



Econ 208

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Marek Kapicka

Lecture 10

Ramsey Optimal Taxation



# Where are we?

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- Introduction: A model with no Government
- The Effects of Government Spending
- Government Taxation and Government Debt
  - Labor Taxation
  - Capital Taxation
  - **Government Debt**
- Fiscal and Monetary Policy
- Optimal Monetary Policy
- Financial Intermediation
- Current Account Determination
- Fiscal Deficits and Current Account



# Announcements

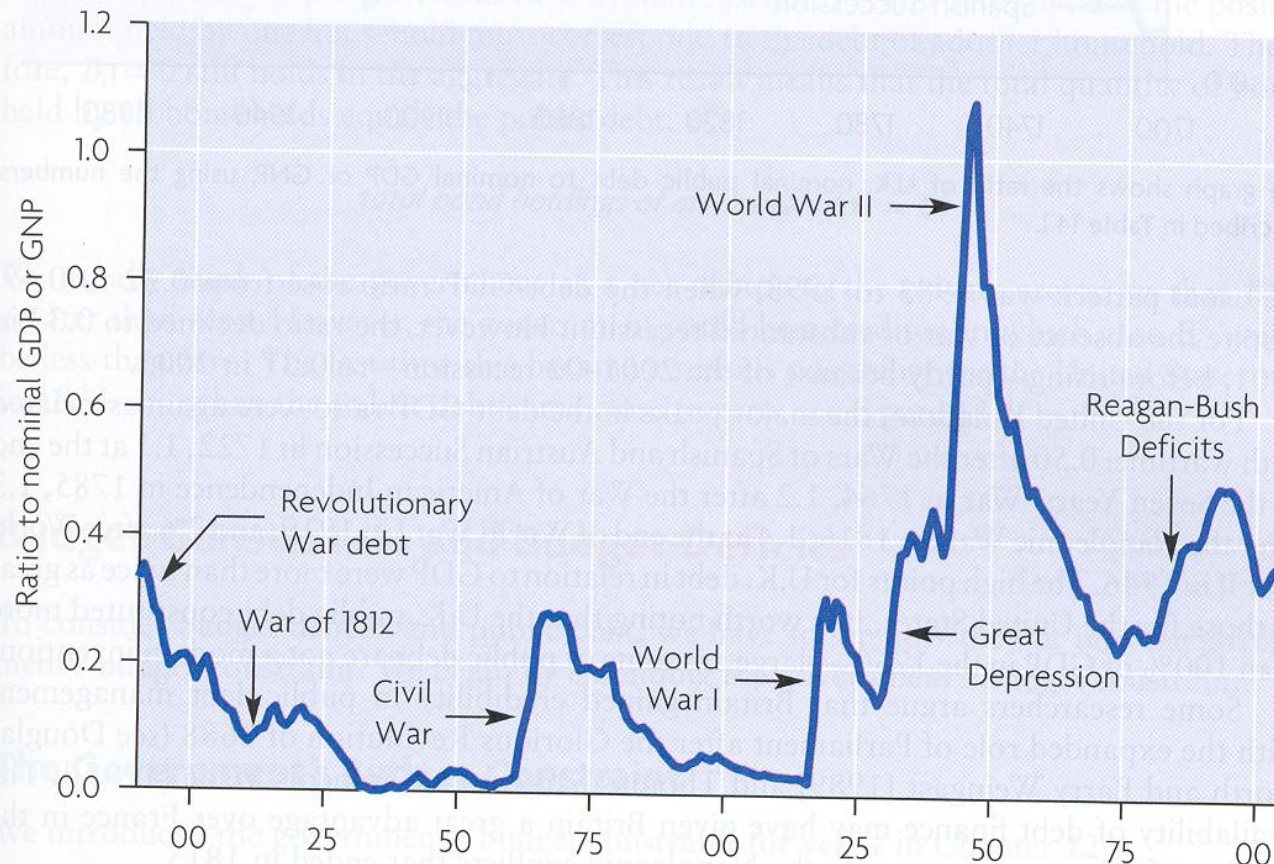
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- PS3 on the web today
- Read 14.3-14.4 for today

# US Government Debt

Figure 14.1

Ratio of U.S. Public Debt to GDP, 1790–2005





# Ramsey Approach to Taxation

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- Choose optimal (welfare maximizing) sequences of taxes (and debt) given that only distortionary tax instruments are available.
  - Tax instruments are given
  - Lump sum taxation not allowed



# Ramsey Taxation

## Main Results

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- Uniform Commodity Taxation
  - Under certain conditions, tax rates should be equated across goods
    - Distortions will be spread evenly
- Applies to dynamic economies:
  - Tax smoothing



