1 Introduction

Renewed debate in empirical economics concerns the evolution of U.S. labor market inequality over the past few decades. The 1980s witnessed concurrent, considerable growth in both the wage premium and relative supply of college educated workers. We might thus reasonably infer that a dramatic increase in relative demand has taken place. Since the most plausible determinant of demand potentially responsible is technology, in the early 1990s a consensus emerged, attributing widening inequality over the previous decade to skill-biased technological change.

However, the longest sustained decline in the real Federal minimum wage on record also occurred over the 1980s. Based on this, the consensus account is now challenged. The crux of the controversy is whether the expansion of inequality beginning circa 1980 was a secular phenomenon reflecting a rise in the price of high skill (the traditionalist position), or whether it was an episodic phenomenon just of the 1980s (the revisionist position)\textsuperscript{1}.

This paper dissects the dispute with a theoretical, general equilibrium perspective. I embed the Roy framework [27] into a dynamic simulation model with endogenous labor supply. The general equilibrium approach thus far missing from the empirical conversation allows for the development of more finely-tuned intuition concerning the many channels through which the minimum wage and SBTC individually influence inequality. Moreover, since the model endogenously generates all the standard inequality metrics, counterfactual

\textsuperscript{1}Representing the revisionists are Card and DiNardo [11], along with Lemieux [26] and others. Autor, Katz, and Kearney [3] have recently issued a traditionalist response.
exercises allow us to isolate the individual effects of SBTC and the minimum wage. The model indicates that most of the growth in earnings inequality at the lower tail of the income distribution is due to the declining minimum wage.

This theoretical approach enables the analysis of general equilibrium and compositional effects invisible in the standard empirical framework. It is often assumed\(^2\) that the Federal minimum wage shouldn’t influence earnings at the upper tail of the income distribution. But this presumes the absence of a general equilibrium response: if the minimum wage unemploys unskilled workers, a drop in the minimum wage should increase the earnings of the highly-paid, skilled workforce. Neverthess, the standard presumption is correct. Although a significant fraction of the potential labor force is unemployed as a consequence of the minimum wage\(^3\), their level of human capital is too low to produce a perceptible shift in the relative supply of low-skill\(^4\). Moreover, this identifies the means through which the minimum wage influences inequality. A drop in the minimum wage increases lower-tail inequality because it enables the least-skilled workers to supply their human capital to the labor market.

The empirical literature on inequality also stresses the importance of controlling for the composition of the workforce. For example, over time the college educated workforce has become more experienced and more diverse. The college premium thus measures the return to higher education rather than composition only provided observations are weighted to reflect a population with constant demography. But this sidesteps a second, more serious compositional problem: the mean level of innate, unobserved skill of both college and high-school educated workers may be changing. It turns out that the mean skill-level of college workers in the youngest cohort has fallen 4.79% since 1970, and the mean level of unskilled human capital supplied by high-school workers entering the labor market has fallen 0.78%\(^5\). Thus, although the college premium has risen 58% for young workers since 1970, and 7.4% for more experienced workers, I estimate that the internal rate of return to college education per unit of human capital has risen only from 3.83% in 1970 to 4.31% in 2008\(^6\).

This paper proceeds as follows. In the next section, I will present the data in question and boil down trends which warrant explanation into a number of stylized facts. Section

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\(^2\)See [3] and [25], for two examples.

\(^3\)The model estimates that 2.58% of the potential labor force is unemployed due to the minimum wage in 2000. Using BLS data for the level of employment, this corresponds to 3,492,136 workers.

\(^4\)Since the continuing rise in inequality at the upper-tail of the income distribution can’t be attributed to the minimum wage, skill-biased technological change still plays an important role.

\(^5\)Families entering the college workforce have less skilled-human capital than the median college worker, but more skilled-human capital than families remaining in the high-school workforce. The mean level of both skilled and unskilled human capital supplied by families entering the college workforce has fallen 0.78%\(^5\).

\(^6\)The modal worker possesses 417 units of unskilled human capital and 346 units of skilled human capital. Because the endowments of both types of human capital differ among workers, the IRR figure above has little practical relevance.
III discusses the research context and explains why the standard empirical framework is an inadequate vehicle for a comprehensive analysis of upper and lower tail inequality. The model is presented in Section III, parameterized in Section IV, and solved along the transition path in Section V. Implications are then considered, and Section VI concludes.

2 The Data

Inequality is a broad term, encompassing both the earnings gap between groups, in addition to the difference in income received at quantiles of the earnings distribution either within groups, or in the population of as a whole. Using data from the March CPS, Figures 1–7 graph the values of relevant statistics for full-time, full-year (FTFY) workers from 1965–2008. This illustrates that the evolution of inequality over the past forty years is an intricate phenomenon. The most significant changes are the stylized facts summarized below. These are the central objects of interest for this study.


2. The college premium for younger workers dropped in the 1970s, and has been expanding ever since. The rate of growth declined noticeably around 1995. Changes in the college premium for older workers are much more modest. Growth in the college premium coincides with higher levels of college enrollment among the college-age population.

3. Experience differentials are much larger for college educated workers than for high school educated workers. The college experience differential grew substantially only in the 1970s, while the high-school experience differential has been growing throughout the past 40 years.

Figure 1 plots indexed values of the 10th percentile, 50th percentile, and 90th percentile of the earnings distribution for all full time, full year workers over the period 1965-2008. Relative uniform growth in earnings across all quantiles from 1965 until 1980 is evident, as is a widening of inequality starting around 1980. But this standard account is open to

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7Experience differentials, the college / high school wage premium, and the gender earnings gap, for example
8Throughout this paper, earnings data is weekly income for FTFY workers. This is the quotient of annual earnings and weeks worked in the March CPS. (See the Data Appendix for more). Although the time period in the model is annual, weekly earnings are used so that earnings are comparable between individuals. Full year employees can work anywhere between 40 and 52 weeks per year.
considerable refinement. Median earnings clearly grow faster than the 10th percentile only during the 1980s, thus lower tail inequality growth is indeed episodic. However, the gap between the 90th percentile and the median grows steadily throughout the period, with some slowdown in the early 2000s9.

We observe a far more substantial widening of the wage distribution when we analyze men and women separately (Figures 2 and 3). To be consistent with Figure 1, this implies that a considerable narrowing of the gender earnings gap must have taken place. The growth in inequality is also considerably greater for women than men. Since the period under consideration coincides with the entry of women in the labor market, this fact is perhaps the basis for further research10. For now, for the benefit of simplicity though at the possible cost of explanatory power, I will assume women and men are perfect substitutes in production with the same prior distribution of skill.

The college premium from 1965–2007 is the subject of a large theoretical literature11. In particular, its drop over the 1970s has received considerable attention. Figure 4 displays the simple college premium: the log difference between the mean wage received by workers with a college education and those with less than college. Figure 5 plots the college premium separately for young and old workers. We see that there only really is a drop in the college premium for younger workers. Subsequent sustained growth in the college premium follows, but only until the mid-1990s. For this reason many authors assume SBTC slows down or stops around 1995. Recent stability in the college premium has been interpreted as support for the revisionist hypothesis.

A third dimension of inequality are experience differentials, or the gap in wages between older and younger workers (Figures 6 and 7). The returns to experience have grown steadily since 1965 for workers with a high school education. But it’s notable that this growth is from a very low base, at the end of the 1960s, median earnings of young high school workers was actually slightly higher than median earnings of their more experienced counterparts. For college workers, the experience premium rises abruptly in the 1970s, and has declined slightly since. Despite this, in 2008 the age gap for educated workers is still much larger.

9Autor, Katz, and Kearney [3] dub this the "lower tail inequality puzzle." They argue that a more nuanced version of SBTC is necessary to explain it. In nuanced SBTC, there are three types of labor: abstract, routine, and manual. Computer technology and routine workers are perfect substitutes. If workers paid median wages predominantly supply routine labor, then as computers become cheaper, median wages fall. If lower paid workers supply manual labor while highly paid workers supply abstract labor, upper tail inequality will rise while lower tail inequality simultaneously falls.

