Econ 208

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Lecture 2

Basic Intertemporal Model

What To Do

- Read: DLS, Chapter 12
- Problem Set 1: On the web, due next Monday in class.

Where are we?

- A Basic Intertemporal Model
  - A) Consumer Optimization
  - B) Market Clearing
  - C) Adding capital stock
  - D) Welfare Theorems
  - E) Infinite horizon

C) Adding Capital Stock

Consumer's problem revisited

- Budget Constraints:

$$C_1 + B_1 = \Pi_1(r)$$

$$C_2 = \Pi_2(r) + B_1(1+r)$$

  - b_1 are savings from period 1 to period 2
  - r is the interest rate

C) Adding Capital Stock

Market Equilibrium

- Market Clearing

$$C_1 + I_1 = F(K_1)$$

$$C_2 = F(K_2) + K_3$$

- Properties of Equilibrium:

$$\begin{aligned} U'(c_1) &= \beta(1+r)U'(c_2) \\ &= \beta[F_K(K_2) + 1 - \delta]U'(c_2) \end{aligned}$$

Where are we?

- A Basic Intertemporal Model
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D) Efficiency of Equilibrium
Pareto Efficiency

- Pareto Efficient Allocation satisfies

$$\max_{C_1,C_2,I_1} U(C_1)+\beta U(C_2) \quad s.t \quad C_1+I_1=F(K_1)$$

$$C_2=F(K_2)+K_3$$

- Properties of Pareto Optimum:

$$U'(c_1)=\beta[F_K(K_2)+1-\delta]U'(c_2)$$

D) Efficiency of Equilibrium

Welfare Theorems

- The allocation is the same as in the competitive equilibrium
- The equilibrium allocation is (Pareto) efficient
- Practical Advantages of this result:
  - Solving for Pareto Optimum is easier
- How to figure out what the prices must be?

Where are we?

- A Basic Intertemporal Model
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E) Infinite Horizon

- Shortcomings of the previous model:
  - 2 periods are arbitrary
- Solution: Infinite number of periods
- Solve the Pareto Problem

$$\max_{\{c_t, k_{t+1}\}} \sum_{t=0}^{\infty} \beta^t U(C_t)$$

$$s.t.$$

$$C_t + K_{t+1} = F(K_t) + (1-\delta)K_t$$

$$K_0 \quad \text{given}$$

E) Infinite Horizon

Euler Equation again and Steady State

- Consumption satisfies:

$$U'(c_t) = \beta[F_K(K_{t+1}) + 1 - \delta]U'(c_{t+1})$$

- Steady State:

