Economics 208 Marek Kapicka
Macroeconomics Spring 2009

Problem Set 3 Answers

1 Elasticity of Labor Supply and the Laffer Curve

Consider consumers with preferences over consumption C and labor supply L given by

$$U(C,L) = C - \frac{L^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}}, \qquad \gamma > 0.$$

The parameter γ is the elasticity of labor supply (elasticity of labor supply measures how responsive the labor supply is to changes in the net wage rate).

1. Suppose that the consumers's consumption is given by $C = w(1-\tau)L$, where τ is the labor tax rate and w is the wage rate. Compute the optimal labor supply as a function of τ and w.

$$\max_L w(1-\tau)L - \frac{L^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}}$$

$$w(1-\tau) = L^{\frac{1}{\gamma}}$$

$$L^s = w^\gamma (1-\tau)^\gamma$$

Partial equilibrium Laffer curve

2. Suppose that the wage rate is exogenously given. Compute the government's tax revenue as a function of τ and w. Find the tax rate that maximizes government's revenue. How does it depend on the elasticity of labor supply? Compute the revenue maximizing tax rate.

$$T = \tau L^s = w^\gamma \tau (1-\tau)^\gamma$$

$$\max_{\tau} w^{\gamma}\tau(1-\tau)^{\gamma}$$
$$(1-\tau)^{\gamma} - \gamma\tau(1-\tau)^{\gamma-1} = 0$$
$$1 - \gamma\frac{\tau}{1-\tau} = 0$$
$$\gamma\frac{\tau}{1-\tau} = 1$$
$$\gamma\tau = 1-\tau$$
$$\tau^* = \frac{1}{1+\gamma}.$$

Thus, the peak of the Laffer curve depends negatively on the elasticity of labor supply. Intuitively, the more elastic the labor supply, the more negative response there is going to be from an increase in the tax rate.

General equilibrium Laffer curve. One potential problem with the answer in part 2 is that the wage rate is exogenously fixed. In reality, wages will respond to changes in labor supply. To examine this effect, set $\gamma = 1$ and assume that the firms in the economy have production function given by $F(L) = 2\sqrt{L}$.

3. Set up the firms profit maximization problem and compute the labor supply as a function of the wage rate.

$$\max_{L} 2\sqrt{L} - wL$$
$$\frac{1}{\sqrt{L}} = w$$
$$L^d = \frac{1}{w^2}.$$

4. Using your answers from parts 1 and 3, compute the equilibrium wage rate as a function of the tax rate. Is it increasing or decreasing in the tax rate? Why? (15 pts)

$$L^d = L^s$$
$$\frac{1}{w^2} = w^{\gamma}(1-\tau)^{\gamma}$$
$$(1-\tau)^{-\gamma} = w^{2+\gamma}$$
$$w^*(\tau) = (1-\tau)^{-\frac{\gamma}{2+\gamma}}$$

for $\gamma = 1$, we have $w^* = (1-\tau)^{-\frac{1}{3}}$.

5. Compute the government's tax revenue as a function of τ when the wage rate is determined endogenously, i.e. given by your answer in part 4. Compute the revenue maximizing tax rate and compare it to your answer in part 2. Is it higher or lower? Explain.

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\begin{aligned}
T &= \tau w^*(\tau)^\gamma (1-\tau)^\gamma = \tau(1-\tau)^{-\frac{\gamma}{2+\gamma}}(1-\tau)^\gamma = \tau(1-\tau)^{\gamma-\frac{\gamma}{2+\gamma}} \\
0 &= (1-\tau)^{\gamma-\frac{\gamma}{2+\gamma}} - (\gamma - \frac{\gamma}{2+\gamma})\tau(1-\tau)^{\gamma-\frac{\gamma}{2+\gamma}-1} \\
1 &= (\gamma - \frac{\gamma}{2+\gamma})\frac{\tau}{1-\tau} \\
1-\tau &= (\gamma - \frac{\gamma}{2+\gamma})\tau \\
\tau &= \frac{1}{1+\gamma-\frac{\gamma}{2+\gamma}}
\end{aligned}
$$

and for $\gamma = 1$, one gets $\tau^* = 60\%$. The peak of the Laffer curve is now reached at a higher tax rate because labor taxation increases the wage rate, which has additional positive impact on the government revenue.