Problem set 5 Solution

1 Bank Runs

Consider an economy with three periods $t = 0, 1, 2$. Individuals are endowed with one unit of the consumption good at $t = 0$. There is a technology that gives a return of $1 + r > 1$ in $t = 2$ if investment was done in $t = 0$. If the investment is interrupted in period $t = 1$ the return is 1 (i.e. you get the same one unit that was invested). In addition, goods can be stored from period 1 to 2.

There are two types of agents. With probability θ the agent will be "impatient" (of type 1) who cares only about consumption at $t = 1$. With probability $1-\theta$, the agent will be "patient" (of type 2) and care only about consumption at $t = 2$. Agents realize their types after investment is done. Preferences can be described be the expected utility

$$U(c_1, c_2) = \theta(-\frac{1}{c_1}) + (1-\theta)(-\frac{1}{c_2}).$$

In the aggregate, the probabilities also represent fractions of each type of consumers.

1. Write down the consumers' problem in case of no banking, i.e without any possibility of insurance. Compute the optimal consumption for patient and impatient agents, and their expected utility EU^{nb}. Also compute the per capita output of the economy Y^{nb} (aggregate over agents and periods 1 and 2).

   Without insurance, everyone invests at time 0. Patient consumers wait until period 2 and consume $1+r$, while impatient consumers interrupt the production in period 1 and consume 1. The expected utility is then $EU^{nb} = -(\theta + \frac{1-\theta}{1+r})$. The output per person is $Y^{nb} = \theta + (1-\theta)(1+r)$.

2. Suppose banks accept deposits at time 0 and pay d_1 if you withdraw your deposits at $t = 1$, and d_2 if you withdraw your deposits at $t = 1$. Solve for the optimal consumption of both types of consumers. How does it compare to consumption from part (a)?

The banking contract solves

$$\max_{c_1,c_2} \theta(-\frac{1}{c_1}) + (1-\theta)(-\frac{1}{c_2})$$

subject to

$$\theta c_1 + \frac{(1-\theta)c_2}{1+r} = 1.$$

The first order conditions imply $c_2 = \sqrt{1+r}c_1$. Solving for c_1 and c_2, we get

$$\begin{aligned} c_1 &= \frac{1}{\theta + \frac{1-\theta}{\sqrt{1+r}}} > 1 \\ c_2 &= \frac{1+r}{\theta\sqrt{1+r} + 1 - \theta} < 1 + r. \end{aligned}$$

Thus, the banking contract provides more consumption in state 1 and less consumption in state 2.

3. Compute the expected utility EU^b and aggregate output Y^b. Do they differ from the case of autarchy? How?

   The expected utility is

$$\begin{aligned} EU^b &= -(\frac{\theta}{c_1} + \frac{1-\theta}{c_2}) \\ &= -\frac{1}{c_1}(\theta + \frac{1-\theta}{\sqrt{1+r}}) \\ &= -(\theta + \frac{1-\theta}{\sqrt{1+r}})^2 \\ &= -\theta^2 - \frac{(1-\theta)^2}{1+r} - 2\frac{\theta(1-\theta)}{\sqrt{1+r}} \\ &= -(\theta + \frac{1-\theta}{1+r}) - \theta(1-\theta)\frac{2\sqrt{1+r} - (2+r)}{1+r} \\ &> -(\theta + \frac{1-\theta}{1+r}) \\ &= EU^{nb} \end{aligned}$$

since $2\sqrt{1+r} < 2+r$. The aggregate output is

$$\begin{aligned} Y^b &= \theta c_1 + (1-\theta)c_2 \\ &= c_1(\theta + (1-\theta)\sqrt{1+r}) \\ &= \frac{\theta + (1-\theta)\sqrt{1+r}}{\theta + \frac{1-\theta}{\sqrt{1+r}}} \\ &= \frac{\theta\sqrt{1+r} + (1-\theta)(1+r)}{\theta\sqrt{1+r} + 1 - \theta} \\ &= 1 + \frac{1-\theta}{\theta\sqrt{1+r} + 1 - \theta} r \\ &< 1 + (1-\theta)r \\ &= Y^{nb}. \end{aligned}$$

Hence the aggregate output decreases.

4. What happen if all agents want to withdraw their deposits at $t = 1$?

   There is not enough resources: the total amount of resources the bank can extract by interrupting the production is 1 per person, but it commits to pay $d_1 > 1$ per person. Hence not everyone will be paid.