

# A Quantitative Analysis of Educational Reforms in a Dynastic Framework

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## Abstract

We study the effects of educational reforms that would implement European educational subsidies in the United States. We find that the effects of such reforms depend critically on how the reform is financed. If higher schooling subsidies are financed by higher taxes, the reform leads to a more efficient allocation of skills across educational levels, higher tertiary attainment rates and increases steady state welfare by 1.52%. If higher schooling subsidies are financed by implementing stricter admission policies, as typically happens in Europe, the reform is less efficient in allocating skills, leads to significantly lower tertiary attainment rates and has steady state welfare costs of 10.28%. Intergenerational mobility in education increases under both reforms. The results of the second reform are consistent with European educational outcomes.

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# 1 Introduction

Educational policies differ widely across developed countries. The differences are particularly striking when one looks at tertiary education in the United States and in Europe. While in the United States only 48% of the costs comes from public sources, Europe finances about 87% of tertiary costs from public sources on average, and in some countries this fraction is as high as 97% (Denmark, Finland, Greece). At the same time, the tertiary educational system in Europe is much more selective and the admission rules and requirements are stricter than in the United States. Our computations show that the admission rates are 66.5% in the United States and only 45.5% in Europe.

Educational outcomes differ across countries as well. The tertiary attainment rates are significantly lower in Europe than they are in the United States. In Europe the tertiary attainment rate is only 23% on average, while in the United States 39% of people attains a tertiary level degree. On the other hand, Europe does better in terms of the intergenerational mobility in education and earnings, which is in general higher in Europe than in the United States, see for instance [Blanden, Gregg, and Machin \(2005\)](#) or [Corak \(2006\)](#).

The goal of this paper is to find out whether higher European educational subsidies combined with stricter admission rules may lead to both of these educational outcomes. More precisely, we analyze an *admission based reform* of the U.S. educational system where the government adopts European educational subsidies and implements stricter admission policies to balance its budget. The admission based reform represents a move toward the European educational system. We also investigate an alternative educational reform called *tax based reform* where the government adopt European educational subsidies as well, but finances them by an increase in taxes, rather than by more restrictive admission rates.

We find that the two reforms have a very different impact on the economy. Under the admission based reform the government must significantly cut the admission rates at the tertiary level to balance the budget. As a result, the tertiary attainment rate drops from 39% to 14.9%. The admission based reform also significantly decreases steady state welfare by 10.28%. On the other hand, steady state welfare under the tax based reform increases by 1.52% overall. This is so even though, in order to balance the budget, the government must decrease the tax exemption level from 23.9% of income to 16.0% of income. The tax based reform is also beneficial for the tertiary attainment, since the attainment rate increases to 46.8%.

Both reforms have the opposite effects on the economy along many other dimensions as well. The admission based reform decreases aggregate output and capital, increases income and asset inequality, and in terms of wages and steady state welfare affects medium skilled people negatively and high skilled people positively. Reverse effects are obtained under the tax based reform. On the other hand, both the admission based reform and the tax based reform increase the intergenerational mobility in earnings. The intergenerational correlation in the present value of earnings is 0.151 before the tax reform, and it decreases to 0.126 under

the tax based reform, and to 0.089 under the admission based reform. Overall, the outcomes of the admission based reform are consistent with the European educational outcomes, both in terms of decreased educational attainment and in terms of increased intergenerational mobility. Our results suggest that a move towards the European educational system would have large welfare costs.

To obtain some insights into how efficiently different educational systems allocate skills, we compare the allocation of skills under the current U.S. system and under both reforms to the first best allocation of skills. In the first best allocation all high skilled agents, 57.8% of medium skilled agents, and none of low skilled agents are assigned to tertiary schooling. The U.S. educational system is inefficient by those standards since only 66.2% of high skilled agents end up at the tertiary schooling level. We find that the tax based reform leads to a more efficient allocation of skills with 76.5% of high skilled agents studying at the tertiary level. On the other hand, the admission based reform leads to a less efficient allocation of skills, and only 50.7% of high skilled agents study at the tertiary level.

We also compare the intergenerational mobility in education and earnings with the first best results. The intergenerational mobility under the current U.S. educational system is much lower than the first best intergenerational mobility. This is to be expected: in the first best allocation the intergenerational mobility is determined purely by the persistence of skills across generations, while the model of the U.S. educational system features market incompleteness and financial constraints, and educational decisions depend on household wealth as well. We find, however, that financial constraints play a relatively minor role in the educational decisions. This is similar to the findings in [Cameron and Heckman \(1998\)](#) that, at least in the U.S., credit constraints are relatively unimportant for schooling decisions. Market incompleteness is thus most likely to cause low intergenerational mobility in education.

The model we build extends the dynastic framework of [Fuster \(1999\)](#) and [Fuster, Imrohoroglu, and Imrohoroglu \(2003\)](#) by incorporating human capital decisions. The model features individuals that are heterogeneous in their age, skills and (endogenously) in their assets, schooling choice and consumption. Those individuals are altruistic, care about utility of their descendants, and make all decisions on a family level. This seems especially important for investments in education since they are typically made at early stages of life, parents are involved in those decisions, and often provide needed financial resources.

In order to quantitatively evaluate the model we use PSID data to estimate two key elements of the model. The first one are the age-earnings profiles. We differentiate them by education and skills. Skills are constant over one's lifetime and represent the observed heterogeneity in earnings conditional on education and age. We identify each individual in our sample with a particular skill level and estimate the intergenerational correlations in skills. The intergenerational correlations in skills are the second key element of the model, and are critical in matching the intergenerational correlations in the present value of earnings that are observed in the data.

By assuming that educational decisions are made at a family level the model is similar to [Becker and Tomes \(1986\)](#). Unlike them, however, we build a general equilibrium model. The importance of general equilibrium effects in evaluation of policy reforms has been stressed only recently, most notably by [Heckman, Lochner, and Taber \(1998\)](#). Educational reforms in a general equilibrium life-cycle framework have been studied first by [Heckman, Lochner, and Taber \(1999b\)](#) and [Heckman, Lochner, and Taber \(1999a\)](#), who consider the effects of an increase in educational subsidies. Their main focus is to determine the magnitude of general equilibrium effects (which appear to be large), rather than to analyze various educational policies in detail. [Kryvtsov and Ueberfeldt \(2007\)](#), [Lee \(2005\)](#) and [Abraham \(2004\)](#) also analyze educational reforms in a general equilibrium life-cycle framework. Compared to all those papers, our value added is that we allow for dynastic framework with realistically chosen intergenerational persistence in skills and provide a much more comprehensive set of policy reforms. Compared to the last three papers, we use a carefully estimated heterogeneity in skills to evaluate distributional consequences of the educational reforms. In a parallel research, [Gallipoli, Meghir, and Violante \(2007\)](#) study various educational policies in a similar general equilibrium economy with intergenerational links, using NLSY rather than PSID data to estimate the intergenerational correlations in skills.

The paper is organized as follows. Next section describes the differences between European and U.S. educational systems. The model is introduced in Section 3 and the equilibrium is defined in Section 4. Section 5 calibrates the model. Benchmark results are reported in Section 6, and results from the reforms are reported in Section 7. Section 8 concludes.

## 2 U.S. vs. European Educational Systems

This section describes aspects of U.S. and European educational systems<sup>1</sup> that are relevant for our analysis. Those aspects are summarized in Table 1.

The first part of Table 1 shows a fraction of the total costs that is financed from public sources. We define the total costs of education to include both expenditures on educational institutions (both public, e.g. direct public spending, and private, e.g. tuition fees), and public expenditures on education outside educational institutions (e.g. subsidized private spending on living costs). Total costs of education do not include private spending on education outside of educational institutions, e.g. living costs paid by parents (because the data are not available), as well as foregone earnings. The data show that fraction is relatively similar for primary and secondary level, where most of the costs are subsidized. The difference lies at the tertiary level. In the U.S., only 47.7% of the costs is publicly

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<sup>1</sup>The data are taken from [OECD \(2006\)](#). Because of data availability, Europe here includes the following 18 EU countries: Austria, Belgium, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Italy, Ireland, Netherlands, Poland, Portugal, Slovak Republic, Spain, Sweden and United Kingdom.

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**Education Policies and Attainment: U.S. and Europe**

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	U.S.	Europe
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Publicly Financed Costs (% of Total Costs)		
Primary	0.919	0.951
Secondary	0.919	0.951
Tertiary	0.477	0.873
<hr/>		
Public Expenditures (% of GDP)		
Education	0.057	0.055
Total	0.197	0.328
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Educational Attainment		
Primary	0.12	0.33
Secondary	0.49	0.44
Tertiary	0.39	0.23
<hr/>		
Cost per Student (% of GDP per capita)		
Primary	0.224	0.219
Secondary	0.247	0.240
Tertiary	0.709	0.445
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Admission Rates		
Tertiary	0.665	0.455

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Source: [OECD \(2006\)](#). Publicly Financed Costs of Education: Tables B1.1a, B3.2a,b, B5.2, B5.3. Public Expenditures: Table B4.1 (in 2003). Educational Attainment: Tables A1.2a, A1.3a (measured as % of 25-64 year old population). Cost per Student: Tables B1.1a, B2.3a,b, B5.2, B5.3, X2.2.