$$U'(C^{ss}) = \beta[F_K(K^{ss}) + 1 - \delta]U'(C^{ss})$$

$$1 = \beta[F_K(K^{ss}) + 1 - \delta]$$

$$F_K(K^{ss}) = \frac{1}{\beta} + \delta - 1$$

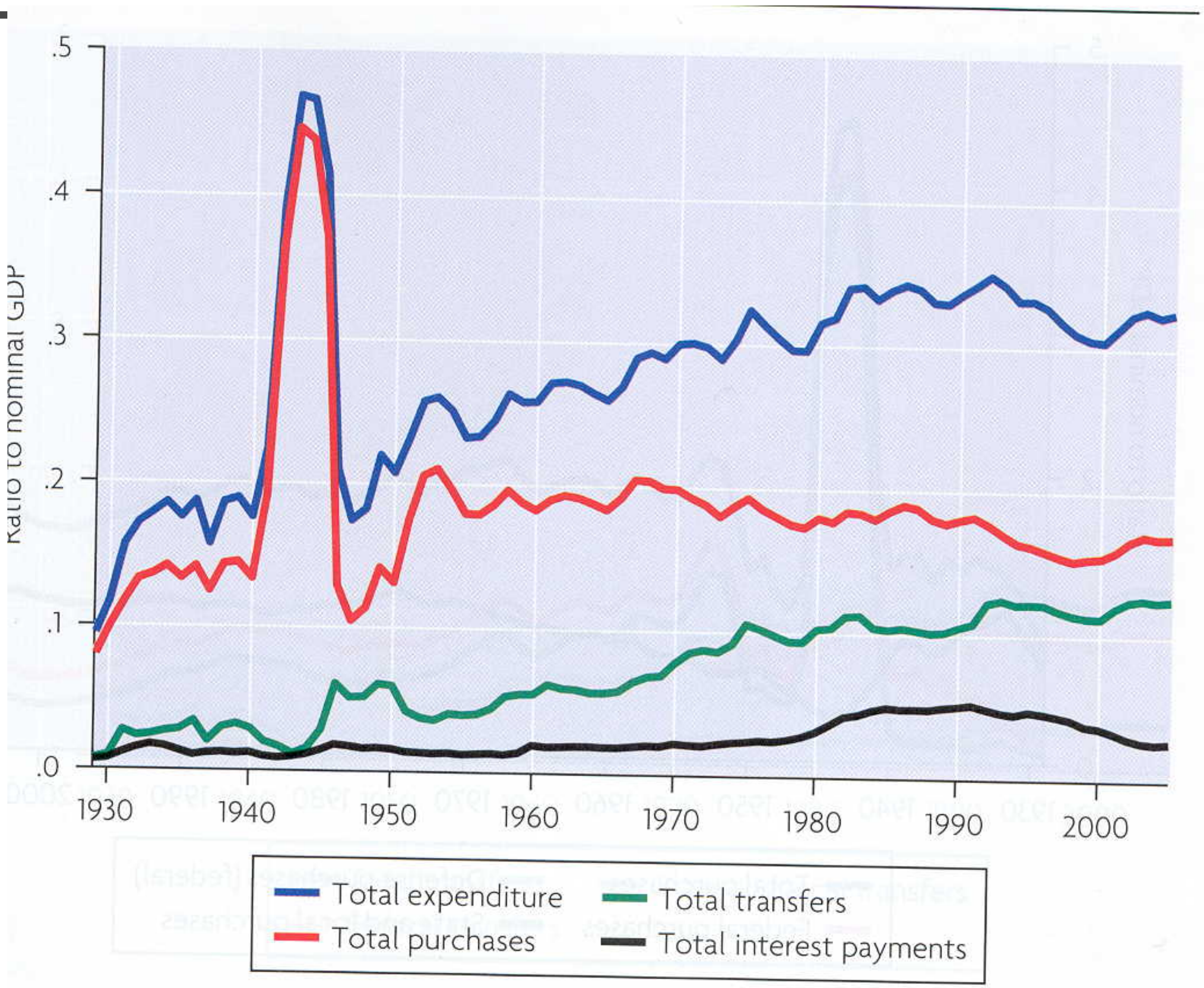
Where are we?

- Introduction: A model with no Government
- The Effects of Government Spending
- Government Taxation and Government Debt
- Fiscal and Monetary Policy
- Optimal Monetary Policy
- Financial Intermediation
- Current Account Determination
- Fiscal Deficits and Current Account

