Econ 208

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Lecture 2

Basic Intertemporal Model

Where are we?

- A Basic Intertemporal Model
  - A) Consumer Optimization
  - B) Market Clearing
  - C) Adding capital stock
  - D) Welfare Theorems
  - E) Infinite horizon

A) Consumer's optimization

- Euler Equation

$$U'(c_1) = (1+r)\beta U'(c_2)$$

- Log utility:

$$\frac{c_2}{c_1} = (1+r)\beta$$

  - Solution:

$$c_1^* = \frac{y_1 + \frac{y_2}{1+r}}{1+\beta}$$

$$b_1^* = y_1 - c_1^*$$

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B) Market Equilibrium

- Suppose that there is N identical agents
- Market clearing condition is

$$Nb_1^*(r^*) = 0$$

- Log utility:

$$y_1 = \frac{y_1 + \frac{y_2}{1+r}}{1+\beta}$$

$$r^* = \frac{1}{\beta}\frac{y_2}{y_1}$$

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C) Adding Capital Stock

- Shortcomings of the previous model
  - Production is not determined within the model
- Solution: Introduce production
  - There is a firm producing output using capital stock it owns
  - Consumers own the firm, get the profits

C) Adding Capital Stock

Firm's Problem

- Production function

$$y_1 = F(K_1)$$
$$y_2 = F(K_2)$$

  - Capital changes according to

$$K_2 = (1-\delta)K_1 + I_1$$
$$K_3 = (1-\delta)K_2$$

  - Initial capital stock K_1 given
  - Capital stock K_3 can be sold at the end of period 2

C) Adding Capital Stock

Firm's Problem

- Profits

$$\Pi_1 = Y_1 - I_1$$

$$\Pi_2 = Y_2 + K_3$$

- Maximize the *present value* of profits

$$\max_I \Pi_1 + \frac{\Pi_2}{1+r}$$

- In the optimum:

$$F_K(K_2) = r + \delta$$

C) Adding Capital Stock
Consumer's problem revisited

- Budget Constraints:

$$C_1 + B_1 = \Pi_1(r)$$
$$C_2 = \Pi_2(r) + B_1(1+r)$$

  - b_1 are savings from period 1 to period 2
  - r is the interest rate

C) Adding Capital Stock

Market Equilibrium

- Market Clearing

$$C_1 + I_1 = F(K_1)$$

$$C_2 = F(K_2) + K_3$$

- Properties of Equilibrium:

$$U'(c_1) = \beta(1+r)U'(c_2)$$

$$= \beta[F_K(K_2) + 1 - \delta]U'(c_2)$$

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D) Efficiency of Equilibrium

Pareto Efficiency

- Pareto Efficient Allocation satisfies

$$\max_{C_1, C_2, I_1} U(C_1) + \beta U(C_2) \quad s.t \quad C_1 + I_1 = F(K_1)$$

$$C_2 = F(K_2) + K_3$$

- Properties of Pareto Optimum:

$$U'(c_1) = \beta[F_K(K_2) + 1 - \delta]U'(c_2)$$

D) Efficiency of Equilibrium
Welfare Theorems

- The allocation is the same as in the competitive equilibrium
- The equilibrium allocation is (Pareto) efficient
- Practical Advantages of this result:
  - Solving for Pareto Optimum is easier
- How to figure out what the prices must be?

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E) Infinite Horizon

- Shortcomings of the previous model:
  - 2 periods are arbitrary
- Solution: Infinite number of periods
- Solve the Pareto Problem

$$\max_{\{c_t, k_{t+1}\}} \sum_{t=0}^{\infty} \beta^t U(C_t)$$

$$s.t.$$

$$C_t + K_{t+1} = F(K_t) + (1-\delta)K_t$$

$$K_0 \quad \text{given}$$

E) Infinite Horizon

Euler Equation again and Steady State

- Consumption satisfies:

$$U'(c_t) = \beta[F_K(K_{t+1}) + 1 - \delta]U'(c_{t+1})$$

- Steady State:

$$U'(C^{ss}) = \beta[F_K(K^{ss}) + 1 - \delta]U'(C^{ss})$$

$$1 = \beta[F_K(K^{ss}) + 1 - \delta]$$

$$F_K(K^{ss}) = \frac{1}{\beta} + \delta - 1$$