Table 1: Educational Policies in the United States and Europe

provided. In contrast, Europe subsidizes a much larger fraction of total costs. The fraction ranges from 70% in Poland to 97% in Denmark, Finland and Greece, and equals to 87.3% on average.

Despite that, the total educational expenditures as a fraction of GDP are roughly comparable in the U.S. (5.7%) and Europe (5.5%). There are two reasons that account for that. First, the educational attainment is significantly higher in the U.S. than it is in Europe. 39% of the population between ages 25 and 64 has tertiary education in the U.S. In Europe, this fraction ranges from 11% in Italy to 35% in Sweden, and is only 23% on average. Second, the costs of tertiary education per student are lower in Europe than in the U.S. For instance, at the tertiary level, the U.S. costs per student are 70.9% of GDP per capita, while in Europe the costs are only 44.5% of GDP per capita. The differences are even more pronounced in absolute levels. This paper takes the schooling costs as exogenous, and will not provide any insight into why the educational costs or quality differ between U.S. and Europe. Instead, it focuses on the role of different educational policies in explaining differences in educational attainment.

A second dimension of the educational policies are admission policies. We define the admission policies very broadly. In our view they include direct admission procedures, but also screening during the first year of the university study and eligibility requirements for secondary level students. We compute the implied admission rates as follows. In the United States, 66.61 % of high schools students apply to college.<sup>2</sup> Since 88% of population obtains at least secondary degree, 58.61% of population applies to college. Since 39% of population attains at college, the implied admission rate is 66.5%. For Europe, we compute the admission rates using the data reported by [Jallade \(1992\)](#) for Germany, France, United Kingdom and Spain. As an example, in Germany 45% of population has a secondary degree and is qualified to enter a tertiary education level. Given that 83% of population attains at least secondary degree ([OECD \(2006\)](#)), 54.2% of secondary level students are qualified to enter a tertiary education level. Since the fraction of secondary level students who enter a tertiary system (transfer rate) is 73%, the implied admission rate is 39.6%. We obtain admission rates for France, United Kingdom and Spain in the same way and average. The average implied admission rate is 45.5%, and is significantly lower than in the United States. Hence, Europe appears to have stricter admission policies.

### 3 The Model

We consider three overlapping generations of grandparents, parents and children. They constitute a single decision unit, pool their resources, and maximize the same objective function at the time when their lives overlap. This decision unit is called “a household”. A dynasty is a family line of ancestors and descendants.

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<sup>2</sup>See [U.S. Department of Education \(2001\)](#), Table 186.

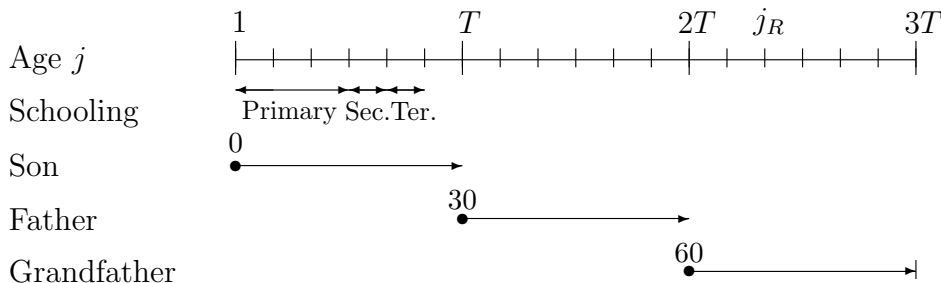


Figure 1: Timeline

A dynasty cares about the utility of all its members. The preferences over a sequence of consumptions per person  $\{c_t\}_{t \geq 0}$  are given by

$$E_0 \sum_{t=0}^{\infty} (\beta(1 + \gamma)^{1-\sigma})^t N_t \frac{c_t^{1-\sigma}}{1-\sigma}, \quad 0 < \beta < 1,$$

where  $\beta$  is discount factor,  $\gamma$  is technology growth rate, and  $N_t$  is dynasty size, which is stochastic and is described later. We abstract from the disutility of labor, as well as the cost of schooling in terms of leisure or any psychological damage.

**Timing.** A model period is 5 years. Each individual is born at age zero and lives no more than 90 years. The lifespan of an individual is divided into three stages of life, each of them equal to  $T = 6$  model periods. The first stage in model periods  $j = 1, \dots, T$  (actual age 0 to 29 years) is youth, and an individual in this stage is called a son. During this stage the family decides whether a son should start working right after the elementary school, continue to secondary level and start working after that, or continue to the tertiary level. In the second stage in model periods  $j = T + 1, \dots, 2T$ , the person is an adult (actual age 30 to 59), and will be called a father. The father spends all his time working. In the last stage in model periods  $j = 2T + 1, \dots, 3T$ , the person is called a grandfather (actual age 60 to 89), and will be retired from period  $j_R = 2T + 2$  (actual age 65). Each period, household members face a probability of dying. The probability of surviving between age  $j$  and  $j + 1$  is given by  $\hat{\psi}_j$ .

The population grows exogenously at rate  $n > 0$ . Following [Fuster, Imrohoroglu, and Imrohoroglu \(2003\)](#), a household consists of  $m = (1 + n)^T$  sons, one father, and  $m^{-1}$  grandfathers. The age of a household is indexed by the age of the sons,  $j = 1, \dots, T$ .

A timeline for a typical household is depicted in [Figure 1](#). Age-1 household consists of  $m$  newborn sons, father of age  $T + 1$  (calendar age 30), and  $m^{-1}$  grandfathers of age  $2T + 1$

(calendar age 60). Age- $T$  household consists of  $m$  sons of age  $T$  (calendar age 25), father of age  $2T$  (calendar age 55), and  $m^{-1}$  grandfathers of age  $3T$  (calendar age 85). If at the end of period  $T$  the son is still alive, the household splits into  $m$  new age-1 households, each with  $m$  newborn sons, one father (one of the  $m$  sons), and  $m^{-1}$  grandfathers (out of one father). All former grandfathers die with certainty at this time. If a son dies before age  $T$  then the dynasty is broken at some point because no new child can be born. When the dynasty dies out, it is replaced by a new dynasty with zero assets.

**Age-Earnings Profiles.** Productivity of an individual is given by the age-earnings profile  $\{\varepsilon_j(h, z)\}_{j=1}^{3T}$ . The age-earnings profile depends on the skill of an individual and on his human capital. The age-efficiency profile is zero for retirees ( $j \geq j_R$ ).

There are four levels of human capital,  $h \in H = \{\bar{h}_0, \bar{h}_P, \bar{h}_S, \bar{h}_T\}$ . Each son is born with a basic level of human capital,  $\bar{h}_0$ . When the son is young, the family can decide to invest in his human capital. The human capital level  $\bar{h}_P$  corresponds to primary school (P) which includes a compulsory pre-school and elementary education in periods  $j = 1, 2, 3$  (age 0-14). Secondary (S) level of education  $\bar{h}_S$  can be achieved in period  $j = 4$  (age 15-19), and tertiary education (T)  $\bar{h}_T$  can be achieved in period  $j = 5$  (age 20-24).

It is assumed that schooling is exclusive so that a person who attends a school cannot simultaneously work. It is also assumed that schooling is sequential: To obtain a human capital level  $\bar{h}_T$ , the son needs to already have human capital level  $\bar{h}_S$ . Finally, the decision to start working is irreversible: once the son starts working, he cannot go back to school. The education is thus completed in period  $j = 5$  (age 20-24) at the latest, and the human capital of an individual is constant since then.

The costs of schooling consist of forgone earnings and direct costs of schooling. The direct costs  $x(h^s)$  depend on the education level.

The idiosyncratic skill level takes four possible values: zero, low, medium, and high,  $z \in Z = \{0, \bar{z}_L, \bar{z}_M, \bar{z}_H\}$ , where zero skill denotes a dead individual. Skills are exogenous and, conditionally on being alive, constant for the whole lifespan of an individual. In addition, the skill of a son is partially inherited from its father in the following way. If a son survives to age  $T$ , he becomes a father, and all his  $m$  children share the same skill  $z^s$  which follows a first-order Markov process,

$$Q(\bar{z}_i, \bar{z}_j) = Prob(z^s = \bar{z}_i, | z^f = \bar{z}_j), \quad i, j \in (L, M, H),$$

where  $z^f$  is the skill of their father. Note that all newborns inside a single household have the same skills which may be different from that of their cousins in the other households in the same dynasty.

**Household States.** In what follows, we will define  $h = (h^s, h^f, h^g) \in H^3$  to be a collection of household members' human capital levels, and  $z = (z^s, z^f, z^g) \in Z^3$  to be their skills. The state of a household of a given age is completely described by vectors  $h$  and  $z$ , and by household assets  $a$ .