10It would be interesting to extend the model to a three sector Roy framework which includes home production. We could assume that women and men possess a different prior distribution of the three skills, or alternatively that discrimination raises the cost to women of supplying skill in the labor market. This would add a third factor to the accounting exercise.

11See, for example, Guvenen and Kuruscu [15], He [16], and Krusell, et. al. [24], for three notable examples.
than the comparable figure for less educated workers. (This masks some interesting gender
effects\(^\text{12}\), but the basic story is the same if we analyze men and women separately.)

Much of this points to a notable shift in the U.S. labor market at the beginning of the
80s. Revisionists identify this change as the eroding Federal minimum wage, traditionalists
identify it as skill biased technical change. Since it is a potential determinant of inequality,
the real Federal minimum wage is plotted in Figure 8.

The minimum wage fell 27.1% over the 1980s, and this corresponds with the greatest rise
in lower tail inequality on record. Sustained drops in the minimum wage also occur from
1968–1973, and from 1997–2006. With reference to Figure 1, there is visual evidence that
this corresponds with slower growth in the 10th percentile of the earnings distribution than
the other quantiles. In a simple regression of the log 90-10 wage gap on the log minimum
wage, the OLS coefficient is −0.82, with a t-statistic of −5.5, and an \( R^2 \) of 0.42. The log
90-10 gap, along with predicted values from this regression, are presented in Figure 9\(^\text{13}\). Thus
empirically we have some evidence for the significance of the minimum wage.

However, this is complicated by the presence of a large number of FTFY workers who
report earning less than the minimum wage. Sub-minimum wage workers constitute a sur-
prisingly large fraction of the labor force: more than 20% in 1965, just over 5% in 2008.
Figure 10 plots the fraction of workers who report weekly earnings below the legal minimum.
The red lines indicate years during which the nominal minimum wage rose – whenever the
minimum wage rises, a visibly larger fraction of the workforce reported earning less than the
minimum. This may reflect either the many legal exemptions to the minimum wage, or loose
enforcement\(^\text{14}\). To address that problem, in this paper I assume that lifetime exemptions
from the minimum wage are distributed randomly to a changing fraction of agents in each
cohort. These exemption probabilities add a the third exogenous variable to the model,
along with technology and the minimum wage itself.

\(^\text{12}\)The median weekly wage in 1969 for high school educated workers FTFY aged 25 – 30 was $503.27 in
constant 2000 U.S. dollars. FTFY workers aged 46 – 60 earned $497.15. Underneath this is a substantial
gender gap, a small experience premium for men ($550.69 versus $573.64) and an experience penalty
for women ($344.18 versus $321.24).

\(^\text{13}\)In a similar regression, Card and DiNardo [11] calculate an \( R^2 \) above 0.9. Also, their measure of overall
inequality shows a much clearer flattening around 1990. The difference is for two reasons: (1) they use
the CPI-U to measure inflation, rather than the GDP deflator, and (2) they use CPS outgoing rotation
group data, which asks respondents directly about their hourly or weekly pay. Given March CPS data it is
necessary to construct weekly pay indirectly by dividing annual earnings of full time workers by the number
of weeks they report having worked. I use the March CPS because coverage begins about a decade earlier.

\(^\text{14}\)Alternatively, it could indicate a problem with the data. Weekly earnings are calculated as the quotient
of annual income from all sources and the number of weeks spent working for individuals who report working
full-time. Starting in 1990, the March CPS includes a variables for the hourly wage of workers paid by the
hour, and the weekly wage of all other workers. Using these variables we calculate far fewer sub-minimum
wage wokers, thus there may be some inaccuracies built into the data used to impute weekly earnings.
3 Related Literature

For a review of the economic literature on technology and inequality up to 2002, see Acemoglu [1]. Given the stylized facts above, it is natural to compare the measures of between-group inequality – the college premium and experience differentials – with changes in relative supply. If supply is assumed to be exogenous, then the shift in the relative demand for more educated or more experienced labor can be isolated. This method was pioneered by Katz and Murphy [23]. Since many papers are anchored on the Katz and Murphy approach\(^\text{15}\), I label it "the standard partial equilibrium empirical framework".

Katz and Murphy assume an exogenous, SBTC-induced constant growth rate in the relative demand for highly educated labor. Given a CES aggregate production function, this growth rate is recovered via a regression of the college premium on a linear trend and relative supply. Over the 1970s, due to the baby-boom and Vietnam-era draft laws, the relative supply of college-educated workers grew faster than relative demand. With the passage of the baby boom into mid-career in the 1980s, the growth rate of relative supply fell. Accordingly, the college premium shrank in the 1970s, and rose again in the 1980s. Noting that the growth in the college premium occurred mostly among young workers, Card and Lemieux [12] extend the KM framework so that college and high-school educated workers within different age groups are imperfect substitutes. They conclude that the increase in the college premium is due mostly to slower growth in the relative supply of young male college-educated workers.

The standard partial equilibrium framework thus has some success explaining changes in between-group inequality. But it is much less informative in an analysis of upper and lower-tail inequality. This is because these models lack within-group heterogeneity\(^\text{16}\). They class individuals into cells distinguished by education, experience, sex, and race. All individuals within the cells are assumed identical\(^\text{17}\). The theoretical earnings distribution produced by the standard framework is thus simply a ranking of the median earnings of different demographic groups. However, according to Katz and Autor [22], demography can only account for about one third of the changes in wage structure since the mid 1960s. Thus an analysis of upper and lower tail inequality lacking within-group heterogeneity is necessarily incomplete.

\(^{15}\)See, for instance, Berman, Bound, and Griliches [8], Berman, Bound, and Machin [9], and Autor, Katz, and Kreuger [4].
\(^{16}\)Juhn, Murphy, and Pierce [21] is a notable exception.
\(^{17}\)In the standard partial equilibrium framework, the ratio of median log wages between two cells is regressed on time trends and relative supply terms. Since median log wages is the dependent variable, the residual is not a proxy for unobserved heterogeneity. Instead, the residual is due to exogenous shocks in the productivity of an entire demographic group.
A second strand of the inequality literature focuses on institutional factors. These include declining union power and looser restrictions on employment and international trade, in addition to the minimum wage. Changes in industry wage premia and shifts in product demand unfavorable to industries that were the traditional employes of blue-collar, male labor are analyzed in Bound and Johnson [10]. Bound and Johnson find that these changes are too small to account for the relative drop in the earnings of male high school graduates. They conclude that a sustained positive shock to the productivity of educated workers, namely SBTC, must have taken place.

Lee’s 1999 study [25] focuses on the minimum wage. Using cross-state wage variation to identify the effect of the minimum wage\(^{18}\), Lee determines that the declining real minimum wage explains virtually all of the growth in lower tail inequality over the 80s. However, since an important assumption in [25] is that the minimum wage doesn’t affect wages at the upper tail of the income distribution, Lee doesn’t dispute the occurrence of SBTC.

The challenge to the consensus was issued in 2002 by Card and DiNardo [11]. Using both the 90-10 log wage gap and the standard deviation in earnings as metrics, they show that both overall and residual inequality\(^{19}\) rose only during the 1980s, and was stable both before and after. They also criticize the practice of declaring that the coefficients of a linear time trend in a KM-style regression represent technological change. They argue that it is preferable to measure technological change directly from observable data. In that spirit, it is suggestive that the growth of inequality allegedly slowed over the 1990s, when advances in information technology were continuing to take place. In their response, Autor, Katz and Kearney [3] assert that Card and DiNardo are simply using the wrong inequality metric. Upper tail inequality continued to rise throughout that decade.

4 The Model

The theoretical scaffolding of this paper is the Roy model of sectoral choice [27]. The economic environment is Roy’s setup is very simple. There are two sectors, and a continuum of agents endowed with two skills. All agents share the same prior distribution of skills, and each skill is useful only in one sector. Heckman uses versions of this model to analyze diverse topics like the home production versus labor supply decision [17], the union wage differential

\(^{18}\)In states where the minimum wage is high relative to the median wage, lower tail inequality should on average be low.