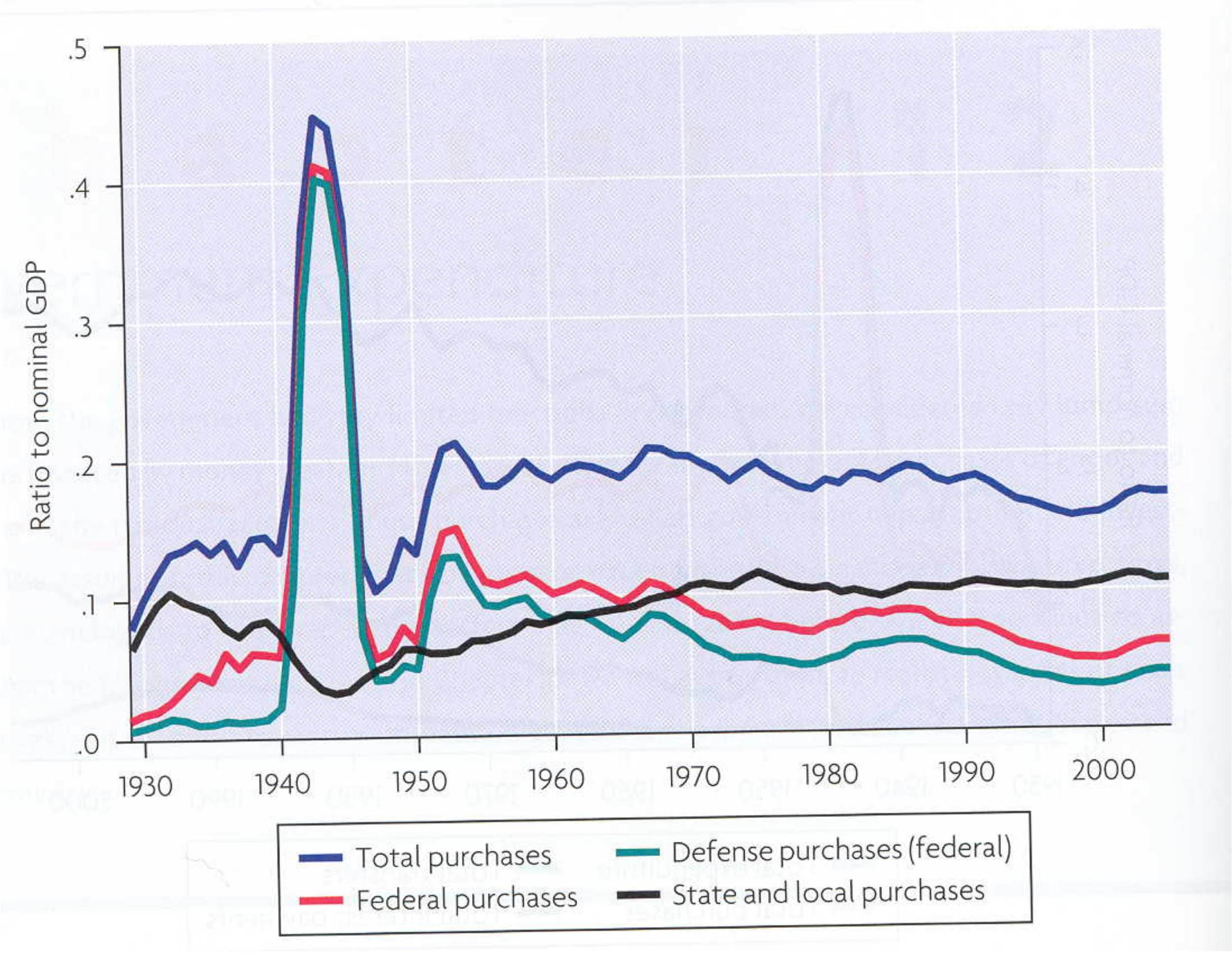
Roadmap

- Government Expenditures
  - A) Data on Govt Expenditures
  - B) Permanent Changes in Gov't Spending
  - C) Temporary Changes in Gov't Spending
  - D) Social Security

