Economics 204b
Spring 2009

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Problem Set 3

Due Friday May 2

1 Consumer Durables

Consider a consumer who derives utility from both nondurable consumption c_t and a flow of services from consumer durables. The stock of consumer durables d_t evolves according to

$$d_{t+1} = (1-\delta)d_t + e_t$$

where δ is depreciation and e_t are current purchases of consumer durables. The purchases of consumer durables are allowed to be negative (that is, the consumer is allowed to sell durables). It is assumed that durables purchased in the current period yield services starting in the next period and so the preferences are given by

$$E_0 \sum \beta^t U(c_t, d_t) \qquad 0 < \beta < 1,$$

where U is period utility.

Each period, the consumer receives exogenous income y_t. The income is i.i.d. and can only take I values, with probability that $y = y_i$ given by π_i, $i = 1..I$. The consumer can also save assets that earn a rate of interest r, exogenously given. The budget constraint of the consumer in period t is thus given by

$$c_t + e_t + a_{t+1} = y_t + (1+r)a_t,$$

where a_t are the assets at the beginning of period t. Note that the budget constraint incorporates an assumption that the relative price of nondurables and durables is equal to one.

1. Formulate the Bellman equation corresponding to this problem.

2. Modify the problem so as to incorporate the assumption that durables purchased in the current period provide services starting in the current period. Rewrite the Bellman equation.

3. Now suppose that the investment in consumer durables is partially irreversible in a sense that durables sell at a discount. In particular, the price of buying durables is equal to one as before, but the durables can only be sold at a price $p < 1$.

   Formulate the Bellman equation for this problem. (Hint: think of the consumer as first solving two subproblems: how much durables to buy conditional on buying and how much durables to sell conditional on selling, and then selecting the better alternative.)

2 Asset Pricing in an Endowment Economy

Consider an economy where the representative agent's preferences are given by

$$\sum_{t=0}^{\infty}\sum_{z^t}\beta^t U(c_t(z^t))P(z^t),$$

where z_t is an i.i.d. shock in period t, drawn from some finite set Z. The probability of a shock z_t is $\pi(z_t)$. Hence, the probability of a sequence z^t is given by $P(z^t) = \pi(z_0)\pi(z_1)....\pi(z_t)$. The utility function is strictly increasing, strictly concave and continuously differentiable. The output of the firm in period t is given by z_t (no capital or labor is needed to get z_t).

1. Define the Arrow-Debreu Equilibrium in this economy. (Hint: modify the definition given in class to take into account that there is no production.)
2. Specify the competitive equilibrium price system. The formula for the price of a commodity should be in terms of the data that define the economy.
3. Price the following assets:
   (a) The unconditional delivery of one unit of the date t good.
   (b) Ownership of a firm with dividend stream $\{d_t(z^t)\}$.
   (c) Ownership of a firm at date t (pre dividend price) in terms of the date t good at date t if the history of shocks is z^t.