Based on individual survival probabilities  $\hat{\psi}$ , on the transition matrix  $Q$ , and on our notational convention that skill is zero if an individual dies, we can completely summarize the survival dynamics of the household by a function  $\psi_j(z', z)$ ,  $j = 1 \dots T$ , that defines the conditional probability of the family having a skill profile  $z'$  next period given  $z$ .<sup>3</sup>

We also define three functions  $\phi_s(z^s)$ ,  $\phi_f(z^f)$  and  $\phi_g(z^g)$  that denote the size of each generation within the dynasty:  $\phi_s(z^s) = m$  if sons are alive and zero otherwise,  $\phi_f(z^f) = 1$  if father is alive and zero otherwise, and  $\phi_g(z^g) = m^{-1}$  if grandfathers are alive and zero otherwise. The total size of the living family is then given by  $\phi(z)$  defined as

$$\phi(z) = \phi_s(z^s) + \phi_f(z^f) + \phi_g(z^g).$$

### 3.1 Aggregate Production Function

We assume that the aggregate production technology is represented by a standard Cobb-Douglas production function,

$$F(K, AL) = K^\alpha (AL)^{1-\alpha},$$

where  $K$  is aggregate capital stock,  $A$  is technology level growing at an exogenous rate  $\gamma$ , and  $L$  is a constant returns to scale aggregator of primary, secondary, and tertiary aggregate human capital  $H_P$ ,  $H_S$ ,  $H_T$ . The aggregator is given by

$$L(H_P, H_S, H_T) = (\theta_P H_P^\rho + \theta_S H_S^\rho + \theta_T H_T^\rho)^{1/\rho}.$$

Different human capital levels are therefore imperfect substitutes, with the elasticity of substitution between them given by  $1/(1-\rho)$ . The share parameters  $\theta_P$ ,  $\theta_S$  and  $\theta_T$  are constant over time and have been normalized to sum to one. The physical capital stock depreciates at rate  $\delta \in (0, 1)$ .<sup>4</sup>

Competition among firms results in prices of inputs being equal to their marginal products,

$$r = F_K(K, AL) - \delta, \tag{1}$$

$$w_i = AF_L(K, AL)L_{H_i}(H_P, H_S, H_T), \quad i = (P, S, T). \tag{2}$$

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<sup>3</sup>The construction is fairly straightforward. For example, if  $z^s$ ,  $z^f$  and  $z^g$  are all nonzero (meaning that everyone is alive, then for  $j < T$ ,  $\psi_j(z', z)$  is given by  $\hat{\psi}_j \hat{\psi}_{j+T} \hat{\psi}_{j+2T}$  if  $z' = z$ , by  $(1 - \hat{\psi}_j) \hat{\psi}_{j+T} \hat{\psi}_{j+2T}$  if  $z' = (0, z^f, z^g)$ , and so on. The transition is different for  $j = T$ , when  $\psi_T(z', z)$  is given by  $Q(z^{s'}, z^s) \hat{\psi}_T \hat{\psi}_{2T}$  for  $z' = (z^{s'}, z^s, z^f)$ ,  $Q(z^{s'}, z^s)(1 - \hat{\psi}_T) \hat{\psi}_{2T}$  for  $z' = (z^{s'}, 0, z^f)$ , and so on.

<sup>4</sup>We omit time subscripts on the quantities because we will only consider steady state allocations.

## 3.2 Government Policies

Government policies consists of educational and tax policies. The educational policies in our model have two dimensions. First, the educational policies are defined by the amount of schooling subsidies. Second, a novel feature of our model is that we explicitly model the admission system at the secondary and tertiary education by allowing for a possibility that people prefer to study, but are not accepted. We allow the admission rates to depend on applicant's skill to model the existing usage of standardized test scores, admission tests or other demonstration of quality by the applicant.

The government provides a schooling subsidy  $d(h^s)$ . Since the skills are assumed to be observed by the government, the schooling subsidy can possibly depend on a student's skill. The private direct cost  $b(h^s)$  to the household is then the difference between the total direct cost and the educational subsidy:

$$b(h^s) = x(h^s) - d(h^s).$$

The other educational policy instrument of the government is the ability to institute admission policies, i.e. to restrict the number of students enrolled at a given level. The probability of being admitted at a school of level  $\bar{h}_i$  can depend on applicant's skill  $z^s$ , and is given by  $\pi_i(z^s)$ . In the aggregate,  $\pi_i(z^s)$  also represents the fraction of applicants of skill  $z^s$  being admitted. Equivalently, we can think of the government as choosing separate capacities for each skill level  $\bar{\chi}_i(z^s)$  that sum to  $\bar{\chi}_i$ .

Tax instruments of the government include a flat tax rate on consumption  $\tau_c$ , a flat tax rate on capital income  $\tau_k$ , a social security tax at rate  $\tau_s$ , and a nonlinear tax schedule  $t(I)$  on taxable income  $I$ . The government uses the tax revenue to finance its consumption  $G$ , social security benefits  $SS$ , as well as educational subsidies. The nonlinear tax system taxes each household member individually, with household assets  $a$  divided equally among all living family members. We define the taxable income of a son, father and grandfather as

$$\begin{aligned} I_j^s(a, h^s, z, s) &= ra/\phi(z) + \varepsilon_j(h^s, z^s)(1-s)w - I_0, \\ I_j^f(a, h^f, z) &= ra/\phi(z) + \varepsilon_{j+T}(h^f, z^f)w - I_0, \\ I_j^g(a, h^g, z) &= ra/\phi(z) + \varepsilon_{j+2T}(h^g, z^g)w - I_0, \end{aligned}$$

where  $I_0$  is tax deduction and the variable  $s \in \{0, 1\}$  indicates whether the sons attends a school ( $s = 1$ ) or not ( $s = 0$ ). The total income tax payments of the whole household in period  $j$  is

$$t_j(a, h, z, s) = \phi_s(z^s)t(I_j^s(a, h^s, z, s)) + \phi_f(z^f)t(I_j^f(a, h^f, z)) + \phi_g(z^g)t(I_j^g(a, h^g, z)), \quad (3)$$

The social security benefits  $SS_j(h^g, z^g)$ ,  $j = j_R, \dots, 3T$  are received by the retired grandfather, and depend, in general, on his human capital and skill. All additional details on the social security benefits are described in detail in Section 5.

### 3.3 Household's Problem

The individual state of an age- $j$  household is given by  $(a, h, z)$ , where  $a$  is the household's joint asset holdings. The after-tax earnings of an age- $j$  household are given by<sup>5</sup>

$$e_j(a, h, z, s) = \begin{cases} \phi_g(z^g)SS_{j+2T}(h^g, z^g) + \\ \quad [\phi_f(z^f)\varepsilon_{j+T}(h^f, z^f)w_{hf} + \phi_s(z^s)\varepsilon_j(h^s, z^s)(1-s)w_{hs}] (1 - \tau_{ss}) - t_j(a, h, z, s) & \text{if } j + 2T \geq j_R, \\ \phi_g(z^g)\varepsilon_{j+2T}(h^g, z^g)w_{hg} + \phi_f(z^f)\varepsilon_{j+T}(h^f, z^f)w_{hf} + \phi_s(z^s)\varepsilon_j(h^s, z^s)(1-s)w_{hs} & \\ (1 - \tau_{ss}) - t_j(a, h, z, s) & \text{otherwise.} \end{cases}$$

#### 3.3.1 Value Function for Age $j = 1, 2, 3$

Let  $V_j(a, h, z)$  be a steady state value function of an age- $j$  household. The value function satisfies for  $j = 1, 2, 3$ ,

$$V_j(a, h, z) = \max_{c, a'} \left\{ \phi(z) \frac{c^{1-\sigma}}{1-\sigma} + \beta(1+\gamma)^{1-\sigma} \sum_{z'} \psi_j(z', z) V_{j+1}(a', h', z') \right\},$$

subject to the budget constraint

$$(1 - \tau_c)\phi(z)c + \phi_s(z^s) b(\bar{h}_P) + (1 + \gamma)a' = (1 + (1 - \tau_k)r)a + e_j(a, h, z, s) + \phi(z)\xi, \quad (4)$$

where  $c$  is consumption of each household member,  $a'$  are savings of the whole household, and  $\xi$  is a lump-sum transfer from accidental bequests from deceased dynasties that are distributed by the government. Finally,  $h'$  is equal to  $h$  in periods 1, 2, and to  $(\bar{h}_P, h^f, h^g)$  in period 3, reflecting the fact that it takes three periods to obtain the primary educational level  $\bar{h}_P$ .

#### 3.3.2 Value Function for Age $j = 4, 5$

In periods  $j = 4, 5$ , sons' human capital may increase, depending on household's choices and on the admission policies. Denote  $\hat{V}_j(a, h, z; s)$  to be the value of making a schooling decision  $s \in \{0, 1\}$ .  $\hat{V}_j$  satisfies

$$\hat{V}_j(a, h, z; s) = \max_{c, a'} \left\{ \phi(z) \frac{c^{1-\sigma}}{1-\sigma} + \beta(1+\gamma)^{1-\sigma} \sum_{z'} \psi_j(z', z) V_{j+1}(a', h', z') \right\},$$

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<sup>5</sup>There are households in which both father and grandfather have died and sons must be supported by the government during their compulsory education. We assume that the support for each orphan son equals the average earnings of a low-skill father with secondary education. This government expenditure is included in its exogenous consumption,  $G$ . It is a minor detail in the model and is not further specified in the budget constraints or in the definition of equilibrium.

subject to the budget constraint

$$(1 + \tau_c)\phi(z)c + s\phi_s(z^s)b(h_+^s) + (1 + \gamma)a' = (1 + (1 - \tau_k)r)a + e_j(a, h, z, s) + \phi(z)\xi, \quad (5)$$

where  $h_+^s$  denotes next level of human capital, defined as  $h_+^s = \bar{h}_S$  if  $h^s = \bar{h}_P$  and  $h_+^s = \bar{h}_T$  if  $h^s = \bar{h}_S$ . Next period human capital  $h'$  is given by  $h' = h$  if  $s = 0$  and  $h' = (h_+^s, h^f, h^g)$  if  $s = 1$ .