\(^{19}\)Consider a Mincer regression of log wages on a cubic in experience and indicator variables for education, gender, and race. Residual inequality measured alternatively as the 90-10 gap in the distribution of the estimated residuals [3] or the unexplained portion in the variance of wages [11]: \(\sigma^2 (1 - R^2)\). Intuitively, it is inequality remaining after observable covariates like education and experience are accounted for.
[18], and the effect of minimum wages [20], among others. Wages are usually exogenous, but that is not appropriate in an analysis of SBTC. Hence I embed the Roy model in a general equilibrium framework.

4.1 Production

The two factors of production are two types of human capital: that which may be supplied by low skilled, high-school educated labor, and that which requires college-educated labor. Agents are hired to supply their human capital by a single competitive, representative firm with a CES production function:

\[ Y_t = \left\{ [g_H(t)N_H(t)]^\psi + [g_L(t)N_L(t)]^\psi \right\}^{\frac{1}{\psi}} \quad (1) \]

\( Y_t \) represents aggregate weekly production, which is assumed to be constant over the course of year \( t \). \( N_H(t) \) and \( N_L(t) \) is the aggregate weekly supply of type-\( H \) and type-\( L \) human capital over year \( t \), \( g_s^s \) represents the state of technology favoring workers in sector \( s \).\(^{20}\) The elasticity of substitution between the two types of skill is \( \frac{1}{1-\psi} \). SBTC is an increase in \( \frac{g_H}{g_L} \). Wages are endogenously determined at the intersection of the supply and demand curves for skill. In particular, the inverse demand for each skill is its marginal product:

\[ w_H(t) = \chi_t \left( g_H N_H^{\psi-1} \right), \quad w_L(t) = \chi_t \left( g_L N_L^{\psi-1} \right) \quad (2) \]

where \( \chi_t = \left[ (g_H N_H)^\psi + (g_L N_L)^\psi \right]^{\frac{1-\psi}{\psi}} \)

Recall here that \( w_H \) is compensation from the representative firm for each unit of type-\( H \) human capital supplied per week, and similarly for \( w_L \).

4.2 Households

There is a continuum of infinitely lived families characterized by their unchanging skill endowments \( \theta = (\theta_H, \theta_L) \), where \( \theta \) is a random variable with probability density function \( f(\cdot; \nu) \), and \( \nu \) is a vector of parameters. Each family consists of an infinite stream of overlapping generations\(^{21}\). A family member lives \( T + 1 \) periods, so an agent born at time \( t \) dies at time \( t + T \). A cohort of mass \( \frac{1}{T+1} \) is born each period, so the population is always of size

\(^{20}\)In what follows I’ll often suppress time subscripts for legibility

\(^{21}\)This formulation was inspired by Stokey’s [28] study of the effect of free trade on human capital accumulation.
Given a deterministic sequence \( \{I_t\}_{t=0}^{\infty} \) of family income and interest rates \( r \), each family chooses savings \( S_t \) and consumption \( C_t \) to maximize lifetime family utility:

\[
V(I_t, r) = \max_{\{S_t\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma}}{1-\sigma} \right)
\]

s.t. \( C_t = I_t + (1 + r_t) S_t - S_{t+1} \)  

At birth, a family member selects his or her sector of employment. This decision is irreversible: agents may never switch sectors. Decisions are made to maximize family utility, so an agent will supply skill to the sector in which discounted expected income is higher. I assume an individual’s human capital stock rises with experience, but declines with age. This allows the model to capture experience differentials, and is consistent with the standard Mincer framework. An agent born at time \( t - i \) with skill endowments \( \theta \) and \( x \) years of labor market experience can supply \( \theta_L \exp \{\alpha_1 x - \alpha_2 i^2\} \) units of type-\( L \) human capital, or \( \theta_H \exp \{\beta_1 x - \beta_2 i^2\} \) units of type-\( H \) human capital, at time \( t \). To work in sector \( H \), however, an agent must first spend four years attending college. There is no tuition, the only cost of college attendance is forgone income in sector \( L \).

An agent may supply skill provided either earnings exceeds the minimum wage, or earnings are less than the minimum wage and the agent was granted an exemption. Exemptions are permanent – an agent exempt from the minimum wage at time \( t \) is exempt in all periods thereafter. Because individuals must choose their sector of employment before learning if they are exempt, the minimum wage is a source of potential uncertainty.

Let \( V_t^S(\theta) \) be discounted lifetime income for an agent born at time \( t \) with skill parameters \( \theta \) who supplies skill in sector \( S \in \{L, H\} \). Let the exogenous sequences \( \{W_t\}_{t=0}^{\infty} \) and \( \{p_t\}_{t=0}^{\infty} \) give minimum wages and exemption probabilities, respectively. To write the functions \( V_t^S(\theta) \), let the mappings \( \omega^L_t : \theta \to \{0, 1\}^{T+1} \) give the periods in which a non-exempt agent born at time \( t \) with skill parameters \( \theta \) who chooses sector \( L \) is allowed to work. Define \( \omega^H_t : \theta \to \)

\[22\] Since we have constant returns to scale technology, this assumption might seem innocuous. But with endogenous schooling, the demographic structure of the population is potentially relevant. For example, suppose a baby boom occurs at time \( t \). Low skill workers immediately enter the labor market, but high skill workers must first spend four years in college. Hence at the prevailing level of relative wages following the baby boom at time \( t \), the relative supply of low skill workers will be temporarily higher, thereby pushing up the college premium and encouraging college enrollment. Hui He [16] examines this, but finds that demographic change does not have a significant effect on college enrollment.

\[23\] The \( \alpha \) and \( \beta \) parameters are calibrated to have positive values.
\[ \{0, 1\}^{T-3} \text{ similarly. We will have} \]

\[
\omega_t^L(\theta, 0) = 1 \iff w_L(t) \theta_L \geq W_t, \text{ and} \\
\omega_t^L(\theta, i) = 1 \iff w_L(t + 1) \theta_L \exp\left[ \alpha_1 \sum_{j=0}^{i-1} \omega_t^L(\theta, j) - \alpha_2 i^2 \right] \geq W_{t+i}
\]

Then expected income is the weighted sum of discounted income with and without an exemption. For notational convenience, let \( R_t^i \) be the gross \( i \)-period return on savings, \( R_t^i = \prod_{j=1}^{i} (1 + r_{t+j}) \). Define \( R_t = R_t^1 \), and \( R_t^0 = 1 \).

\[
V_t^L(\theta) = \theta_L \left( \frac{\pi_t}{T} \sum_{i=0}^{T} \left[ \frac{w_L(t+i)}{R_t^i} \exp (\alpha_1 i - \alpha_2 i^2) \right] + (1 - \pi_t) \sum_{i=0}^{T} \left[ \frac{w_L(t+i)}{R_t^i} \omega_{t-i}^L(\theta, i) \exp \left( \alpha_1 \sum_{j=0}^{i-1} \omega_t^L(\theta, j) - \alpha_2 i^2 \right) \right] \right) \\
V_t^H(\theta) = \theta_H \left( \frac{\pi_t}{T} \sum_{i=4}^{T} \left[ \frac{w_H(t+i)}{R_t^i} \exp [\beta_1 (i - 4) - \beta_2 i^2] \right] + (1 - \pi_t) \sum_{i=4}^{T} \left[ \frac{w_H(t+i)}{R_t^i} \omega_{t-i}^H(\theta, i - 4) \exp \left( \beta_1 \sum_{j=0}^{i-5} \omega_t^H(\theta, j) - \beta_2 i^2 \right) \right] \right)
\]

(4)

We can represent the schooling choice in this notation:

\[
s_i(\theta) = 1 \iff V_t^H(\theta) > V_t^L(\theta), \ t \geq 0
\]

(5)

Thus we may write the aggregate supply of both types of human capital. To conserve space, it is written as the sum of human capital supplied by each cohort.

\[
N_L(t) = \sum_{i=0}^{T} N_L^i(t), \quad N_H(t) = \sum_{i=4}^{T} N_H^i(t), \quad \widehat{i} \equiv i - 4
\]
The sequence of events over the course of year \( t \) is as follows. At the beginning of year \( t \), 18 year old new family members are born, and the family receives the gross interest on last year’s savings. New family members immediately decide whether or not to spend the next four periods attending college, and subsequently learn if they are exempt from the minimum wage. Next, all family members not attending school receive their expected labor market income for the period, where the expectations were made at birth. This may be rationalized by assuming the presence of a perfectly competitive insurance industry which eliminates the risk of unemployment due to the minimum wage. The rest of the period is spent working, studying, and consuming.

