Economics 208 Marek Kapicka
Macroeconomics Spring 2008

Midterm Answers

You have 75 minutes. This is a closed book/closed notes exam. You can use a calculator, but no other tools. No cell phones or calculators are allowed. Anyone who violates the rules will have 0 points. Please write all your answers on the exam paper. There is 200 points in total. Good luck!

Your name:_Marek Kapicka_(5 points)

1 Multiple Choice (5 pts each).

There is only one correct answer for each question.

1. In response to a tax cut, the consumption of a consumer who is borrowing constrained _____, whereas the consumption of an unconstrained consumer acting in accord with Ricardian equivalence _____

   A) increases; increases

   B) increases; remains unchanged

   C) remains unchanged; increases

   D) remains unchanged; remains unchanged

2. In response to a tax cut, the savings of a consumer who is borrowing constrained (both before and after the tax cut) _____, whereas the savings of an unconstrained consumer acting in accord with Ricardian equivalence _____

   A) increase; increase

   B) increase; remain unchanged

   C) remain unchanged; increase

   D) remain unchanged; remain unchanged

3. Suppose that the productivity of government expenditures is unaffected by private capital stock. Temporary increases in government spending _____the equilibrium interest rate, whereas permanent increases in government spending _____the equilibrium interest rate.

   A) decrease; decrease

   B) decrease; do not change

   C) do not change; decrease

   D) do not change; do not change

4. According to the Ramsey theory of taxation,

   A) wars should be financed mostly by an increase in capital taxes.

   B) wars should be financed mostly by an increase in labor taxes.

   C) wars should be financed mostly by an increase in government debt.

   D) it does not matter how wars are financed.

5. Suppose the government is running large persistent budget deficits. When the central bank is not fully dominant,

   A) it is always unable to choose low inflation today.

   B) Lower inflation today implies higher inflation in the future.

   C) Lower inflation today implies even lower inflation in the future.

   D) it cannot afford to have inflation.

6. Government revenue from income taxes, as a fraction of nominal GDP,

   A) has been stable over 20th century.

   B) has sharply decreased during WWII.

   C) has sharply increased during WWII.

   D) has sharply decreased in 1980's.

7. If the interest rate exceeds the population growth rate,

   A) fully funded social security system is more efficient than PAYG social security system.

   B) PAYG social security system is more efficient than fully funded social security system.

C) both systems are equally efficient.

D) It cannot be determined which system is more efficient.

2 Credit Markets (80 points)

Consider an economy with two consumers who live for two periods. Consumer 1 is very impatient, and therefore places a high weight on utility in the first period. The utility function for the first consumer is

$$u_A(c_1, c_2) = \ln c_1 + \frac{1}{2} \ln c_2.$$

The second consumer is patient and values both periods equally. His utility function is:

$$u_B(c_1, c_2) = \ln c_1 + \ln c_2.$$

Both consumers have a fixed income of one unit of consumption good in each period. The consumption good cannot be stored, but the two consumers can trade with each other in the credit market. The interest rate on savings is denoted by r.

1. Set up the optimization problem of each consumer and write down the market clearing conditions in the goods market and in the credit market. (25 pts)

   Household A:

   $$\max_{c_1,c_2} \ln c_1 + \frac{1}{2} \ln c_2 \quad s.t. \quad c_1 + \frac{c_2}{1+r} = 1 + \frac{1}{1+r}$$

   Household B:

   $$\max_{c_1,c_2} \ln c_1 + \ln c_2 \quad s.t. \quad c_1 + \frac{c_2}{1+r} = 1 + \frac{1}{1+r}$$

   Market clearing in the consumption goods market:

   $$c_1^A(r) + c_1^B(r) = 2.$$
   $$c_2^A(r) + c_2^B(r) = 2.$$

   If savings are denoted by $s^A(r)$ and $s^B(r)$ then market clearing in the credit market is given by

   $$s^A(r) + s^B(r) = 0.$$

2. Find the consumption of both consumers in the two periods as a function of the interest rate. (20 pts)

   The first order conditions are

$$\frac{1}{c_1^A} = \frac{1+r}{2}\frac{1}{c_2^A}$$
$$\frac{1}{c_1^B} = (1+r)\frac{1}{c_2^B}$$

   and by using budget constraints, one gets

$$c_1^A + \frac{c_2^A}{1+r} = \frac{3}{2}c_1^A = 1 + \frac{1}{1+r}$$
$$c_1^B + \frac{c_2^B}{1+r} = 2c_1^B = 1 + \frac{1}{1+r}.$$

   Hence

$$c_1^A(r) = \frac{2}{3}(1 + \frac{1}{1+r}), \quad c_2^A(r) = \frac{1}{3}(2+r)$$
$$c_1^B(r) = \frac{1}{2}(1 + \frac{1}{1+r}), \quad c_2^B(r) = \frac{1}{2}(2+r)$$

3. Find the equilibrium interest rate (20 pts).

   Market clearing condition $c_1^A(r) + c_1^B(r) = 2$ implies that

$$\frac{2}{3}(1 + \frac{1}{1+r}) + \frac{1}{2}(1 + \frac{1}{1+r}) = 2$$
$$1 + \frac{1}{1+r} = \frac{12}{7}$$
$$\frac{1}{1+r} = \frac{5}{7}$$
$$r^* = \frac{2}{5}$$

   and so the equilibrium interest rate is 40%.