In period 4 the value function satisfies

$$V_4(a, h, z) = \max \left\{ \hat{V}_4(a, h, z; 0), \pi(\bar{h}_S, z^s) \hat{V}_4(a, h, z; 1) + [1 - \pi(\bar{h}_S, z^s)] \hat{V}_4(a, h, z; 0) \right\},$$

because people who apply for the school (choose  $s = 1$ ) may be rejected due to admission policies. In period 5, conditional on  $h^s = \bar{h}_S$ , the household can choose whether the son obtains tertiary education.

$$V_5(a, h, z) = \max \left\{ \hat{V}_5(a, h, z; 0), \pi(\bar{h}_T, z^s) \hat{V}_5(a, h, z; 1) + [1 - \pi(\bar{h}_T, z^s)] \hat{V}_5(a, h, z; 0) \right\}.$$

However, if the son has not attended secondary education in period 4 then  $s = 0$  must be chosen:

$$V_5(a, h, z) = \hat{V}_5(a, h, z; 0).$$

### 3.3.3 Value Function for Age $j = T$

At the end of period  $T$ , the household transforms itself into an age-1 household in the following way: the grandfathers reach the end of their life, fathers become grandfathers, sons become fathers, and new sons are born. Since there is no schooling in period  $j = T$ , the value function  $V_T$  is given by

$$V_T(a, h, z) = \max_{c, a'} \left\{ \phi(z) \frac{c^{1-\sigma}}{1-\sigma} + \beta(1 + \gamma)^{1-\sigma} m \sum_{z'} V_1(a', h', z') \psi_T(z', z) \right\},$$

subject to the budget constraint

$$(1 - \tau_c)\phi(z)c + m(1 + \gamma)a' = (1 + (1 - \tau_k)r)a + e_T(a, h, z, s) + \phi(z)\xi. \quad (6)$$

The vector of skills of the new age-1 household is given by  $z' = (z^{s'}, z^s, z^f)$ , where  $z^{s'}$  is newborns' skill draw. The vector of human capital levels of the new age-1 household is given by  $h' = (\bar{h}_0, h^s, h^f)$ . In words, the newborn sons partially inherit their father's skill and start with the basic human capital level, while each of the current sons becomes a father and the current father becomes grandfather, both keeping their skills and human capital.

## 4 Recursive Competitive Equilibrium

Let  $(a, h, z) \in (A \times H^3 \times Z^3)$  be an individual household's state vector. The optimal policy functions are given by  $\{a_j, h_j, s_j\}_{j=1}^T$ , where we define  $s_j = 0$  whenever no schooling choice is possible, and  $s_3 = 1$  and  $h_j \in H^3$  denotes the vector of next period human capital for the whole household. Let  $\{\lambda_j\}_{j=1}^T$  be an age-dependent measure of households over the individual states. Its law of motion for each  $(a', h', z') \in (A \times H^3 \times Z^3)$  and for  $j = 1, 2, 3$ , is

$$\lambda_{j+1}(a', h', z') = \sum_{\{(a,h,z):a'=a_j(a,h,z),h'=h_j(a,h,z)\}} \psi_j(z', z) \lambda_j(a, h, z).$$

For  $j = 4, 5$ , the law of motion is given by

$$\begin{aligned} \lambda_{j+1}(a', h', z') = & \sum_{\{(a,h,z):a'=a_j(a,h,z;1)\}} s(a, h, z) \pi(h_+^s, z^s) \psi_j(z', z) \lambda_j(a, h, z) \\ & + \sum_{\{(a,h,z):a'=a_j(a,h,z;0)\}} [1 - s(a, h, z) \pi(h_+^s, z^s)] \psi_j(z', z) \lambda_j(a, h, z). \end{aligned}$$

The law of motion for the measure of age-1 households, for each  $(a', h', z') \in (A \times H^3 \times Z^3)$  with  $z' = (z^{s'}, z^s, z^f)$ , is, for  $h' = (\bar{h}_0, h^s, h^f)$

$$\lambda_1(a', h', z') = \sum_{\{(a,h,z):a'=a_T(a,h,z)\}} \phi_T(z^s) \psi_T(z', z) \lambda_T(a, h, z),$$

and zero otherwise.

Broken dynasties are replaced by newborn dynasties with zero assets and with a representative composition of skills and human capital of  $j = 1$  households.

The Stationary Recursive Competitive Equilibrium is defined as follows:

**Definition 1** *Given educational policies  $(d, \pi, \bar{\chi})$ , tax policies  $(SS, t, \tau_k, \tau_c, \tau_{ss})$ , and government consumption  $G$ , a stationary recursive competitive equilibrium is a set of value functions  $\{V_j(\cdot)\}_{j=1}^T$ , household policy functions  $\{c_j(\cdot), a'_j(\cdot), s_j(\cdot)\}_{j=1}^T$ , factor prices  $(w_P, w_S, w_T, r)$ , accidental bequest transfers  $\xi$ , measures  $\{\lambda_j\}_{j=1}^T$  such that:*

1. *given government policies and prices, household policy functions solve problems (4), (5) and (6);*
2. *the prices  $(w_P, w_S, w_T, r)$  are competitive, i.e. given by (2);*
3. *aggregate levels of capital  $K$ , human capital  $H_i, i \in \{P, S, T\}$ , consumption  $C$ , school enrollment  $S(\bar{h}_i, \bar{z}_i)$ , schooling subsidies  $D$ , schooling costs  $X$ , social security benefits*

$SS$  and tax revenues  $T$  are

$$\begin{aligned}
K &= \sum_{j,a,h,z} a \lambda_j(a, h, z) (1+n)^{1-j}, \\
H_i &= \sum_{j,a,h,z} [\phi_g(z^g) \varepsilon_{j+2T}(\bar{h}_i, z^g) 1_{h^g=\bar{h}_i} + \phi_f(z^f) \varepsilon_{j+T}(\bar{h}_i, z^f) 1_{h^f=\bar{h}_i} \\
&\quad + \phi_s(z^s) \varepsilon_j(\bar{h}_i, z^s) 1_{h^s=\bar{h}_i}] \lambda_j(a, h, z) (1+n)^{1-j}, \\
C &= \sum_{j,a,h,z} \phi(z) c_j(a, h, z) \lambda_j(a, h, z) (1+n)^{1-j}, \\
S(\bar{h}_P, z^s) &= \sum_{j=1}^3 \sum_{a,h,z} \phi_s(z^s) \lambda_j(a, h, z) (1+n)^{1-j}, \\
S(\bar{h}_i, z^s) &= \sum_{a,h,z^s} \phi_s(z^s) s_j(a, h, z) \pi(\bar{h}_i, z^s) \lambda_j(a, h, z) (1+n)^{1-j} \text{ for } i = S, T, \\
D &= \sum_{\bar{h}_i, z^s} d(\bar{h}_i, z^s) S(\bar{h}_i, z^s), \\
X &= \sum_{\bar{h}_i, z^s} x(\bar{h}_i) S(\bar{h}_i, z^s), \\
SS &= \sum_{j=j_R}^{3T} \sum_{a,h,z} \phi_g(z^g) SS_j(h^g, z^g) \lambda_j(a, h, z) (1+n)^{1-j}, \\
T &= \sum_{j,a,h,z} t_j(a, h, z, s_j(a, h, z)) \lambda_j(a, h, z) (1+n)^{1-j};
\end{aligned}$$

4. accidental bequests are

$$(1+n)\xi = (1+r) \sum_{j,a,h,z} \phi_j(z) (1 - \psi_j(z', z)) a_j(a, h, z) \lambda_j(a, h, z) (1+n)^{1-j};$$

5. the measures  $\{\lambda_j\}_{j=1}^T$  are time invariant;

6. the government's budget is balanced

$$G + D + SS = \tau_k r K + \tau_c C + \tau_{ss} (w_P H_P + w_S H_S + w_T H_T) + T;$$

7. and the aggregate feasibility constraint holds,

$$C + (1+n)(1+\gamma)K + G + X = F(K, L) + (1-\delta)K.$$

## 5 Data and Calibration

This section describes how we choose the age-efficiency profile  $\varepsilon$ , intergenerational correlations in skills  $Q$ , survival probabilities  $\hat{\psi}$ , government tax and educational policies, and how we calibrate the remaining model parameters  $\sigma$ ,  $\beta$ ,  $\gamma$ ,  $n$ ,  $\delta$ ,  $\alpha$  and  $\theta_i$ .

### 5.1 The Age-Efficiency Profile of Earnings

Our data source for estimation of the age-efficiency profile of earnings  $\varepsilon$  is the PSID data for years 1968-2005. To each individual in our sample<sup>6</sup> we assign a primary schooling level if she/he completed less than 12 grades, secondary schooling level if she completed at least 12 but less than 16 grades, and a tertiary schooling level if she completed 16 grades or more. Overall, 11.7% of people in the sample has primary schooling level, 57.8% has secondary schooling level, and 30.5% has tertiary schooling level. Those fractions differ somewhat from the nationwide educational attainment data, mainly because the composition of our sample does not exactly replicate the composition of U.S. population.