### 4.3 Equilibrium

Given the exogenous sequences \( \{g_{t}^{H}\}_{t=0}^{\infty} \) and \( \{g_{t}^{L}\}_{t=0}^{\infty} \), and initial conditions \( \{s_{t}(\theta)\}_{t=-T}^{0} \) and \( S_{0}(\theta) \) an equilibrium consists of sequences of prices \( r_{t}, w_{t}^{H}, w_{t}^{L} \), human capital supply \( N_{H}(t) \), \( N_{L}(t) \), schooling decision rules \( s_{t} : \Theta \rightarrow \{0, 1\} \), savings \( S_{t} : \Theta \rightarrow \mathbb{R} \), consumption levels \( C_{t} : \Theta \rightarrow \mathbb{R}_{+} \) such that

1. Given prices and technology levels \( \{g_{t}^{H}\}_{t=0}^{\infty}, \{g_{t}^{L}\}_{t=0}^{\infty} \), the firm’s hiring decisions \( N_{H}(t) \) and \( N_{L}(t) \) maximize its period profit. Equivalently, the firm’s decisions satisfy the first order conditions (2).

2. Schooling decisions are made to maximize discounted household income. Equivalently, schooling decisions for agents in family \( \theta \) and cohort \( t \) satisfy (5).

3. Given schooling, labor supply, and interest rates \( r_{t} \), households choose levels of consumption \( C_{t}(\theta) \) and savings \( S_{t}(\theta) \) to maximize family utility (3).

4. The human capital, bond, and goods markets clear. Specifically, \( N_{H}(t) \) and \( N_{L}(t) \) are given by (6), aggregate consumption equals aggregate production, and bonds are in

\[
N_{L}^{t} = \pi_{t} \int_{\{s_{t}(\theta) = 0\}} \theta_{L} \exp \left( \alpha_{1} i - \alpha_{2} i^{2} \right) f(\theta; \nu) \, d\theta
\]

\[
+ (1 - \pi_{t}) \int_{\{s_{t}(\theta) = 0\}} \theta_{L} \omega_{t-i}^{L}(\theta, i) \exp \left( \alpha_{1} \sum_{j=0}^{i-1} \omega_{j}^{L}(\theta, j) - \alpha_{2} i^{2} \right) f(\theta; \nu) \, d\theta
\]

\[
N_{H}^{t} = \pi_{t} \int_{\{s_{t}(\theta) = 1\}} \theta_{H} \exp \left( \beta_{1} i - \beta_{2} i^{2} \right) f(\theta; \nu) \, d\theta
\]

\[
+ (1 - \pi_{t}) \int_{\{s_{t}(\theta) = 1\}} \theta_{H} \omega_{t-i}^{H}(\theta, i) \exp \left( \beta_{1} \sum_{j=0}^{i-1} \omega_{j}^{H}(\theta, j) - \beta_{2} i^{2} \right) f(\theta; \nu) \, d\theta
\]
zero net supply:

$$\int \Theta C_t(\theta) f(\theta; \nu) d\theta = Y_t, \quad \int \Theta S_t(\theta) f(\theta; \nu) d\theta = 0$$

5 Identifying the Parameters

To calibrate the model, the economy is assumed to be on a balanced growth path until 1969. Under balanced growth, the technology parameters $g_t^H$ and $g_t^L$ are assumed to grow at the common, constant rate $\gamma$:

$$\frac{g_{t+1}^H}{g_t^H} = \frac{g_{t+1}^L}{g_t^L} = (1 + \gamma)$$

Since they enter multiplicatively with the calibrated initial values of $N_H$ and $N_L$, the initial values of $g_H$ and $g_L$ at the base year $t = 0$ (1969) can be chosen arbitrarily as a normalization. Accordingly, values of $g_H(0)$ and $g_L(0)$ are selected to ensure that $w_H(0) = w_L(0) = 1$. The balanced growth path also features constant exemption probabilities, and a constant real minimum wage: $\pi_t = \pi$, and $W_{t+1} = (1 + \gamma) W_t$. Now we can solve for the balanced growth path with constant, common growth in wages: $w_H(t + 1) = (1 + \gamma) w_H(t)$, $w_L(t + 1) = (1 + \gamma) w_L(t)$, no debt or credit $S_t(\theta) = 0 \forall t \geq 0$, and a constant interest rate $R_t = (1 + r)$. First, extracting the Euler Equation from the family utility maximization problem:

$$\beta (1 + r_{t+1}) = \left( \frac{C_{t+1}}{C_t} \right)^\sigma$$

If consumption grows at the same rate as income, then

$$\beta (1 + r) = (1 + \gamma)^\sigma \quad (7)$$

If agents all expect constant growth in both wages at the rate $\gamma$, the schooling decision becomes much simpler. A simple induction argument shows that provided $\frac{a_1}{a_2} > T$ and $\frac{b_1}{b_2} > T + 4$, then the functions $\omega_L^t(\theta)$ and $\omega_H^t(\theta)$ are stationary, and take only two values – a vector of ones or a vector of zeros. Specifically, let $b_L = \frac{W_t}{w_L(t)}$, and $b_H = \frac{W_t}{w_H(t)} \exp(\beta_2 A^2)$. Then,

$$\omega_L(\theta) = \begin{cases} (1, \ldots, 1) & \text{if } \theta_L \geq b_L \\ (0, \ldots, 0) & \text{otherwise} \end{cases}$$
\[ V_t^L (\theta) = \theta_L \pi_{1(L \geq \pi L)} \sum_{i=0}^{T} \left[ \frac{w_L(t+i)}{R} \exp \left( \alpha_1 i - \alpha_2 i^2 \right) \right] \]

\[ V_t^H (\theta) = \theta_H \pi_{1(H \geq \pi H)} \sum_{i=4}^{T} \left[ \frac{w_H(t+i)}{R} \exp \left( \beta_1 i - \beta_2 i^2 \right) \right] \]

Note that given expected steady growth in both wages, decisions are stationary – every member of family \( \theta \) makes the same schooling decision regardless of birth time. This implies that \( N_H(t) \) and \( N_L(t) \) are constant (see equation 6), so wages indeed do grow at \( \gamma \) in equilibrium. Labor income for both college and no-college families then grows at \( \gamma \), so \( S_t(\theta) = 0 \) and \( C_{t+1}(\theta) = (1 + \gamma) C_t(\theta) \) is consistent with equilibrium.

### 5.1 Identifying \( \beta, \gamma, \pi, \sigma, \psi, W_t, \) and \( T \)

Assuming full-time workers supply their skills for 35 hours per week on the labor market, the sequence of minimum wages \( W_t \) is 35 times the prevailing minimum wage at time \( t \), adjusted for inflation using the NIPA GDP deflator. The coefficient of relative risk aversion \( \sigma \) is taken from Gourinchas and Parker [14]. \( \gamma = 0.02 \) approximately matches per-capita output growth from 1947 to the base year 1968, according to NIPA data. \( \beta \) is assumed equal to 0.98, which from (7) implies a steady 3.08% real interest rate until 1968. With the focus on workers aged 18 to 64, \( T = 46 \). Finally, the value for the production function parameter \( \psi \), which represents the elasticity of substitution between high and low skilled human capital, is taken from Heckman, Lochner, and Taber [19]. The values for these parameters are given in Table 1.