4. Compute the equilibrium savings of both agents. Provide an intuition for the equilibrium pattern of savings. (15 pts)

Consumption in the first period is given by

$$\begin{aligned}
c_1^{A*} &= \frac{2}{3}\frac{12}{7} = \frac{8}{7} \\
c_1^{B*} &= \frac{1}{2}\frac{12}{7} = \frac{6}{7} \\
c_2^{A*} &= \frac{1+r^*}{2}c_1^{A*} = \frac{7}{10}c_1^{A*} = \frac{8}{10} \\
c_2^{B*} &= (1+r^*)c_1^{B*} = \frac{7}{5}c_1^{B*} = \frac{12}{10}.
\end{aligned}$$

Hence savings are given by $s^{A*} = -\frac{1}{7}$ and $s^{B*} = \frac{1}{7}$. The impatient guy wants to consume more today, and so he borrows from the patient guy.

3 Elasticity of Labor Supply and the Laffer Curve (80 points)

Consider consumers with preferences over consumption C and labor supply L given by

$$U(C, L) = C - \frac{L^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}}, \qquad \gamma > 0.$$

The parameter γ is the elasticity of labor supply (elasticity of labor supply measures how responsive the labor supply is to changes in the net wage rate).

1. Suppose that the consumers's consumption is given by $C = w(1-\tau)L$, where τ is the labor tax rate and w is the wage rate. Compute the optimal labor supply as a function of τ and w. (15 pts)

$$\begin{aligned}
&\max_L w(1-\tau)L - \frac{L^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} \\
w(1-\tau) &= L^{\frac{1}{\gamma}} \\
L^s &= w^\gamma(1-\tau)^\gamma
\end{aligned}$$

Partial equilibrium Laffer curve

2. Suppose that the wage rate is exogenously given. Compute the government's tax revenue as a function of τ and w. Find the tax rate that maximizes government's revenue. How does it depend on the elasticity of labor supply? Compute the revenue maximizing tax rate if $\gamma = 1$. (20 pts)

$$T = \tau L^s = w^\gamma \tau (1-\tau)^\gamma$$

$$\begin{aligned}
&\max_\tau w^\gamma \tau (1-\tau)^\gamma \\
(1-\tau)^\gamma - \gamma\tau(1-\tau)^{\gamma-1} &= 0 \\
1 - \gamma\frac{\tau}{1-\tau} &= 0 \\
\gamma\frac{\tau}{1-\tau} &= 1 \\
\gamma\tau &= 1-\tau \\
\tau^* &= \frac{1}{1+\gamma}.
\end{aligned}$$

Thus, the peak of the Laffer curve depends negatively on the elasticity of labor supply. Intuitively, the more elastic the labor supply, the more negative response there is going to be from an increase in the tax rate. If $\gamma = 1$ then the peak is at 50%.

General equilibrium Laffer curve. One potential problem with the answer in part 2 is that the wage rate is exogenously fixed. In reality, wages will respond to changes in labor supply. To examine this effect, set $\gamma = 1$ and assume that the firms in the economy have production function given by $F(L) = 2\sqrt{L}$.

3. Set up the firms profit maximization problem and compute the labor supply as a function of the wage rate. (10 pts)

$$\begin{aligned}
&\max_L 2\sqrt{L} - wL \\
\frac{1}{\sqrt{L}} &= w \\
L^d &= \frac{1}{w^2}.
\end{aligned}$$

4. Using your answers from parts 1 and 3, compute the equilibrium wage rate as a function of the tax rate. Is it increasing or decreasing in the tax rate? Why? (15 pts)

$$\begin{aligned} L^d &= L^s \\ \frac{1}{w^2} &= w^\gamma (1-\tau)^\gamma \\ (1-\tau)^{-\gamma} &= w^{2+\gamma} \\ w^*(\tau) &= (1-\tau)^{-\frac{\gamma}{2+\gamma}} \end{aligned}$$

for $\gamma = 1$, we have $w^* = (1-\tau)^{-\frac{1}{3}}$.

5. Compute the government's tax revenue as a function of τ when the wage rate is determined endogenously, i.e. given by your answer in part 4. Compute the revenue maximizing tax rate and compare it to your answer in part 2. Is it higher or lower? Why? (20 pts)

$$\begin{aligned} T &= \tau w^*(\tau)^\gamma (1-\tau)^\gamma = \tau (1-\tau)^{-\frac{\gamma}{2+\gamma}} (1-\tau)^\gamma = \tau (1-\tau)^{\gamma - \frac{\gamma}{2+\gamma}} \\ 0 &= (1-\tau)^{\gamma - \frac{\gamma}{2+\gamma}} - (\gamma - \frac{\gamma}{2+\gamma}) \tau (1-\tau)^{\gamma - \frac{\gamma}{2+\gamma} - 1} \\ 1 &= (\gamma - \frac{\gamma}{2+\gamma}) \frac{\tau}{1-\tau} \\ 1 - \tau &= (\gamma - \frac{\gamma}{2+\gamma}) \tau \\ \tau &= \frac{1}{1 + \gamma - \frac{\gamma}{2+\gamma}} \end{aligned}$$

and for $\gamma = 1$, one gets $\tau^* = 60\%$. The peak of the Laffer curve is now reached at a higher tax rate because labor taxation increases the wage rate, which has additional positive impact on the government revenue.