Labor earnings of each individual are defined as total labor earnings of the family she/he is heading. We prefer to use an extensive definition of earnings, rather than labor earnings of the head only, because we believe that family earnings are more relevant to the schooling choices of children. We deflate labor earnings by the Consumer Price Index and express all variables in terms of 2005 dollars.

#### 5.1.1 Decomposing Individual Earnings

As a first step in the computation of the age-efficiency we decompose the earnings profile of each individual in our PSID sample into a deterministic component that depends only on age and education, an idiosyncratic fixed effects component, and a transitory error term. We associate the fixed effect component with the permanent skill shock in the model.

To obtain a life-cycle profile of earnings by education and age, we assign an individual of age  $a$  to five age bins  $a - 2$ ,  $a - 1$ ,  $a$ ,  $a + 1$  and  $a + 2$ . Let  $y_{\iota a t}^h$  be time  $t$  logarithm of labor earnings of an individual  $\iota$  who has attained education level  $h$  and is currently of age  $a$ . We

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<sup>6</sup>We restrict the data sample to heads of household between 18 and 65 years. We exclude observations where earnings are top coded, earnings are less than one half of the minimum wage prevailing in that year and when an individual works less than 520 and more than 5096 hours per year. We also require that all these restrictions must be satisfied in at least two consecutive years. If an individual satisfies these criteria for two consecutive years several times, we treat him as two separate individuals. Finally, we exclude all individuals who belong to the SEO subsample of PSID. After these adjustments, our final sample contains 82 593 individual/year observations.

control for the time effects<sup>7</sup>, and assume that  $y_{iat}^h$  takes the following form:

$$y_{iat}^h = b_a^h + v_t^h + u_{iat}^h,$$

where  $b_a^h$  is the age effect,  $v_t^h$  is the time effect, and  $u_{iat}^h$  is the residual. For each educational level we run a regression of  $y_{iat}^h$  on the full set of age and time dummies and estimate the coefficients  $b_a^h$  and  $c_t^h$ .

The residual term  $u_{iat}^h$  is decomposed into a fixed effect component and a transitory error term. Although the residual  $u_{iat}^h$  has zero mean by construction, it does not necessarily have the same distribution over time and age. We assume that the distribution of the residual does not depend directly on time and so  $u_{iat}^h$  does not depend directly on time as well. The residual can then be written as

$$u_{ia}^h = \phi_a^h \eta_i + \sigma_a^h \epsilon_{ia}, \quad (7)$$

where  $\eta_i$  is the idiosyncratic fixed effect with mean zero and unitary variance,  $\epsilon_{ia}$  is the idiosyncratic life-cycle transitory shock when the individual is of age  $a$ , with mean zero and unitary variance.<sup>8</sup> The fixed effect factor loadings  $\phi_a^h$  determine the impact of the fixed effect on one's earnings in age  $a$ , while  $\sigma_a^h$  determines the impact of the transitory term.

Equation (7) implies that for any age bin  $a$  between 20 and 63,

$$Var(u_{ia}^h) = (\phi_a^h)^2 + (\sigma_a^h)^2, \quad (8)$$

$$Cov(u_{ia}^h, u_{ia+k}^h) = \phi_a^h \phi_{a+k}^h, \quad 0 \leq k \leq 63. \quad (9)$$

We estimate 88 parameters  $\phi_a^h$  and  $\sigma_a^h$  using the empirical values for 44 variances (8) and  $44 * 43/2 = 946$  covariances (9). We use the minimum distance estimator to obtain the estimates. The constants  $\sigma_a^h$  increase with both age and education. The fixed effect factor loadings peak around the age of 50, when they are between 0.22-0.26, depending on the education. They are approximately of the same magnitude as in [Storesletten, Telmer, and Yaron \(2004\)](#), who estimate it to be 0.21.

### 5.1.2 Assigning Individual Fixed Effects

Consistently with the model assumption that there are three skill levels we assume that the fixed effect  $\eta$  can take values  $\eta_L < 0$  (low shock),  $\eta_M$  (medium shock) and  $\eta_H > 0$  (high

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<sup>7</sup>Both cohort effects and time effects cannot be identified separately (see e.g. [Heathcote, Storesletten, and Violante \(2005\)](#)). Consistently with the literature we choose to work with time effects and ignore cohort effects.

<sup>8</sup>Since we assume that  $\epsilon_{ia}$  is iid, our statistical model of earnings is significantly simpler than the statistical model of earnings in [Storesletten, Telmer, and Yaron \(2004\)](#). We provide a simpler decomposition because our model does not focus on the role of  $\epsilon_{ia}$ . It is therefore meaningful to have a setup where the transitory component is as simple as possible. On the other hand,  $\epsilon_{ia}$  cannot be set identically to zero and all variation in earnings attributed to the fixed effect, because earnings in data fluctuate over time.

shock). We assign a fixed effect to each individual according to the following procedure. For an individual  $\iota$  we compute the mean of transitory shocks, conditional on the value of the fixed effect being low, medium and high:

$$\zeta_{\iota}^{h,i} = \sum_a [u_{\iota a}^h - \eta_i \phi_a^h], \quad i = (L, M, H).$$

Conditionally on the permanent shock being  $\eta_i$ , the expected value of  $\zeta_{\iota}^{h,i}$  is zero, while the expected values of  $\zeta_{\iota}^{h,\hat{i}}$ ,  $\hat{i} \neq i$  is not. We therefore compare the absolute values of  $\zeta_{\iota}^{h,L}$ ,  $\zeta_{\iota}^{h,M}$  and  $\zeta_{\iota}^{h,H}$  for each agent  $\iota$ , and assign the permanent shock depending on which value is closest to zero.

The values of  $\eta_L$ ,  $\eta_M$  and  $\eta_H$  are chosen to be such that a fraction of people with each of the fixed effects will be equal to one third, and that the expected value of the shock is zero. The resulting values are  $\eta_L = -1.1056$ ,  $\eta_M = -0.7402$  and  $\eta_H = 1.8458$ . The age-efficiency profiles are then computed by equating  $\varepsilon_j(h, \bar{z}_i)$  to the mean of  $e^{b_a^h + \phi_a^h \eta_i}$  over ages  $a$  that correspond to the model age  $j$ , and normalized by average earnings. Figure 2 shows the age profiles of earnings by skill level and education.

The age-earning profiles are shown for ages 4 to  $j_R$  in case of primary education, 5 to  $j_R$  in case of secondary education, and 6 to  $j_R$  in case of tertiary education. They are normalized so that the average earnings are equal to 1.

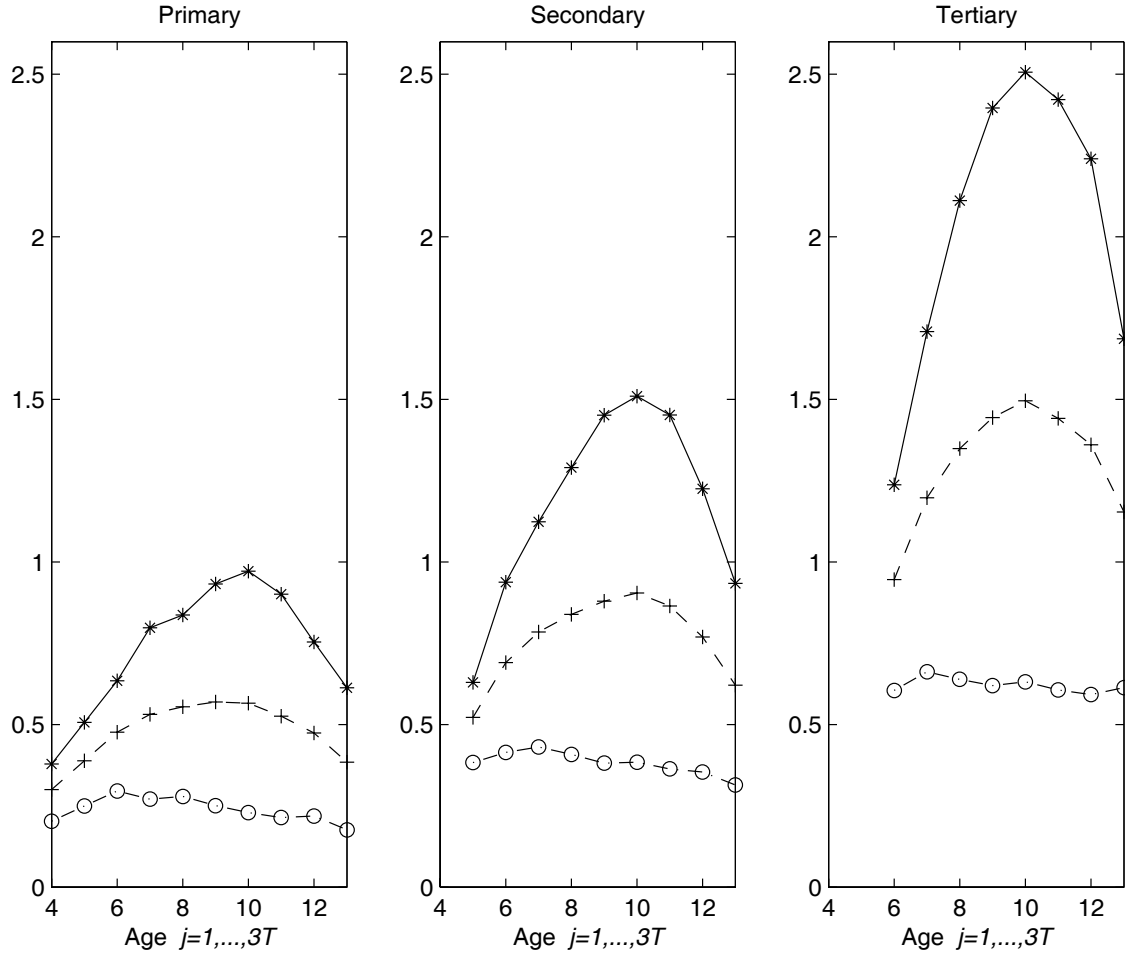
The figure shows that there is a significant dispersion in earnings across age, skills, and education. Maximum earnings are achieved by high skilled individual with tertiary education. At its peak, such an individual earns 2.5 times the average earnings. On the other hand, the lowest earnings achieved by low skilled individuals with primary education. At the beginning and end of the life cycle they are equal to about 0.25 times the average earnings. The age-earning profiles peak around model age 10, which corresponds to actual age 50 for high and medium skills. For low skills they peak at earlier ages.

