

# Problem Set 1

Due Monday April 13 in class

## 1 Intertemporal choice

Consider an economy with 2 households. Denote them household  $a$  and household  $b$ . Both households live for two periods. They care only about consumption and have the utility function

$$u = \sqrt{c_1} + \frac{2}{3}\sqrt{c_2}$$

where  $c_1$  is consumption in period 1 and  $c_2$  is consumption in period 2. The discount factor  $\beta$  was set equal to  $\frac{2}{3}$ . Denote  $c_1^a, c_2^a$  a consumption of household  $a$  and similarly for household  $b$ .

Household  $a$  has a fixed income of two units of consumption good in the first period, and zero units in the second period. Consumer  $b$  is endowed with one unit of consumption good in each period.

The households cannot store the consumption good, but can trade with each other in the credit market. Denote the savings as  $s^a$  for household  $a$  and  $s^b$ .

Since there is nobody else in the economy, total savings in the economy have to be equal to zero:

$$s^a + s^b = 0 \tag{1}$$

Denote the market interest rate as  $r$ . We are ultimately interested in how much each household consumes in each period and how much they save. Thus, we need to find 6 numbers  $c_1^a, c_2^a, s^a, c_1^b, c_2^b, s^b$  and an equilibrium interest rate  $r$ .

1. Write down the present value budget constraint for both households.
2. Let  $c_1^a(r)$  and  $c_2^a(r)$  be consumption choices of consumer  $a$ . Define  $c_1^b(r)$  and  $c_2^b(r)$  similarly. Solve for  $c_1^a(r), c_2^a(r), c_1^b(r), c_2^b(r)$  as functions of  $r$ .

3. Use the first period flow budget constraint to express savings  $s^a(r)$  and  $s^b(r)$  as functions of  $r$ .
4. Now use your answers and the market clearing condition 1 to solve for the equilibrium interest rate.
5. Which household is borrowing and which one is saving? Compute the savings of each household, by plugging the equilibrium interest rate into the expressions  $s^a(r)$  and  $s^b(r)$ .