### 5.2 Identifying the Mincer Parameters, \( \nu, \) and \( \pi_t \)

The natural logarithm of the skill parameters \( \theta \) are assumed to follow Azzalini’s bivariate skew-normal distribution (see [5] and [6]). More conventional distributions such as the log-

---

Table 1: Parameters Taken from Other Sources

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( \gamma )</th>
<th>( \sigma )</th>
<th>( \psi )</th>
<th>( T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.98</td>
<td>0.02</td>
<td>0.5</td>
<td>0.306</td>
<td>46</td>
</tr>
</tbody>
</table>

---

24 From 1954 to 1968, the geometric mean yield on three month Treasury bills was 3.11% This is calculated using the annualized yield. The data is available at the website of the Fed, www.federalreserve.gov.
normal or log-Cauchy generally calibrate perfectly correlated skill parameters. This is turn typically implies uniform schooling choice within generations: everyone in a given generation makes the same decision. The problem is that in this Roy setting, over the balanced growth path observed earnings of each agent (adjusted for lifetime skill accumulation) is the maximum of potential earnings in the two sectors. Thus under a lognormal distribution, with less than perfect correlation the distribution of observed earnings should have rightward skew, each skill provides a hedge on earnings in case innate skill in the other sector is low. Since such rightward skew isn’t present in the data, either the two skill parameters are perfectly correlated, or that the initial distribution of at least one skill itself is skewed to the left.

The skew-normal distribution is a generalization of the normal. Suppose \((Z_L, Z_H)\) is jointly skew normal. Then its pdf is parameterized by a vector \(\nu = (\mu_H, \mu_L, \sigma_H, \sigma_L, \rho, \alpha_L, \alpha_H)\), where \(\alpha_L\) and \(\alpha_H\) measure skew. Specifically, the pdf \(f(Z_L, Z_H; \nu)\) is given by

\[
f(Z_L, Z_H; \nu) = \frac{2}{\sigma_L \sigma_H} \phi \left( \frac{Z_L - \mu_L}{\sigma_L}, \frac{Z_H - \mu_H}{\sigma_H}; \rho \right) \Phi \left[ \alpha_L \left( \frac{Z_L - \mu_L}{\sigma_L} \right) + \alpha_H \left( \frac{Z_H - \mu_H}{\sigma_H} \right) \right]
\]

where \(\phi(x, y; \rho)\) is the bivariate normal pdf with correlation \(\rho\), and \(\Phi()\) is the normal cdf. The skew-normal density is obtained as follows. Let \((Z_1, Z_2, Z_3) \sim N(\mu, \Omega)\). Azzalini [6] shows that \(\left( \frac{Z_1 - \mu_1}{\sigma_1}, \frac{Z_2 - \mu_2}{\sigma_2}, \frac{Z_3 - \mu_3}{\sigma_3} \right) > 0\) is distributed skew-normal. This suggests a theoretical justification for the skew-normal environment presented here. Imagine extending the model to include a third sector which represents home production. Agents choose to supply their skills on the labor market provided maximum labor-market earnings exceed the unobserved level of potential home production. Lognormally distributed human capital endowments \((\theta_L, \theta_H, \theta_P)\), where \(\theta_P\) represents the agents ability to engage in home production, would imply the skew-lognormal distribution of labor-market skill used here. Extending this model in that direction is a topic for further research.

Now under the skew-normal formulation, it is necessary to consistently estimate the Mincer parameters \((\alpha, \beta)\), the skill distribution parameters \(\nu\), and the exemption probabilities \(\pi_t\). Available data include earnings levels, age, and schooling choice for full-time, full year workers sampled in the CPS. Using 1969 CPS data (which covers the base year 1968), \((\alpha, \beta), \nu, \pi\) are identified using the following hybrid maximum likelihood - OLS technique. Throughout, let \(Z_L = \log \theta_L\), and \(Z_H = \log \theta_H\). Let \(y_j\) indicate log earnings for sample member \(j\), and \(I_j\) his or her age. \(s_j\) indicates the schooling choice of sample member \(j\), \(s_j = 1\) if he attended college, and \(s_j = 0\) otherwise.
First, for a set value of $\nu$, $\alpha$ and $\beta$ are estimated via OLS. Note that

\[
e [y_j | I_j, s_j = 0] = E [Z_L | s_j = 0] + \alpha_1 I_j - \alpha_2 I_j^2
\]

\[
e [y_j | I_j, s_j = 1] = E [Z_H | s_j = 1] + \beta_1 (I_j - 4) - \beta_2 I_j^2
\]

(9)

Given $\nu$ and the steady state criterion for schooling choice from (8), and a trial value of $(\alpha, \beta)$, it is possible to compute $E [Z_L | s_j = 0]$ and $E [Z_H | s_j = 1]$. Then by separately regressing the difference between log earnings and this calibrated mean skill level in both sectors on age and its square, an updated set of Mincer parameters is produced. Thus given $\nu$ we have a function which maps a particular value of the Mincer parameters onto a new value. For each $\nu$, a compatible set of Mincer parameters is a fixed point of this function. Altogether, calibration proceeds as follows. For each trial value of $\nu$, a compatible set of Mincer parameters is found numerically. The likelihood function (which is derived in the appendix) is then evaluated for $\nu$. the parameters presented in Table 2 are those that numerically maximize the likelihood function, confirmed over a large grid of starting values.

To check the parameterization, steady state calibrated values of key statistics, along with their values in the data where available, are presented in Table 3. Figures 9 and 10 compare mean log earnings given age for workers in both sectors (from the 1969 March CPS) with the calibrated values.

Figure 13 is a contour plot of the joint density of the log skill parameters – the skew normal distribution with parameters $\nu$ given by Table 2. Over the initial balanced growth path, agents with skill parameters to the northwest of the red line attend college. This illustrates the channels through which an increase in the minimum wage could influence equilibrium quantities. First, there is a college attendance distortion effect. Workers potentially constrained by the minimum wage in one sector, but not in the other, may change their attendance decision to avoid the chance of constraint. In Figure 11, this is represented by the red notch. Workers with skill parameters within the notch are potentially constrained in the high skill sector but not the low skill sector, and thus artificially are compelled to avoid college. The second channel is a truncation effect represented by the green box in Figure 13. Agents with skill parameters within the green box would be sub-minimum wage workers, and are thus excluded from the labor market without an exemption. From Table 3, this represents 15.56% of all workers, of whom (from Table 2) 29.42% are denied an exemption. Thus 4.58% of the labor force is unemployed in the initial steady state due to the minimum wage, nearly all of whom would otherwise supply low skill. Finally, this implies that there may be general equilibrium effects to the extent that as the minimum wage rises, the supply of low skill decreases. Through general equilibrium effects the minimum wage
Skill Distribution Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model</th>
<th>Data (1969 CPS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_H$</td>
<td>5.8575</td>
<td></td>
</tr>
<tr>
<td>$\mu_L$</td>
<td>5.8416</td>
<td></td>
</tr>
<tr>
<td>$\sigma_H$</td>
<td>0.6250</td>
<td></td>
</tr>
<tr>
<td>$\sigma_L$</td>
<td>0.5229</td>
<td></td>
</tr>
<tr>
<td>$\alpha_H$</td>
<td>-8.4657</td>
<td></td>
</tr>
<tr>
<td>$\alpha_L$</td>
<td>7.8303</td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.8603</td>
<td></td>
</tr>
<tr>
<td>$\pi_0$</td>
<td>0.7058</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Calibrated Parameters in the Benchmark SBTC Framework

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model</th>
<th>Data (1969 CPS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Earnings, College Workers</td>
<td>$826.35</td>
<td>$854.05</td>
</tr>
<tr>
<td>Mean Earnings, H.S Workers</td>
<td>$511.75</td>
<td>$533.87</td>
</tr>
<tr>
<td>Median Earnings, College</td>
<td>$724.86</td>
<td>$764.85</td>
</tr>
<tr>
<td>Median Earnings, High School Students</td>
<td>$445.54</td>
<td>$489.50</td>
</tr>
<tr>
<td>Variance of Earnings, College</td>
<td>465.96</td>
<td>537.47</td>
</tr>
<tr>
<td>Variance of Earnings, High School</td>
<td>289.37</td>
<td>299.57</td>
</tr>
<tr>
<td>Fraction of Workers with College</td>
<td>14.39%</td>
<td>14.55%</td>
</tr>
<tr>
<td>Education</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fraction of Workers earning less than</td>
<td>15.56%</td>
<td>15.21%</td>
</tr>
<tr>
<td>the Minimum Wage</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Aggregate Variables in the initial steady state, constant 2000 USD

is then expected to decrease upper tail inequality, particular if workers paid median wages predominantly supply type L labor.