As might be expected, education is the most beneficial to high skilled individual, who tend to realize the largest gains. Low skilled individuals, on the other hand, gain relatively little from being educated. Even if they achieve tertiary education their earnings are significantly below average.

## 5.2 Intergenerational Correlations in Skills

Each individual in the data set is now paired with a particular value of the skill shock  $z$ . PSID allows one to identify parents and sons who are both in the sample. Our sample includes 3435 father/son pairs. The implied transition matrix for skills  $Q(z^s, z^f)$  is in Table 2.

The intergenerational correlations in skills have, in general, the expected pattern: son's skills are positively correlated with father's skills. For instance, 52% of low skilled fathers have a low skilled son, while only 32.1% of high skilled fathers have a low skilled son. Or,



Notes: Life-cycle labor productivity profiles in efficiency units by primary, secondary, and tertiary education attainment and skills (low ( $\circ$ ), medium ( $+$ ) and high ( $*$ )).

Figure 2: Efficiency Profiles by Education Attainment and Skills

only 23.3% of low skilled fathers have a high skilled son, while 37.3% of high skilled fathers have a high skilled son.

### 5.3 Mortality Rates

Since the economywide frequencies of sex, race and education differ from their frequencies in the PSID sample, economywide mortality rates are not a good choice for calibrating the survival probabilities  $\hat{\psi}$ . We therefore use the mortality rates from [Brown, Liebman, and Pollet \(2002\)](#), who estimate them by age (25-100), sex, education and race using data from the National Longitudinal Mortality Survey. We aggregate the mortality rates by sex, race

U.S. Intergenerational Correlations in Skills			
Father's Skill	Son's Skill		
	Low	Medium	High
Low	0.519	0.248	0.233
Medium	0.402	0.300	0.298
High	0.321	0.306	0.373

Table 2: U.S. Intergenerational Correlations in Skills

and education, using frequencies from the PSID sample.<sup>9</sup>

## 5.4 Tax Policies

The tax function  $t(I)$  is taken to be a piecewise linear function with marginal income tax rates equal to the U.S. statutory rates for 2004. The income tax brackets are described in Table 3. As in Ventura (1999), we define the income tax brackets to be the ratios of total income to average income. We do not make assumptions about the tax deduction  $I_0$ . Instead we find such a level of tax deduction that clears the government budget constraint in the benchmark steady state. The resulting tax exemption in the benchmark steady state is 23.87% of the average earnings.<sup>10</sup> The capital tax rate  $\tau_k = 0.36$  is taken from Mendoza, Razin, and Tesar (1994).

The replacement rate of retirement benefits is equal to 44% of the average earnings. The marginal replacement rate is 90% for earnings below 20% of the average, 33% for earnings above that and below 125%, and 15% for earnings above 125% and below 246% of the average earnings. The social security tax is set at  $\tau_{SS} = 0.124$  up to the maximum level of earnings 87,900 USD in 2004, which is 1.78 of the average earnings.<sup>11</sup> Any difference between social security expenditures and receipts are included in government expenditures  $G$ .

## 5.5 Educational Policies

The parametrization of the U.S. costs of education and of schooling subsidies is based on the data in Table 1. In particular, the costs of schooling  $x(h_i)$  are equated to the costs per student as a fraction of GDP per capita, and are equal to 0.224 for primary, 0.259 for

<sup>9</sup>They consider three educational categories: people with less than high school, people with high school, but not with college, and people with at least college degree. This categorization is consistent with our definition of educational groups.

<sup>10</sup>We have experimented with clearing the government budget constraint by adjusting the marginal tax rates instead of the exemption level. The results were almost identical.

<sup>11</sup>See data in Social Security Old-Age, Survivors, and Disability Insurance (OASDI).

<b>Marginal Tax Rates</b>				
Tax Rate $\tau_m$	U.S. Tax System Taxable Income		Model Fraction of Average Income	
	>	$\leq$	>	$\leq$
.10	\$0	\$ 7,300	$\iota_0 = 0$	$\iota_1 = 0.2I^*$
.15	\$7,300	\$29,700	$\iota_1 = 0.2I^*$	$\iota_2 = 0.81I^*$
.25	\$29,700	\$71,950	$\iota_2 = 0.81I^*$	$\iota_3 = 1.97I^*$
.28	\$71,950	\$150,150	$\iota_3 = 1.97I^*$	$\iota_4 = 4.11I^*$
.33	\$150,150	\$326,450	$\iota_4 = 4.11I^*$	$\iota_5 = 8.93I^*$
.35	\$326,450	$\infty$	$\iota_5 = 8.93I^*$	$\infty$

Notes:  $\iota_0, \iota_1, \dots, \iota_5$  are tax brackets,  $I^*$  is the average income.

Table 3: U.S. and Model Marginal Tax Rates.

secondary, and 0.709 for tertiary education. The schooling subsidies  $d(h_i)$  are equal to the fraction of costs that is publicly financed, i.e. to 0.919, 0.919, and 0.477 for the three levels of education.

We calibrate the admission rates  $\pi(\bar{h}_S, z)$  and  $\pi(\bar{h}_T, z)$  to be such that the attainment of secondary and tertiary education is the same as in the United States. In choosing the admission rates we assume that they depend on skills, and that higher skill individuals have priority in the admission process. That is, medium skill applicants are admitted only if all high skill applicants have already been accepted, and similarly for low skill applicants.

## 5.6 Remaining Parameters

We set the government expenditures to be 19.7% of GDP. This includes expenditures on education, which are equal to 5.7% of GDP and government consumption, which is equal to 14% of GDP.<sup>12</sup> The discount factor  $\beta$  is chosen to yield the capital output ratio of 3.00. The coefficient of relative risk aversion  $\sigma$  is set to 2, and the capital share  $\alpha$  is 0.34. The parameter  $\rho$  is assumed to be 0.5, which is in the middle of the estimates in [Gallipoli, Meghir, and Violante \(2007\)](#), who estimate it to be between 0.32 and 0.68. This implies that the elasticity of substitution between various human capital levels is 2. The share parameters  $\theta_P, \theta_S, \theta_T$  are calibrated to be such that the equilibrium earning profiles in the benchmark model coincide with the estimated age-efficiency profiles. Depreciation  $\delta$  is 0.04. The annual technology growth rate is 1.65% and the population growth rate is 1.2%. All parameters are summarized in table 4.

<sup>12</sup>The data are taken from [OECD \(2006\)](#), Table B4.1, and other OECD sources.

<b>Parameters</b>		
<b>Population</b>		
$j_{3T}$	= 18	Maximum lifetime (90 years)
$j_R$	= 14	Retirement age (65 years)
$\bar{n}_{USA}$	= 0.012	Population growth rate U.S.
$\psi$		Survival probabilities
<b>Utility</b>		
$\beta$	= 0.99	Annual discount factor
$\sigma$	= 2.0	Relative risk aversion
<b>Production</b>		
$\gamma$	= 0.0165	Annual technology growth
$\delta$	= 0.04	Annual depreciation rate
$\alpha$	= 0.34	Capital share
$\rho$	= 0.50	Elasticity of substitution
$[\theta_P, \theta_S, \theta_T]$	= [0.135, 0.375, 0.490]	Share Parameters
$e$		Earnings profiles
<b>Fiscal Policy</b>		
$\tau_k$	= 0.36	Capital income tax rate
$\tau_c$	= 0.055	Consumption tax rate
$G$	= 0.14	Government consumption, (% GDP)

Table 4: Parameters

<b>Educational Outcomes</b>			
	U.S.	Educational Reforms	
	Benchmark	Admission based	Tax based
Admission Rates for Tertiary Schooling			
Low	0	0	0
Medium	0.907	0	0.907
High	1.000	0.512	1.000
Educational Attainment			
Primary	12.0	12.6	12.0
Secondary	49.0	72.5	41.2
Tertiary	39.0	14.9	46.8

Table 5: Educational Outcomes

## 6 Benchmark Results

The calibrated tertiary admission rates are such that all high skilled individuals are accepted, 90.7% of medium skilled applicants are accepted, and none of low skilled applicants is accepted. On average, the probability of being accepted at tertiary level is 62.75%, which is very close to the U.S. acceptance rate of 66.5% discussed earlier. At a secondary level all applicants with medium and high skill and 71.46% of low skill applicants are accepted.