# Ramsey Taxation

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- We will analyze a problem of a government that
  - Face a given sequence of expenditures  $\{G_t\}_{t \geq 0}$
  - Choose a sequence of consumption (sales) taxes  $\{\tau_t\}_{t \geq 0}$
- Similar logic applies to labor taxation



# Ramsey Taxation

## Household Problem

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- Maximize lifetime utility

$$\max_{\{C_t\}} \sum_{t=0}^{\infty} \beta^t \ln(C_t)$$

- Subject to PVBC

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t (1 + \tau_t) C_t = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t Y_t$$

# Ramsey Taxation

## Household Problem

- The Lagrangean

$$\max_{\{C_t\}} \sum_{t=0}^{\infty} \beta^t \ln(C_t) + \lambda \left[ \sum_{t=0}^{\infty} \left( \frac{1}{1+r} \right)^t Y_t - \sum_{t=0}^{\infty} \left( \frac{1}{1+r} \right)^t (1 + \tau_t) C_t \right]$$

- Assume that  $\beta = 1/(1+r)$ :

$$C_t (1 + \tau_t) = \frac{1}{\lambda} = W$$

- where  $W = \frac{r}{1+r} \sum_{t=0}^{\infty} \left( \frac{1}{1+r} \right)^t Y_t$



# Ramsey Taxation

## Household Problem

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- Indirect Utility:

$$\begin{aligned} V(\{\tau_t\}_{t \geq 0}, W) &= \sum_{t=0}^{\infty} \beta^t \ln(C_t^*) \\ &= \sum_{t=0}^{\infty} \beta^t \ln\left(\frac{W}{1 + \tau_t}\right) \\ &= -\sum_{t=0}^{\infty} \beta^t \ln(1 + \tau_t) + \frac{\ln W}{1 - \beta} \end{aligned}$$



# Ramsey Taxation

## Government's Problem

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- PV Budget Constraint

$$\begin{aligned}\sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t G_t &= \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \tau_t C_t^* \\ &= \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \frac{\tau_t}{1+\tau_t} W\end{aligned}$$

- Define  $G = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t G_t$



# Ramsey Taxation

## Government's Problem

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- Ramsey Problem: Choose a sequence of tax rates to maximize agent's utility, subject to the government's budget constraint

$$\max_{\{\tau_t\}_{t \geq 0}} - \sum_{t=0}^{\infty} \beta^t \ln \tau_t + \frac{\ln W}{1-\beta} + \mu \left[ \sum_{t=0}^{\infty} \left( \frac{1}{1+r} \right)^t \frac{\tau_t}{1+\tau_t} - \frac{G}{W} \right]$$

- First Order Condition:

$$\tau_t = \mu - 1$$

# Ramsey Taxation

## Government's Problem

- Solution to the Ramsey Problem: taxes are constant over time, regardless of the time path of government expenditures
- Solving for the optimal tax rate:

$$\frac{\tau^*}{1 + \tau^*} = \frac{r}{1 + r} \frac{G}{W}$$

$$\tau^* = \frac{\frac{r}{1 + r} G}{W - \frac{r}{1 + r} G}$$



# Ramsey Taxation

## Implications for Government Debt

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- Example:

$$G_t = 1, t = 0, \quad G_t = 0, t > 0$$

$$Y_t = 1, t \geq 0$$

- Hence  $W = G = 1$
- The optimal tax rate

$$\tau^* = r$$



# Ramsey Taxation

## Implications for Government Debt

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- Tax collection each period:  $r / (1+r)$

- Core Deficit

$$G_0 - T_0 = \frac{1}{1+r}$$

$$G_t - T_t = -\frac{r}{1+r}, t > 0$$

- Government Debt:

$$B_t^g = \frac{1}{1+r}, t \geq 0$$

# Ramsey Taxation

## WWII vs. Korean War

- WWII financed differently than Korean War

	% OF EXPENDITURES FINANCED BY	
	Direct Taxes	Debt and seignorage
World War II	41%	59%
Korean War	100%	0%

- Marginal Taxes

	% TAX RATES BEFORE/DURING THE WAR	
	Labor	Capital
World War II	9/18	44/60
Korean War	16/20	52/63



# Ramsey Taxation

## WWII vs. Korean War

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- What if WWII were financed like Korean War (taxes only)?
  - Labor taxes would be 64% rather than 18%
  - Capital taxes would be 100% rather than 60%
  - Welfare costs are 3% of consumption



# Ramsey Taxation

## WWII vs. Korean War

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- What if Korean War was financed like WWII (both taxes and debt)?
  - Labor taxes would be 23% rather than 20%
  - Capital taxes would be 50% rather than 62%
  - Welfare gains are 0.4% of consumption