Nevertheless, analytically it turns out that in the absence of the minimum wage, the relative increase in the supply of low skill in the new steady state is very small. There is a nearly imperceptible increase in $\frac{w_L}{w_H}$, and any change in the inequality metrics is due to the inclusion of previously constrained workers. Thus in practice, there is only a truncation effect. To understand this, recall that the 4.58% of the potential labor force excluded in the initial steady state has the lowest skill endowment. The increase in the relative supply of type-L human capital is therefore much smaller than the increase in the relative supply of low-skilled workers.

6 Results

To determine the ability of this model to explain the various trends in the inequality metrics, it is necessary, given a sequence of exogenous variables $\{g_H(t), g_L(t), W_t, \pi_t\}$, to solve for the endogenous variables $\{w_L(t), w_H(t), p_t, N_L(t), N_H(t), C_t\}$ over a transition path. Counterfactual exercises then can isolate the respective effects of SBTC and the minimum wage. The algorithm used to find the equilibrium endogenous variables along the transition path...
path is an adaptation of the procedure laid out by Auerbach and Kotlikoff [2], and is detailed in the Appendix.

But first, we must identify the exogenous variables over the 40 year period beginning 1968. The sequence of real minimum wages $W_t$ is simply the prevailing Federal minimum wage times 35, under the assumption that employees are compensated for 35 hours of work over the course of a week. The initial values $g_H(0)$ and $g_L(0)$ are an arbitrary normalization, and are selected to ensure $w_L(0) = w_H(0) = 1$. Following Guvenen and Kuruscu [15], I assume 2.5% SBTC from 1970–2000, and none thereafter. Thus the levels of $g_H(t)$ and $g_L(t)$ are chosen so that $\frac{g_H(t)}{g_L(t)}$ grows at a 2.5% rate over that period, and so that the model approximately matches the growth rate in GDP per capita\textsuperscript{25}. From 1995-2007, $g_H(t)$ and $g_L(t)$ are chosen to grow at a steady 1.5% rate, approximately in line with per-worker GDP growth. Finally, as a parameter in the BGP distribution of observed earnings, $\pi_0$ is estimated by maximum likelihood as part of the initial calibration. Afterwards, $\pi_t$ is assumed to decline at a linear rate to 4% over the period from 1970-1995.

Results from the benchmark model are displayed in Figures 14–21. The model matches earnings growth in both the median and the 90th percentile (Figures 15 and 16) very well. The model however does not capture the drop in median earnings beginning around 2000. Thus supports Autor, Katz, and Kearney’s hypothesis [3] that the drop in median earnings requires an alternative, nuanced view of SBTC.

Slower relative growth in earnings at the bottom of the income distribution over the 1980s are often attributed to the minimum wage. It is therefore interesting that the model significantly overestimates the growth in wages in the 10th percentile of the earnings distribution (Figure 14), despite the inclusion of the minimum wage in the model. We therefore have a lower-tail earnings growth puzzle, which perhaps reflects institutional factors not built into the model\textsuperscript{26}.

The model’s estimates of the between-group inequality statistics are displayed along with the data in Figures 16–21. Now in both the model and the data, the college premium for young workers drops over a ten year period, and subsequently rises. The growth rate then abruptly flattens\textsuperscript{27}. A drop in the college premium may be due to either a drop in the relative price of high skill, or to composition – a decline in the skill level of the college-educated workforce. Comparing the behavior of the college premium with the behavior

\textsuperscript{25}Real GDP is taken from NIPA Table 1.1.6, and the size of the work force comes from BLS data online at www.bls.gov.

\textsuperscript{26}Declining unionization or less restrictions on international trade, for two examples.

\textsuperscript{27}The drop and subsequent rise occur five years too late in the model. In the model, SBTC took place from 1970–1995. This assumption follows the literature (see [15]). However, the model suggest that the college premium should fall five years after SBTC begins, so perhaps the onset of SBTC should be pushed back five years.
of relative wages (Figure 22) and the mean skill level of college and high-school workers produces an interpretation of the behavior of college premium for young workers.

The college premium for young workers rises from 1970–1975. During this period in the model, SBTC has taken place. But recall that the schooling decision is made when agents are 18 years old, and this decision is irreversible. Thus the relative change in the supply of college and high-school human capital is determined at least four years prior. Until 1975, the entering cohort of college-educated workers made their schooling decision before SBTC was anticipated. Thus at first, SBTC encounters an inelastic labor supply curve, so the relative price of high-skill rises, pushing up the college premium rises.

The college premium subsequently falls because SBTC reduces the average relative skill level of college-educated workers. The higher relative price of high skill induces workers with less type-$H$ human capital to enroll in college. Since both prices and labor supply are endogenous, this is a deeper explanation for the drop in the college premium over the 1970s than the exogenous supply-driven story of the standard partial equilibrium framework. To wit, the drop in the college premium in the 1970s is the consequence of composition, which in turn is the consequence of SBTC – skill biased technological change encourages relatively lower skilled workers to enroll in college, who subsequently earn less on the labor market. The college premium begins to rise again because the relative price of high skill continues rising, in the early 1980s the effect of rising prices outweighted the effect of composition. The slowdown in rise in the college premium in the mid 1990s is due to the (exogenous) end of SBTC. In the model, when SBTC stops, the relative demand for skill falls, and due to the short-run inelasticity of supply, the relative price of skilled labor falls.

The model replicates a falling college premium for young workers only. For older workers, there is no discernable drop in the college premium in the 70s or 80s, but there is a clear break starting in 1995 (Figure 18). Since the supply of college-educated skill of older workers was determined at least 30 years prior, the supply of skill for older workers is far more inelastic in the short run than the supply of skill for young workers. Thus the gradual rise in the college premium over the 1970s and 1980s was due entirely to the rise in the relative price of high skill. Now starting in 1995, SBTC stops. Shortly afterwards, the relatively lower-skilled generation of college-educated workers reaches age 46. Both lower the observed college premium for experienced workers, which is what we observed in the model. Finally, the model’s performance with regards to experience differentials is plotted in Figures 20 and 28.

---

28 The composition effect is illustrated in Figure 19. This is a scatter plot of a random sample from the calibrated earnings distribution. Agents with blue skill parameters attend college in neither 1970 or 1980. Agents with red skill parameters attend in both periods, and those with black skill parameters attend in 1980 but not in 1970. The mean level of type-$H$ skill in the black region is lower than the mean level in the red region.
Now to determine the effect of the minimum wage, consider a counterfactual exercise. The endogenous variables can be determined along an alternate transition path with a different sequence of minimum wages. Specifically, I'll consider a sequence of minimum wages fixed at the real 1969 level until 2000, and then increasing at a steady 1.5% rate. (The alternate sequence cannot be fixed forever, since in the steady state the minimum wage rises at the same rate as the other exogenous variables.)

The values of the endogenous variables in this counterfactual case are almost identical to the benchmark case, never differing by more than 0.4%. Any change in the inequality metrics is due to the exclusion workers now bound by the higher minimum wage. This might seem puzzling, considering that 4.58% of the potential labor force is excluded from the labor force in the initial 1968 steady state (Table 3). Moreover, numerically the fraction of the labor force excluded over the transition path never falls below 1.5% – it is 2.58% in 2000 – and these workers would have almost all supplied unskilled human capital.