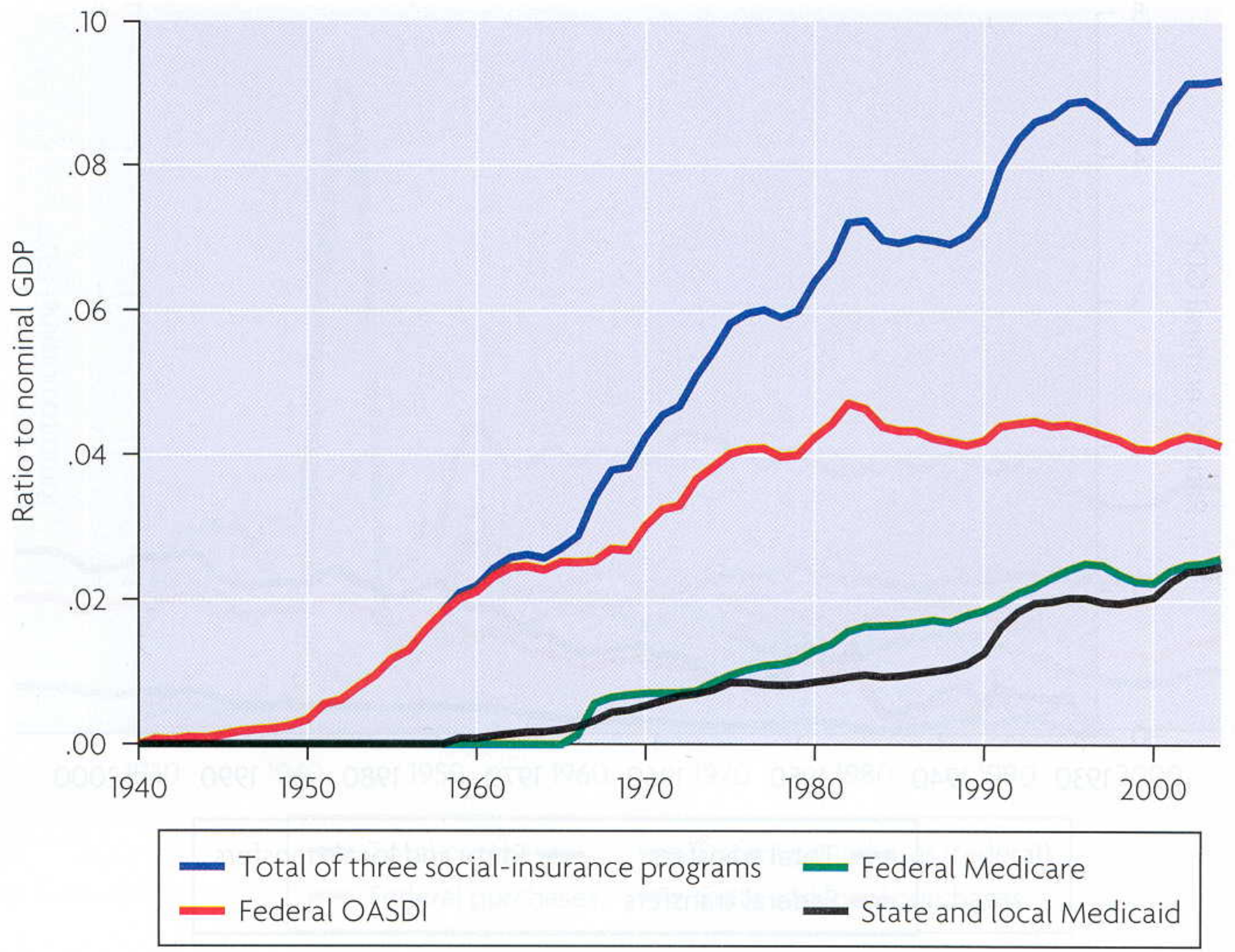
A1) Government Expenditures



A2) Government Purchases



A3) Transfers



B) The Effects of Government Purchases

- Questions:
  - How do changes in government purchases affect
    - Consumption, output, capital stock?
    - Interest rates?
  - Crowding out?

B) The Effects of Government Purchases

- How are govt Purchases useful?
  - Productive services (legal system)
  - Consumption services (public parks)
  - Focus on the first role
- 2 alternatives

$$Y_t^G = \phi G \quad (Assumption\,1)$$

$$Y_t^G = \phi KG \; (Assumption\,2)$$

B1) Analysis with Assumption 1

Consumers

- Preferences

$$\max_{\{C_t, K_{t+1}\}} \sum_{t=0}^{\infty} \beta^t U(C_t)$$

- Taxes are Lump-Sum: $T=G$
- Budget Constraint in each period

$$C_t + B_{t+1} = \Pi_t + (1+r)B_t - T_t$$

B1) Analysis with Assumption 1

Firms

- Invest in Capital Stock
- Production Function

$$Y_t^P = K_t^\alpha$$

- Law of motion for capital

$$K_{t+1} = (1-\delta)K_t + I_t$$

- Profits

$$\Pi_t = K_t^\alpha - I_t$$

B1) Analysis with Assumption 1

Government

- Budget Constraint

$$G_t = T_t$$

- Production:

$$Y_t^G = \phi G_t$$

B1) Analysis with Assumption 1

Pareto Problem

- Pareto Problem:

$$\max_{\{c_t, k_{t+1}\}} \sum_{t=0}^{\infty} \beta^t U(C_t)$$

$$s.t.$$

$$C_t + K_{t+1} + G_t = K_t^{\alpha} + \phi G + (1-\delta)K_t$$

$$K_0 \quad \text{given}$$

B1) Analysis with Assumption 1

Euler Equation

- Consumption satisfies:

$$U'(C_t) = \beta[\alpha K_{t+1}^{\alpha-1} + 1 - \delta]U'(C_{t+1})$$

B1) Analysis with Assumption 1

Steady State

- In a Steady State,

$$K_t = K_{t+1} = K_{ss}$$
$$C_t = C_{t+1} = C_{ss}$$
$$I_t = I_{t+1} = I_{ss}$$

- Solve for the steady state capital stock:

$$K^{ss} = \left(\frac{\alpha}{1/\beta + \delta - 1}\right)^{\frac{1}{1-\alpha}}$$

  - Independent of Government Spending!

B1) Analysis with Assumption 1

Steady State

- Output, Consumption and Investment

$$Y_{ss} = K_{ss}^{\alpha} + \phi G$$

$$C_{ss} = K_{ss}^{\alpha} - \delta K_{ss} - (1-\phi)G$$

$$I_t = \delta K_{ss}$$

- Steady State Output *increases* in gov't spending
- Consumption *decreases* in gov't spending (crowding out)