The remaining aggregate statistics is presented in table 6. The model generates the Gini coefficient of 0.332 for income, and 0.532 for assets. The model matches inequality in income that has been found in the data reasonably well. The Gini coefficient of asset inequality found in the data is around 0.79, see [Castañeda, , Díaz-Giménez, and Ríos-Rull \(2003\)](#). This is higher than what the model predicts, but the model does a better job than basic incomplete market models with heterogeneous agents (see [Aiyagari \(1994\)](#)).

The tax deduction that clears the government budget constraint equals to 23.9% of average earnings.

### 6.1 Efficiency of Educational Decisions

We do not attempt to define any notion of “second best” efficiency since any such candidate would be extremely hard to compute and would take us too far away from the main goal of the paper. Nevertheless, it is useful to compare the schooling decisions with some measure of efficiency. To do so, we look at how well the schooling system allocates skills across educational levels. We compare the results with the first best allocations of skills across educational levels.

In the first best allocation the social planner assigns education on the basis of comparative advantage. The comparative advantage is measured by the expected present discounted value

<b>Aggregate Levels and Prices</b>			
	U.S.	Educational Reforms	
	Benchmark	Admission based	Tax based
Capital	0.279	0.287	0.269
Labor Supply	1.512	1.386	1.569
Primary	0.069	0.072	0.067
Secondary	0.534	0.795	0.424
Tertiary	0.909	0.518	1.080
Output	0.464	0.427	0.468
Consumption	0.263	0.236	0.267
Capital-Output Ratio	3.00	3.36	2.88
Interest Rate	4.46	3.81	4.73
Wage			
Primary	0.203	0.195	0.204
Secondary	0.203	0.164	0.226
Tertiary	0.203	0.265	0.184
Inequality (Gini)			
Assets	0.532	0.606	0.509
Income	0.332	0.430	0.309
Tax Exemption	0.239	0.239	0.160

Table 6: Aggregate Allocations and Prices

Efficiency of Educational Decisions				
Skill	First Best	U.S.	Educational Reforms	
		Benchmark	Admission based	Tax based
% of Skill Population Enrolled at Primary Level				
All	100.0	100.0	100.0	100.0
% of Skill Population Enrolled at Secondary Level				
Low	88.5	71.4	71.4	71.4
Medium	100.0	100.0	99.6	100.0
High	100.0	99.4	98.9	100.0
% of Skill Population Enrolled at Tertiary Level				
Low	0	0	0	0
Medium	57.8	66.7	0	85.8
High	100.0	66.2	50.7	75.8

Table 7: Efficiency of Educational Decisions

of earnings, with the interest rate being equal to the discount rate. The first column of table 7 shows that in the first best allocation all individuals with high skills are assigned tertiary education. In the optimum, the social planner is indifferent between assigning secondary and tertiary schooling to medium skilled agents, and 57.8% of agents with medium skills are assigned tertiary education. No low skilled agent is assigned to tertiary education.

As one can see from the second column of table 7, the benchmark allocation is quite inefficient: only 66.2% of high skill individuals apply and enroll at the tertiary level. On the other hand, 73.4% of medium skilled individuals apply and 66.7% are accepted at the tertiary level. As in the first best allocation, none of low skilled agents is accepted at the tertiary level. The benchmark results thus show that the U.S. schooling system leads to too few high skilled agents and too many medium skilled agents with tertiary schooling.<sup>13</sup>

## 6.2 Intergenerational Mobility in Education and Earnings

The intergenerational mobility in education is summarized by a transition matrix relating father's and son's education.<sup>14</sup> The first panel of table 8 shows the intergenerational mobility in education found in our PSID sample. There is a substantial correlation of educational decisions across generations. For instance, 61.9% of households where the son has tertiary

<sup>13</sup>Note that we obtain those results despite the assumption that the admission system admits higher skilled applicants first. If the admission system did not discriminate among different skills, the results would be even further away from the first best allocation.

<sup>14</sup>To simplify the analysis, when discussing changes in inequality we omit other dimensions like grandfather's education.

education have father with tertiary attainment, while only 13.7% have a father with primary education.

The benchmark model matches well the U.S. human capital mobility at tertiary sons' level. The benchmark data differ by no more than 2% from what was found in the data. The results at primary and secondary level match U.S. data less well. The model predicts that, conditional on father's education, son's secondary attainment is 5-10% higher than what we see in the data, and primary attainment is 5-10% lower.

To assess the intergenerational mobility in earnings we compute the intergenerational correlation in the present value of earnings. The model matches the U.S. data extremely well. In both cases, the intergenerational correlation is 0.151.

The persistence of educational decisions across generations can be a result of either the persistence of skills across generations, or of market incompleteness, or of the financial constraints. If wealth is highly persistent across generations (as one would expect) then both market incompleteness and financial constraints may decrease the intergenerational mobility in education, since they make the household's educational decisions dependent on household's wealth. In the third panel of table 8 we show that the intergenerational correlation in the present value of earnings in the first best allocation is indeed smaller than in the benchmark model, and is equal to 0.118. This correlation comes purely from the autocorrelation of skills across generations.

To assess whether the difference between the first best allocation and the benchmark allocation is due to market incompleteness or due to the financial constraints, we analyze the importance of credit constraints. We define the percentage of credit constrained households as in [Carneiro and Heckman \(2002\)](#): It is the gap between the percentage enrollment in the highest income quartile for each ability level and the percentage enrolled in the other income quartiles. We find that 16.6% of households are constrained by this definition. These results are somewhat higher than the estimates of [Carneiro and Heckman \(2002\)](#), where only 5.15% students' households are financially constrained. Nevertheless, the vast majority of non-applicants in the benchmark U.S. economy decides to start working because the cost of schooling plus the opportunity cost of son's forgone earnings are too high, or put differently, the expected return on tertiary education is too low.

## 7 European Educational Policies

We now consider a reform where United States adapt European schooling subsidies. In Europe, 95.1% of costs at the primary and secondary level, and 87.3% of the costs at the tertiary level are government paid (see Table 1).

We consider the following two variants of the educational reform. In the first one, called *admission based reform*, we fix the tax system and let the government balance its budget by adjusting the admission rates. That is, the government will subsidize schooling more but for budgetary reason decrease the admission rates. In the second reform, called *tax based*

<b>Intergenerational Mobility in Education</b>			
Father's Attainment	Son's Attainment		
	Primary	Secondary	Tertiary
U.S. Data			
Primary	0.167	0.696	0.137
Secondary	0.046	0.700	0.254
Tertiary	0.000	0.381	0.619
<i>Intergenerational Correlation of Earnings: 0.151</i>			
U.S. Benchmark Model			
Primary	0.202	0.649	0.148
Secondary	0.146	0.601	0.252
Tertiary	0.116	0.284	0.599
<i>Intergenerational Correlation of Earnings: 0.151</i>			
First Best			
Primary	0.006	0.564	0.376
Secondary	0.057	0.545	0.399
Tertiary	0.040	0.438	0.522
<i>Intergenerational Correlation of Earnings: 0.118</i>			
Admission based Reform			
Primary	0.220	0.677	0.103
Secondary	0.139	0.718	0.143
Tertiary	0.105	0.707	0.188
<i>Intergenerational Correlation of Earnings: 0.089</i>			
Tax based Reform			
Primary	0.190	0.569	0.241
Secondary	0.152	0.495	0.354
Tertiary	0.117	0.286	0.598
<i>Intergenerational Correlation of Earnings: 0.126</i>			

Table 8: Intergenerational Mobility

*reform*, we fix the admission rates at the benchmark level and adjust tax exemption level to balance the government budget.<sup>15</sup> The admission based reform constitutes a move toward the European educational policies. The tax based reform is therefore a hybrid that adopts European educational subsidies but not European admission policies.

Both reforms impact the economy very differently. In the admission based reform the government must significantly cut the admission rates at the tertiary level in order to balance the budget. Table 5 shows the changes in the educational attainment. Only 51.2% of high skilled individuals are accepted, and no low or medium skilled individuals are accepted. As a result, tertiary educational attainment decreases sharply from 39% to only 14.9%. The secondary attainment is almost unaffected. In the tax based reform the government must decrease the tax deduction to 16% of average income to balance the budget. Tertiary educational attainment increases from 39% to 46.8%, while the secondary educational attainment stays again virtually unchanged.

The results of the admission based reform are consistent with the fact that tertiary educational attainment is lower in Europe. However, the model predicts tertiary educational attainment that is lower than the European ones: in Europe the tertiary attainment is 23% on average, while the model predicts 14.9%.<sup>16</sup>

The second and third column of table 6 shows the steady state aggregate allocations and equilibrium outcomes under both reforms. Since investment in human education is a substitute for investment in assets, the capital-output ratio substantially increases in the admission based reform and falls in the tax based reform. Steady state output falls in the admission based reform by 8.0% and increases in the tax based reform by 0.8%.

Since the educational attainment varies significantly in each steady state, differences in labor supply and equilibrium wages are large. As tertiary attainment significantly falls in the admission based reform, the scarcity of this type of labor increases its wage by 30.7% compared to the benchmark steady state, and lowers the wage for secondary attainment. As a consequence, inequality in assets and income significantly increases.<sup>17</sup> On the other hand, the tax based reform increases the equilibrium wage for individuals with secondary education by 11.3%. Increased tertiary attainment also lowers the wage for graduates from tertiary institutions by 8.9%. Changes in wages contribute to an overall decline in inequality, and both income and asset inequality falls.

