Economics 208 Marek Kapicka
Macroeconomics Spring 2009

Problem Set 2 Solution

1 Social Security in General Equilibrium

One of the weaknesses of the model of social security we have studied in class is that it takes the interest rate as given. In the real world, we have to allow for the possibility that the social security system might affect aggregate savings and investment, which in turn would have an effect on interest rates.

Consider a model in which people live for two periods. They work when young, but they do not consume, and they consume when old, but do not work. Each period a new young generation is born, and for simplicity we assume that all generations are of equal size. A consumer born at time t has the following preferences over labor l_t when young and consumption c_{t+1} when old:

$$u(c_{t+1}, l_t) = 2\sqrt{c_{t+1}} - l_t.$$

The consumer has to pay a proportional social security tax τ on labor income in the first period. Since there is no consumption in the first period, all after-tax income will be saved:

$$s_t = (1-\tau)w_t l_t,$$

where w_t is the wage rate in period t. The social security benefits *per dollar earned* will be denoted by b_t. Depending on the pension system, b_t will take different values. Consumption in the second period is then given by

$$c_{t+1} = (1 + r_{t+1})s_t + b_t w_t l_t,$$

where r_{t+1} is the interest rate on period t savings.

There is a single firm in the economy that produces the consumption good using the production function $y_t = 2\sqrt{k_t l_t}$. The firm problem is to maximize profits by choosing how much capital to rent (at an interest rate r_t) and how much labor to demand (at a wage rate w_t) :

$$\max 2\sqrt{k_t l_t} - r_t k_t - w_t l_t.$$

The equilibrium conditions are given by

$$\sqrt{\frac{k_t}{l_t}} = w_t \tag{1}$$

$$\sqrt{\frac{l_t}{k_t}} = r_t. \tag{2}$$

1. Solve for the optimal labor supply l_t and savings s_t, both as a function of the interest rate r_{t+1}, wages w_t, and parameters of the social security system τ and b_t.

   Maximization problem:

$$\max_{l_t} \sqrt{[(1+r_{t+1})(1-\tau)+b_t]w_t l_t} - l_t$$

$$\begin{aligned} 1 &= \frac{[(1+r_{t+1})(1-\tau)+b_t]w_t}{\sqrt{[(1+r_{t+1})(1-\tau)+b_t]w_t l_t}} \\ \sqrt{l_t} &= \sqrt{[(1+r_{t+1})(1-\tau)+b_t]w_t} \\ l_t &= [(1+r_{t+1})(1-\tau)+b_t]w_t \end{aligned}$$

$$\begin{aligned} s_t &= (1-\tau)w_t l_t \\ &= (1-\tau)[(1+r_{t+1})(1-\tau)+b_t]w_t^2 \end{aligned}$$

2. Consider now a fully funded system. In the fully funded system, both the savings of the household and the social security contributions are invested and earn the same interest r_{t+1}. Therefore, the return b_t on social security is given by $b_t = (1+r_{t+1})\tau$. In addition, next period capital stock is the sum of private and social security contributions:

$$k_{t+1} = s_t + \tau w_t l_t.$$

   (a) Use your answer to part (1) to derive the expression for labor supply and next period capital stock under the fully funded pension system.

$$\begin{aligned} l_t &= [(1+r_{t+1})(1-\tau)+(1+r_{t+1})\tau]w_t \\ &= (1+r_{t+1})w_t \\ k_{t+1} &= s_t + \tau w_t l_t \\ &= (1-\tau)w_t l_t + \tau w_t l_t \\ &= w_t l_t \\ &= (1+r_{t+1})w_t^2 \end{aligned}$$

(b) Assume now that the system is in steady state. Use the firm's equilibrium conditions (1) and (2) together with the expression for labor supply and next period capital stock under the fully funded pension system to compute the equilibrium steady state capital stock and interest rate.

$$\frac{k}{l} = w = \sqrt{\frac{k}{l}}$$

Hence $\frac{k}{l} = 1$. Then,

$$\begin{aligned} k &= (1+r)w^2 = (1+\sqrt{\frac{l}{k}})\frac{k}{l} = 2 \\ l &= 2 \\ r &= \sqrt{\frac{l}{k}} = 1. \end{aligned}$$

3. Consider now a pay-as-you-go social security system. Since all generations are of the same size, the benefits are given by $b_t = \tau$. In addition, since social security benefits are not invested, next period capital stock is given by private savings only:

$$k_{t+1} = s_t.$$

(a) Derive the expression for labor supply and next period capital stock under the pay-as-you-go system.

$$\begin{aligned} l_t &= [(1+r_{t+1})(1-\tau)+\tau]w_t \\ k_{t+1} &= s_t \\ &= (1-\tau)w_t l_t \\ &= (1-\tau)[(1+r_{t+1})(1-\tau)+\tau]w_t^2 \end{aligned}$$

(b) Assume that the system is in steady state. Use the firm's equilibrium conditions (1) and (2) together with the expression for labor supply and next period capital stock under the pay-as-you-go social security system to compute the equilibrium steady state capital stock and interest rate.

$$
\begin{aligned}
\frac{k}{l} &= (1-\tau)w = (1-\tau)\sqrt{\frac{k}{l}} \\
(\frac{k}{l})^2 &= (1-\tau)^2\frac{k}{l} \\
\frac{k}{l} &= (1-\tau)^2.
\end{aligned}
$$

Hence

$$
\begin{aligned}
l &= [(1+r)(1-\tau)+\tau]w \\
&= [(1+\sqrt{\frac{l}{k}})(1-\tau)+\tau]\sqrt{\frac{k}{l}} \\
&= [(1+\frac{1}{1-\tau})(1-\tau)+\tau](1-\tau) \\
&= 2(1-\tau) \\
k &= (1-\tau)^2 l = 2(1-\tau)^3 \\
r &= \sqrt{\frac{l}{k}} = \frac{1}{1-\tau}
\end{aligned}
$$

4. Compare the capital stock, labor supply, and interest rate under both systems.

   Both capital stock and labor supply are lower under the PAYG system. Interest rates are higher under PAYG.