings limit that affected their eligibility for Social Security. Specifically, if they earned less than $L$ in a year, their Social Security benefits of $S$ per year were unaffected. For every dollar they earned above $L$, their Social Security benefits were reduced by $t$ cents per dollar of earnings above the limit, until their Social Security benefits were all gone.

a) Show a retiree’s budget constraint both in the presence and in the absence of the Social Security system described above in a diagram.

b) In a separate diagram, illustrate the predicted effects of raising the earnings limit ($L$) on labor supply, and show how it depends on the person’s initial labor supply before this policy change. Assuming that leisure is a normal good, show that labor supply

i) will not change for people initially earning less than the earnings limit, L.

ii) will either stay the same or increase for persons initially at the earnings limit, L, (i.e. at the kink point in the budget constraint)

iii) is likely to decrease for persons initially earning more than the earnings limit, L.

Explain your answers in terms of income and substitution effects.

II. MATHEMATICAL QUESTIONS:

1. Suppose an individual’s utility function takes the Cobb-Douglas form $U(y, ℓ) = y^αℓ^{1-α}$. Show that this person’s labor supply curve has the form: $h = aT − (1-α)(G/w)$. In what direction do $G$ and $w$ affect utility-maximizing labor supply? Suppose that $G = 0$. What is the effect of $w$ on labor supply now? Why?
2. In class, we showed that the substitution effect on labor supply was given by the term \( \frac{U_1}{-B} > 0 \). When drawing graphs, we identified this “substitution effect” with the change in labor supply that would occur if we raised the wage rate \((w)\), but simultaneously cut nonlabor income \((G)\) by an amount sufficient to keep utility constant. In other words, we asserted that:

\[
\frac{dh}{dw} \text{(holding } dU = 0) = \frac{U_1}{-B}.
\]

Your job is to prove this assertion.

3. Consider a two-person household in which Alice’s utility is given by \( U(q^A) \) and Bruce’s utility is given by \( U(q^B) \). (You can think of \( q^A \) and \( q^B \) as apple pie and beer if you like). Alice’s income is \( y^A \) and Bruce’s is \( y^B \); total household resources are \( Y = y^A + y^B \). Pie and beer can be purchased for prices \( p^A \) and \( p^B \) respectively. Efficient Nash bargaining between Alison and Bruce implies that their consumption choices maximize \( U(q^A) + \lambda U(q^B) \), where \( \lambda \) indexes Bruce’s bargaining power in the relationship.

a) If \( U(\cdot) = \log(\cdot) \), derive the household’s demands for \( q^A \) and \( q^B \) as a function of \( p^A \), \( p^B \), \( Y \) and \( \lambda \). Explain why demand curves written in this form \( q^A = Q^A(p^A, p^B, Y, \lambda) \) and \( q^B = Q^B(p^A, p^B, Y, \lambda) \) are correct even if Bruce’s bargaining power, \( \lambda \), depends on his share of the household’s income. If \( \lambda = y^B - y^A \), what are the effects on \( q^A \) and \( q^B \) of (i) raising \( y^A \) by one dollar; (ii) raising \( y^B \) by one dollar; and (c) shifting one dollar of income from Bruce to Alison, keeping their total income, \( Y \), unchanged?

b) Now suppose that Bruce’s bargaining power is given by the expression: \( \lambda = [y^B - y^A] + \alpha z \), where \( z \) is determined by factors other than income received during the marriage. For example, \( z \) might be the amount of Alison’s income that Bruce would continue to receive even if he divorced her. Verify that the testable conditions for Pareto efficiency in household allocation (equation 20 in the course notes) hold for this household.