Economics 208
Macroeconomics

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Problem Set 1 Answers

1 Intertemporal choice

Consider an economy with 2 households. Denote them household a and household b. Both households live for two periods. They care only about consumption and have the utility function

$$u = \sqrt{c_1} + \frac{2}{3}\sqrt{c_2}$$

where c_1 is consumption in period 1 and c_2 is consumption in period 2. The discount factor β was set equal to $\frac{2}{3}$. Denote c_1^a, c_2^a a consumption of household a and similarly for household b.

Household a has a fixed income of two units of consumption good in the first period, and zero units in the second period. Consumer b is endowed with one unit of consumption good in each period.

The households cannot store the consumption good, but can trade with each other in the credit market. Denote the savings as s^a for household a and s^b.

Since there is nobody else in the economy, total savings in the economy have to be equal to zero:

$$s^a + s^b = 0 \tag{1}$$

Denote the market interest rate as r. We are ultimately interested in how much each household consumes in each period and how much they save. Thus, we need to find 6 numbers $c_1^a, c_2^a, s^a, c_1^a, c_2^a, s^b$ and an equilibrium interest rate r.

1. Write down the present value budget constraint for both households.

$$\begin{aligned} c_1^a + \frac{c_2^a}{1+r} &= 2 \\ c_1^b + \frac{c_2^b}{1+r} &= 1 + \frac{1}{1+r} \end{aligned}$$

2. Let $c_1^a(r)$ and $c_2^a(r)$ be consumption choices of consumer a. Define $c_1^b(r)$ and $c_2^b(r)$ similarly. Solve for $c_1^a(r)$, $c_2^a(r)$, $c_1^b(r)$, $c_2^b(r)$ as functions of r.

- For household a:

$$\max_{c_2} \sqrt{2 - \frac{c_2}{1+r}} + \frac{2}{3}\sqrt{c_2}$$

$$\begin{aligned}
\frac{1}{2}\frac{1}{\sqrt{2-\frac{c_2}{1+R}}}\frac{1}{1+r} &= \frac{2}{3}\frac{1}{2}\frac{1}{\sqrt{c_2}} \\
\frac{2}{3}(1+r)\sqrt{2-\frac{c_2}{1+r}} &= \sqrt{c_2} \\
\frac{4}{9}(1+r)^2(2-\frac{c_2}{1+r}) &= c_2 \\
\frac{8}{9}(1+r)^2 &= c_2(1+\frac{4}{9}(1+r))
\end{aligned}$$

Solving for consumption in both periods, one gets

$$\begin{aligned}
c_2^a(r) &= \frac{\frac{8}{9}(1+r)^2}{(1+\frac{4}{9}(1+r))} \\
c_1^a(r) &= \frac{2}{(1+\frac{4}{9}(1+r))}.
\end{aligned}$$

- For household b:

$$\max_{c_2} \sqrt{1 + \frac{1}{1+r} - \frac{c_2}{1+r}} + \frac{2}{3}\sqrt{c_2}$$

$$\begin{aligned}
\frac{1}{2}\frac{1}{\sqrt{2-\frac{c_2}{1+r}}}\frac{1}{1+r} &= \frac{2}{3}\frac{1}{2}\frac{1}{\sqrt{c_2}} \\
\frac{2}{3}(1+r)\sqrt{1+\frac{1}{1+r}-\frac{c_2}{1+r}} &= \sqrt{c_2} \\
\frac{4}{9}(1+r)^2(1+\frac{1}{1+r}-\frac{c_2}{1+r}) &= c_2 \\
\frac{8}{9}(1+r)^2 &= c_2(1+\frac{4}{9}(1+r))
\end{aligned}$$

Consumption is thus given by

$$
\begin{aligned}
c_2^b(r) &= \frac{\frac{4}{9}(1+r)^2(1+\frac{1}{1+r})}{(1+\frac{4}{9}(1+r))} \\
c_1^b(r) &= \frac{1+\frac{1}{1+r}}{(1+\frac{4}{9}(1+r))}.
\end{aligned}
$$

3. Use the first period flow budget constraint to express savings $s^a(r)$ and $s^b(r)$ as functions of r.

$$
\begin{aligned}
s^a(r) &= 2 - c_1^a(r) = 2 - \frac{2}{(1+\frac{4}{9}(1+r))} = 2\frac{\frac{4}{9}(1+r)}{(1+\frac{4}{9}(1+r))} \\
s^b(r) &= 1 - c_1^b(r) = 1 - \frac{1+\frac{1}{1+r}}{(1+\frac{4}{9}(1+r))} = \frac{1}{1+r}\frac{\frac{4}{9}(1+r)^2-1}{(1+\frac{4}{9}(1+r))}
\end{aligned}
$$

4. Now use your answers and the market clearing condition 1 to solve for the equilibrium interest rate.

$$
\begin{aligned}
s^a(r) + s^b(r) &= 0 \\
2\frac{\frac{4}{9}(1+r)}{(1+\frac{4}{9}(1+r))} + \frac{\frac{4}{9}(1+r) - \frac{1}{1+r}}{(1+\frac{4}{9}(1+r))} &= 0 \\
\frac{8}{9}(1+r) + \frac{4}{9}(1+r) &= \frac{1}{1+r} \\
(1+r)^2 &= \frac{9}{12} \\
1+r &= \sqrt{\frac{3}{4}}
\end{aligned}
$$

5. Which household is borrowing and which one is saving? Compute the savings of each household, by plugging the equilibrium interest rate into the expressions $s^a(r)$ and $s^b(r)$.

$$
\begin{aligned}
s^a(r) &= \frac{\frac{8}{9}\sqrt{\frac{3}{4}}}{(1+\frac{4}{9}\sqrt{\frac{3}{4}})} = 0.556 \\
s^b(r) &= \sqrt{\frac{4}{3}}\frac{\frac{4}{9}\frac{9}{12}-1}{(1+\frac{4}{9}\sqrt{\frac{3}{4}})} = -\frac{\frac{2}{3}\sqrt{\frac{4}{3}}}{(1+\frac{4}{9}\sqrt{\frac{3}{4}})} = -0.556
\end{aligned}
$$

Thus, household a is saving (they have no income in period 2 and would like to consume). Household b is borrowing. Their income is spread evenly, but the market induces them to borrow from household a, because the interest rate is very low (negative).