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<sup>15</sup>We have also experimented with an alternative scenario where the government uniformly adjusts the marginal income tax rates, rather than changing the exemption level. The results were almost the same.

<sup>16</sup>One potential explanation is that there are additional differences between European and U.S. educational systems that are beyond the scope of this paper, namely that the tertiary education is cheaper in Europe.

<sup>17</sup>Those results imply that the lower wealth and income inequality found in European data are caused by redistribution provided by other than educational government policies.

## 7.1 Efficiency of the Educational Decisions

The admission based reform increases the inefficiency since only 50.7% of high skilled individuals are admitted. This is so mainly because of the sharp decline in the admission rates: Since the wages at tertiary level increase significantly, almost all high skilled individuals now apply to tertiary education. Interestingly, despite the fact that the admission rates are now so low, the admission based reform leads to more equal opportunity of studying at the tertiary education level.

The tax based reform, on the other hand, increases the efficiency of schooling decisions. 75.8% of high skilled individuals now study at tertiary level, an increase of 9.6% from the benchmark. The tax based reform thus weakens the link between the tertiary schooling decisions and household wealth.

We also look at the importance of credit constraint in both reforms. We find that the fraction of people that are credit constrained significantly decreases in both reforms. Under the admission based reform, only 0.16% of people are credit constrained, while under the admission based reform this fraction is 8.43%. The reason why there is more credit constrained people under the tax based reform is that tax deduction decreases, and the tax system is not as favorable to low income people.

## 7.2 Intergenerational Mobility in Earnings

The admission based reform decreases the autocorrelation of educational decisions even more than in the first best allocation. As table 9 shows, the intergenerational correlation in the present value of earnings is now 0.089, even lower than in the first best allocation. This is caused again by the fact that almost all high skilled individuals apply at the tertiary schooling level, and wealth does not determine their success.

The tax based reform decreases the intergenerational correlation in the present value of earnings from 0.151 to 0.126. This is caused by the fact that, due to lower subsidies, more individuals with father that has achieved only primary or secondary schooling level now obtains a tertiary degree. The link between father's and son's education is therefore weaker.

## 7.3 Steady State Welfare

Both reforms have a very different impact on the steady state welfare.<sup>18</sup> The admission based reform decreases steady state welfare by 10.28%. On the other hand, under the tax based reform the steady state welfare increases by 1.52% in consumption equivalents.

Table 9 shows the distribution of the steady state welfare gains from the two reforms, measured in per household member consumption units. The gains are closely related to general equilibrium effects of wages for each type of human capital attainment and skills.

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<sup>18</sup>The transition cost of the reforms is not taken into account.

<b>Welfare Gains</b>			
Father's Attainment	Son's Attainment		
	Primary	Secondary	Tertiary
Admission Based Reform			
Primary	-11.0	-15.8	+58.0
Secondary	-9.4	-10.2	+43.4
Tertiary	+32.6	+34.0	+61.7
Tax based Reform			
Primary	-2.3	+3.7	-9.6
Secondary	+3.5	+6.4	-9.5
Tertiary	-6.6	-4.0	-7.3

Note: Welfare gains are steady state welfare gains in % of consumption equivalents per household member.

Table 9: Welfare Gains

The tax based reform favors people with primary and secondary education. It lowers tertiary wages and makes all households whose member has tertiary education worse off. The opposite logic applies to the admission based reform. Similarly, households with secondary education of both father and son experience steady state welfare losses in the admission based reform and gains in the tax based reform. Only the primary school households are always worse off. The magnitude of these changes is large.

## 8 Conclusions

This paper analyzes the consequences of educational reforms where United States adopt European educational subsidies. We consider two variants of those reforms, one where admission rates are cut to finance those subsidies, and one where taxes adjust to finance increased educational subsidies. The first, admission based, reform constitutes a move towards the European educational system. we show that those reforms have very different impact on the economy. The admission based reform has significant welfare costs, decreases tertiary educational attainment, and increases asset and income inequality. The second reform has opposite effects. Those results suggest that, while higher schooling subsidies may have potentially significant welfare benefits, the results critically depend on how the reform is financed. If countries are not willing, or unable, to raise taxes to finance those subsidies then the reforms can have disastrous consequences.

We also look at how efficiently skills are allocated both under the benchmark U.S. system, and under both reforms. We compare the allocation of skills to the first best allocation. While it is straightforward that the first best allocation will never be attained in an environment

with incomplete markets and borrowing constraints, the first best results at least give us a metrics how to measure the (in)efficiency of various schooling systems.

We find that the benchmark allocation is very inefficient and that the tax based reform increases efficiency, while the admission based reform decreases efficiency. Those findings suggest that there might be a space for additional increases in efficiency, for instance if schooling subsidies are allowed to depend on detailed household characteristics. Examples would be a system with merit based subsidies that depend on applicants skills, or systems where subsidies depend on household wealth or father's income. In our ongoing work we evaluate the efficiency consequences of those alternative reforms.

## References

- Abraham, A. (2004). Wage inequality and educational policy with skill-biased technological change in OLG setting. Working paper, University of Rochester. [4](#)
- Aiyagari, R. S. (1994). Uninsured idiosyncratic risk and aggregate savings. *Quarterly Journal of Economics* 109(3), 659–684. [22](#)
- Becker, G. S. and N. Tomes (1986). Human capital and the rise and fall of families. *Journal of Labor Economics* 4(3), S1–S39. [4](#)
- Blanden, J., P. Gregg, and S. Machin (2005). Intergenerational mobility in Europe and North America. Working paper, LSE. [2](#)
- Brown, J., J. Liebman, and J. Pollet (2002). Estimating life tablets that reflect socioeconomic differences in mortality. In M. Feldstein and J. Liebman (Eds.), *The Distributional Effects of Social Security Reform* (1 ed.), Chicago, IL, pp. 447–457. University of Chicago Press. [18](#)
- Cameron, S. and J. J. Heckman (1998). Life cycle schooling and dynamic selection bias: Models and evidence for five cohorts of american males. *Journal of Political Economy* 106, 262–333. [3](#)
- Carneiro, Pedro, S. and J. J. Heckman (2002). The evidence on credit constraints in post-secondary schooling. *The Economic Journal* 112(482), 705–734. [25](#)
- Castañeda, A., J. Díaz-Giménez, and J.-V. Ríos-Rull (2003). Accounting for the US earnings and wealth inequality. *Journal of Political Economy* 111(4), 818–857. [22](#)
- Corak, M. (2006). Do poor children become poor adults? Lessons from a cross country comparison of generational earnings mobility. Discussion Paper 1993, IZA, Bonn. [2](#)
- Fuster, L. (1999). Is altruism important for understanding the long-run effects of social security? *Review of Economic Dynamics* 2(3), 616–637. [3](#)
- Fuster, L., A. Imrohoroglu, and S. Imrohoroglu (2003). A welfare analysis of social security in a dynastic framework. *International Economic Review* 44(4), 1247–1274. [3](#), [7](#)

- Gallipoli, G., C. Meghir, and G. L. Violante (2007). Equilibrium effects of educational policies: A quantitative evaluation. working paper, University College London. [4](#), [20](#)
- Heathcote, J., K. Storesletten, and G. Violante (2005). Two views of inequality over the life-cycle. *Journal of European Economic Association* *3*, 543–552. [16](#)
- Heckman, J. J., J. Lochner, and C. Taber (1998). Explaining rising wage inequality: Explorations with a dynamic general equilibrium model of earnings with heterogeneous agents. *Review of Economic Dynamics* *1*, 1–58. [4](#)
- Heckman, J. J., J. Lochner, and C. Taber (1999a). General equilibrium cost benefit analysis of education and tax policies. Working Paper 6881, National Bureau of Economic Research. [4](#)
- Heckman, J. J., J. Lochner, and C. Taber (1999b). Human capital formation and general equilibrium treatment effects: A study of tax and tuition policy. *Fiscal Studies* *20*(1), 25–40. [4](#)
- Jallade, J.-P. (1992). Undergraduate higher education in Europe: towards a comparative perspective. *European Journal of Education* *27*(1-2), 121–144. [6](#)
- Kryvtsov, O. and A. Ueberfeldt (2007). Schooling, inequality and government policy. Working Paper 2007-12, Bank of Canada. [4](#)
- Lee, D. (2005). An estimable dynamic general equilibrium model of work, schooling, and occupational choice. *International Economic Review* *46*(1), 1–34. [4](#)
- Mendoza, E. G., A. Razin, and L. L. Tesar (1994). Effective tax rates in macroeconomics: Cross-country estimates of tax rates on factor incomes and consumption. *Journal of Monetary Economics* *34*(3), 297–323. [19](#)
- OECD (2006). *Education at a Glance: OECD Indicators* (2006 ed.). OECD, Paris. [4](#), [5](#), [6](#), [20](#)
- Storesletten, K., C. Telmer, and A. Yaron (2004). Consumption and risk sharing over the life cycle. *Journal of Monetary Economics* *51*, 609–633. [16](#)
- U.S. Department of Education (2001). *Digest of Education Statistics* (2000 ed.). Washington, DC. [6](#)
- Ventura, G. (1999). Flat tax reform: A quantitative exploration. *Journal of Economic Dynamics and Control* *23*, 1425–1458. [19](#)