To understand this, recall that those workers bound by the minimum wage have the lowest skill endowment. The drop in the relative supply of type-$L$ human capital in the counterfactual is therefore much smaller than the drop in the relative unskilled labor force – too small, in fact, to perceptibly shift the relative supply curve. Thus there are no general equilibrium effects, and the only consequence of the minimum wage is "truncation." The growth rates of the relevant quantiles of the earnings distribution in both the benchmark and the counterfactual case are displayed in Table 4. Notice that the minimum wage only has a significant effect on earnings at the bottom percentile. Moreover, as expected, earnings at the bottom quantile would grow significant faster had the minimum wage retained its value over the 1980s – had the real value of the minimum wage been constant, Table 4 implies that lower-tail earnings would be nearly 10% higher than they were in 2008. The median wage would be nearly unchanged. Given these counterfactual growth rates, the log wage gap between the median and the tenth percentile would be nearly unchanged from 1970. This matches Lee’s result ?? that nearly all the growth in lower-tail inequality is due to the minimum wage.
<table>
<thead>
<tr>
<th></th>
<th>10th Percentile</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Benchmark</td>
<td>Counterfactual</td>
</tr>
<tr>
<td>1970–1980</td>
<td>−0.097%</td>
<td>0.674%</td>
<td>0.721%</td>
</tr>
<tr>
<td>1980–1995</td>
<td>0.574%</td>
<td>1.01%</td>
<td>1.05%</td>
</tr>
<tr>
<td>1995–2005</td>
<td>0.897%</td>
<td>1.39%</td>
<td>1.97%</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>Median</th>
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<tbody>
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<td>Benchmark</td>
<td>Counterfactual</td>
</tr>
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<td>1970–1980</td>
<td>−0.128%</td>
<td>0.686%</td>
<td>0.676%</td>
</tr>
<tr>
<td>1980–1995</td>
<td>1.072%</td>
<td>1.193%</td>
<td>1.211%</td>
</tr>
<tr>
<td>1995–2005</td>
<td>1.011%</td>
<td>1.445%</td>
<td>1.455%</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>90th Percentile</th>
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<tbody>
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<td></td>
<td>Data</td>
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<td>Counterfactual</td>
</tr>
<tr>
<td>1970–1980</td>
<td>0.413%</td>
<td>0.767%</td>
<td>0.784%</td>
</tr>
<tr>
<td>1980–1995</td>
<td>1.785%</td>
<td>1.283%</td>
<td>1.295%</td>
</tr>
<tr>
<td>1995–2005</td>
<td>1.798%</td>
<td>1.427%</td>
<td>1.456%</td>
</tr>
</tbody>
</table>

Table 4: Annual Growth Rates of Earnings Quantiles in the Data, the Benchmark Model, and Counterfactual

7 Conclusions

Through the inclusion of low-skilled workers, the falling minimum wage has altered the composition of the labor force. This, however, is its only consequence. The growth in lower tail inequality over the 1980s was partly an episodic phenomenon which can be attributed to the minimum wage, but only because the falling minimum wage enabled lower skilled workers to supply their skills on the labor market.

Skill-biased technological change is responsible for much of the continued widening of the earnings distribution: in Table 4, top earnings generally grow faster than the median, which in turn grows faster than earnings received at the lower tail. However, the growth rates of the lowest quantile is considerably slower in the data than in the model, even accounting for the minimum wage. An institutional factor besides the minimum wage is likely responsible. The recent drop in median wages also cannot be explained by the model.

The standard measures for between-group inequality are misleading because they cannot control for unobserved heterogeneity in the labor force. The behavior of the college premium may be attributed to composition. In particular, the college premium fell in the 1970s because the mean level of human capital of college educated workers fell. This composition in turn is due to skill-biased technological change.

Further research should analyze other relevant institutional factors, especially deregulated
institutional trade and the entry of women into the labor market.

References


A Data Appendix

The data are taken from the 1962–2006 March Current Population Survey, made available by the Minnesota Population Center at http://cps.ipums.org. The data processing steps are very similar to Autor, Katz, and Kearney [3]. I include in the sample full time, full year (FTFY) wage and salary workers, and self-employed workers employed in incorporated businesses from 1976 on. Full year workers are those who work at least 40 weeks per year, and full time workers are identified using the variable labeled 'fullpart' in data. All workers classified as self-employed before 1976, and self-employed non-incorporated workers post 1975 are excluded, along with unpaid family members.

Annual income for FTFY workers is the sum of four income variables: wages and salary from the longest job held over the year, other wage income, business income, and farm income. Following Katz and Murphy [23], topcoded values are replaced with with 1.5 times the topcode. Weekly income is then the quotient of annual income and estimated weeks worked. Before 1976, weeks worked over the previous year are intervalled: individuals report weeks worked within intervals of 1-13 weeks, 14-26 weeks, 27-29, etc. From 1976 on, individuals report any integer number of weeks between 0 and 52. With one exception, for years before 1976, the intervalled values are replaced with median weeks worked among individuals who report working any number of weeks within the given interval in 1976. The exception is that I manually topcode weeks worked at 50. As in [3], full time earnings are weighted by the product of the sampling weight and weeks worked. Likewise, individuals with full-time weekly earnings below $67 per week in 1982 dollars are dropped from the data.
$67 per week is one-half weekly earnings for a full time worker (40 hours per week) at the 1982 minimum wage ($3.35 per hour).

Data from the 1969 CPS (covering earnings year 1968) is used to calibrate the parameters \( \alpha, \beta, \) and \( \nu = (\mu_H, \mu_L, \sigma_H, \sigma_L, \rho). \) There are 54 observations in the 1969 data of FTFY wage and salary college workers younger than 23 (so younger than 22 in the earnings year). These, along with 37 observations with negative probability weights, are dropped. After this processing we have 38,265 observations on earnings and schooling of FTFY workers. Using probability weights, an estimated 10.2% of base year FTFY workers are sub-minimum wage workers if full time workers are assumed to work 40 hours per week. This figure drops to 6.14% if full time workers are assumed to work 35 hours per week.

I determine whether workers supply high or low skill as follows. Prior to 1992, the CPS includes the number of years of education for each worker reports having completed. Type - \( H \) workers are those who report having completed four years of college, and type - \( L \) workers are all others. Starting in 1992, the CPS instead reports the highest degree attained, and type - \( H \) workers are those who have at least a bachelor’s degree. College enrollment is taken from Table 189 of the 2007 Digest of Education Statistics. This is an estimate of the fraction of the population aged 18–24 enrolled in degree-granting institutions.

**B Maximum Likelihood Estimates**

Given the sample of observations \((y_j, I_j, s_j, w_j)_{j=1}^{31819}\) and Mincer parameters \((\alpha, \beta)\) from the 1969 CPS, where \(y_j\) indicates log earnings for FTFY worker \(j, \) age, \(s_j \in \{0, 1\}\) schooling choice, and \(w_j\) is the CPS sampling weight, the problem is to write the maximum likelihood function for the parameters \((\nu, \pi_0)\) assuming the economy moves along a balanced growth path. To do this, let 

\[ Z_j = y_j - (\alpha_1 I_j - \alpha_2 I_j^2) (1 - s_j) - (\beta_1 (I_j - 4) - \beta_2 I_j^2) s_j \]

\(Z_j\) is an observation of the random variable \(Z(Z_L, Z_H),\)

\[
Z(Z_L, Z_H) = \begin{cases} 
Z_L & \text{for } V_0^L (\exp(Z_L)) > V_0^H (\exp(Z_H)) \\
Z_H & \text{otherwise}
\end{cases}
\]

where \(V_t^k(\theta)\) and \(V_t^H(\theta)\) are given by (8). Define \(b_L = \log(W_0), b_H = \log(W_0) + \beta_2 4^2,\) the function \(b(\alpha, \beta, \gamma, r, T)\) and the set \(C\) as follows

\[
\hat{b}(\alpha, \beta, \gamma, r, T) = \log \sum_{i=0}^{T} \left[ \left( \frac{1 + \gamma}{1 + r} \right)^i \exp(\alpha i - \alpha_2 i^2) \right] - \log \sum_{i=4}^{T} \left[ \left( \frac{1 + \gamma}{1 + r} \right)^i \exp[\beta_1 (i - 4) - \beta_2 i^2] \right]
\]

\[
C = \{(Z_L, Z_H) : Z_H \geq Z_L + \hat{b} \} \cup \{(Z_L, Z_H) : \hat{g} < Z_H - Z_L < \hat{g} - \log \pi, \ Z_L \geq b_L, \ Z_H < b_H \}
\]
Recalling the base year normalization $w_L(0) = w_H(0) = 1$, from (8) it can be shown that

$$Z(Z_L, Z_H) = \begin{cases} Z_L & \text{for } (Z_L, Z_H) \in C \\ Z_H & \text{otherwise} \end{cases} \quad (10)$$

