Problem (practice)

Consider the following distribution for $X_i$, where $X_i$ is an observation of the random sample $X = (X_1, ..., X_N)$:

$$f_{X_i}(x|\lambda) = \begin{cases} 
\lambda \exp(-\lambda(x - 2)) & \text{if } x \geq 2 \\
0 & \text{if } x < 2 
\end{cases}$$

This distribution has mean $\frac{1}{\lambda} + 2$ and variance $\frac{1}{\lambda^2}$.

(a) What is the expected value of $\exp(x - 2)$?

(b) Find the joint distribution for the sample

(c) Find a sufficient statistic for $\lambda$

(d) What is the Maximum Likelihood Estimator (MLE) for $\lambda$? Denote it $\lambda^{MLE}$.

(e) What is the asymptotic distribution of $\frac{1}{\lambda^{MLE}}$?

(f) What is the asymptotic distribution of $\lambda^{MLE}$?

(g) What is the Likelihood Ratio Test (LRT) statistic for

$$H_0 : \lambda = 1$$

$$H_1 : \lambda \neq 1$$