The likelihood of observation $(Z_t, s_t)$, given by $l(Z_t, s_t; v, \pi, \alpha, \beta, b_L)$, is the joint distribution of $(Z_t, s_t)$. It is determined in terms of the functions $F_L(B, Z_L; \nu)$ and $F_H(B, Z_H; \nu)$ below. The analysis of Basu and Ghosh [7] is very helpful here.

$$F_H(B, Z_H) = \int_{-\infty}^{B} f(Z_L, Z_H; \nu) \, dZ_L \quad F_L(B, Z_L) = \int_{-\infty}^{B} f(Z_L, Z_H; \nu) \, dZ_H \quad (11)$$

where $f(Z_L, Z_H; \nu)$ is the bivariate skew-normal with parameter vector $\nu$. The following algebraic result speeds numerical evaluation of the functions (11) considerably:

$$F_H(B, Z_H) = \frac{2}{\sigma_H} \phi \left( \frac{Z_H - \mu_H}{\sigma_H} \right) \Phi_2 \left[ \frac{1}{1 - \rho^2} \left[ \left( B - \mu_L \right) \frac{1}{\sigma_L} - \rho \left( Z_H - \mu_H \right) \frac{1}{\sigma_H} \right] \right]$$

$$\frac{\alpha_L \rho + \alpha_H}{\sqrt{1 + \alpha_L^2 (1 - \rho^2)}} \left( Z_H - \mu_H \right) \frac{1}{\sigma_H} - \frac{\alpha_L \sqrt{1 - \rho^2}}{\sqrt{1 + \alpha_L^2 (1 - \rho^2)}} \right]$$

$\phi()$ is again the normal pdf. $\Phi_2(x, y, \zeta)$ is the bivariate normal cdf with correlation $\zeta$. The BGP schooling choice criterion (10) now implies the following\(^{29}\):

\[
l(\nu, \pi; Z_t, 0, \alpha, \beta) = \begin{cases} \pi^{1_{(Z_t < b_L)}} F_L(Z_t + \tilde{g}, Z_t) & Z_t < b_L \text{ or } Z_t \geq b_H - \tilde{g} \\ F_L(Z_t + \tilde{g} - \log(\pi), Z_t) & Z_t \geq b_L \text{ and } Z_t < b_L - \tilde{g} + \log(\pi) \\ F_L(b_L, Z_t) & Z_t \geq \min\{b_L, b_H - \tilde{g}\} \text{ and } Z_t < b_H \end{cases}
\]

\[
l(\nu, \pi, Z_t, 1, \alpha, \beta) = \begin{cases} \pi^{1_{(Z_t < b_L)}} F_H(Z_t - \tilde{g}, Z_t) & Z_t < b_L + \tilde{g} \text{ or } Z_t \geq b_H \\ \pi F_H(Z_t - \tilde{g} + \log(\pi), Z_t) & Z_t \geq b_L + \tilde{g} - \log(\pi) \text{ and } Z_t < b_H \\ \pi F_H(b_L, Z_t) & Z_t \geq b_L + \tilde{g} \text{ and } Z_t \leq \min\{b_H, b_L + \tilde{g} - \log(\pi)\} \end{cases}
\]

Of course, the second line of both equations is irrelevant if $b_H - \tilde{g} + \log(\pi) < b_L$. The likelihood function is simply

$$L(\nu, \pi; Z_1, s_1\ldots Z_N, s_N) = \sum_{i=1}^{N} \omega_i \log[l(\nu, \pi; Z_i, s_i, \alpha, \beta)]$$

\(^{29}\)Some parameters are being suppressed for legibility, i.e., $l(\nu, \pi; Z_t, 0, \alpha, \beta)$ instead of $l(\nu, \pi; Z_t, 0, \alpha, \beta, \gamma, b_L, b_H)$

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$N = 31819$, and $\omega_i$ is the CPS sampling weight for observation $i$. This is the function maximized numerically to find the parameters in Table 2.

C Algorithm for Computing the Transition Path

I assume steady 1.5\% growth in $g_H$, $g_L$ and $W$ after 2008, and that $\pi$ remains constant. The economy is assumed to converge to the new balanced growth path 150 years after SBTC began. The new BGP endogenous variables are calculated analytically. For an initial guess of the endogenous variables over the transition path, I assume that $w_L$ and $w_H$ grow at a steady rate from 2000 to the ending steady state. The intial guess of $w_L$ and $w_H$ from 1969 to 1999 is then produced by assuming steady growth in both wages. Finally, $r_t$ is selected to produce a constant interest rate throughout equal to the new steady state value. This initial guess of \{$w_L(t), w_H(t), r_t$\} was found to induce rapid convergence.

Next, a random sample of 10,000 observations of the bivariate skew-normal distribution with parameters $\nu$ is drawn\(^{30}\). Then for each cohort that lives during the transition path – the $T$ initial old cohorts and 150 born during the transition path, the set of ages during which work is permitted is calculated. More specifically, define the set $\Omega$ of possible work histories: $\omega \in \Omega \Rightarrow \omega \in \{0, 1\}^{T+1} \forall i$. With $2^{27}$ elements set $\Omega$ is impossibly large, so initially I calculate the subset $\hat{\Omega}$ of relevant elements of $\Omega$, or those which possibly characterize the labor market experience of an agent in the random sample, given the assumed sequence of exogeneous variables and guessed values of the endogenous prices. Then for each simulated transition path agent, the values $\hat{\omega}_H \in \hat{\Omega}$ and $\hat{\omega}_L \in \hat{\Omega}$ which would describe the agent's experience in either sector is stored.

Since the initial old generations are unable to switch sectors, their labor supply can then be immediately calculated. For transition path generations it is necessary to first calculate expected lifetime income from labor in either sector. Given this, the transition path schooling choice and thus labor supply of transition path generations is determined. Finally, given the endogenous prices and each family member's labor supply, discounted family transition path income, and hence consumption, are found. Using (2) we can find the excess demand for both types of labor are found, and using (1) we find the excess demand for goods (or, by Walras’ Law, bonds).

Now the procedure is repeated with an updated guess of the level of the endogenous variables. The new values of $w_L(t)$ and $w_H(t)$ are between the marginal products and the previous guess. Specifically, for the first five years, the new values are the marginal products,

\(^{30}\)Given $\nu$, [5] shows how to construct random sample of any bivariate skew normal from an appropriately parameterized random sample of the trivariate normal.
for the next five, the values are 90% of the marginal product and 10% of the old values, and so on for the next 50 years. Thereafter, the new value is 10% of the marginal product and 90% the old value. The updated interest rate is that which numerically clears the market for goods, when wages are equal to the marginal product. This process is repeated until convergence, which takes 10–15 iterations.

Figure 1: Indexed quantiles of the earnings distribution, FTFY workers

Figure 2: Quantiles, FTFY men

Figure 3: Quantiles, FTFY women
Figure 4: The College Premium

Figure 5: The College Premium for Young and Old Workers
Figure 6: Experience Differentials, College

Figure 7: Experience Differentials, HS workers

Figure 8: Real Federal Minimum Wage
Figure 9: Overall Inequality and the Minimum Wage

Figure 10: The Fraction of the FTFY Workforce Earning Less than Minimum Wage
Figure 11: The Benchmark Calibration

Figure 12: The Benchmark Calibration

Figure 13: Calibrated Distribution of log Skill Parameters.
Figure 14: 10th Percentile, the Benchmark Model

Figure 15: Median Earnings, the Benchmark Model
Figure 16: 90th Percentile, the Benchmark Model

Figure 17: The College Premium for Workers Age 25-30, Benchmark Model
Figure 18: The College Premium for Workers Age 46-60, Benchmark Model

Figure 19: Agents with black parameters attend college in 1980 but not in 1970.
Figure 20: Experience Differentials for College Workers

Figure 21: Experience Differentials for H.S. Workers
Figure 22: The Endogenous Wage Ratio