Real Business Cycles

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Three Extensions of the Basic RBC model

- Indivisible labor supply
  - Why: Labor supply not elastic enough and no extensive margin
- Variable capacity utilization
  - Why: Probability of technological regress too high
- Government spending shocks
  - Why: correlation between output and productivity is too high

6. Extensions of the basic RBC Model

Problems with labor markets

- The basic RBC model cannot capture the labor market dynamics very well
  - the labor supply in the model exhibits too little variability, compared to the data
- Increasing labor supply elasticity might help but is at odds with micro estimates
- In addition, most of the variations in aggregate labor supply is on the **extensive margin** (in/out of employment), rather than on the **intensive margin** (change in hours per person)
  - extensive margin: 2/3 of changes
  - intensive margin: 1/3 of changes

6. Extensions of the basic RBC Model

6.1. Indivisible labor

- Hansen (1985) and Rogerson (1988): incorporate the fact that labor supply is indivisible (fixed shifts)
  - This extension will both account for the extensive margin movement and make the aggregate labor supply more volatile
- Read Hansen, "Indivisible Labor and the Business Cycle" (on the reading list)

4.1. Indivisible Labor

The lottery

- fixed shifts: each agent can work either $n = H \in (0, 1)$ or $n = 0$ hours
- $\{0, H\}$ is a nonconvex set: agents are better off by using lottery:
  - work with probability p, get wage w
  - stay at home with probability $1 - p$, get nothing
- Expected utility

$$Eu = pU(\tilde{c}_1, 1 - H) + (1 - p)U(\tilde{c}_2, 1)$$

  - $\tilde{c}_1 = w + (r + \delta)k$: consumption if employed
  - $\tilde{c}_2 = (r + \delta)k$: consumption if unemployed
- With log-log, the expected utility becomes

$$Eu = p[\alpha \ln \tilde{c}_1 + (1 - \alpha)\ln(1 - H)] + (1 - p)\alpha \ln \tilde{c}_2$$

6.1. Indivisible Labor

Consumption insurance

- Assume that there are insurance companies that can insure against consumption fluctuations
  - pay $w + (r + \delta)k$ if employed, $(r + \delta)k$ if unemployed
  - get c_1 if employed, c_2 if unemployed
- Zero profit condition implies

$$pc_1 + (1 - p)c_2 = pw + (r + \delta)k$$

- The agent's problem in a competitive equilibrium becomes

$$\max_{c_1, c_2, p} p[\alpha \ln c_1 + (1 - \alpha) \ln(1 - H)] + (1 - p)\alpha \ln c_2$$
$$s.t. \qquad pc_1 + (1 - p)c_2 = pw + (r + \delta)k$$

6.1. Indivisible Labor

Complete Consumption insurance

- with log-log utility, there is complete unemployment insurance:

$$c_1 = c_2 = c$$

- The agent's problem becomes

$$\max_{c,p} \alpha \ln c_1 + p(1-\alpha)\ln(1-H)]$$
$$s.t. \qquad c = pw + (r+\delta)k$$

6.1. Indivisible Labor

The Aggregation

- The expected (or aggregate) leisure is $L = 1 - pH$. The representative agent's problem can be written as

$$\max_{c,L} \alpha \ln c + \phi(L-1)$$
$$s.t. \qquad c = \hat{w}(1-L) + (r+\delta)k$$

  where $\phi = -\frac{1-\alpha}{H}\ln(1-H) > 0$ and $\hat{w} = \frac{w}{H}$.
- **The representative agent has utility that is linear in leisure!**

6.1. Indivisible Labor

The implications

- At an individual level, the elasticity of labor supply is zero
- In the aggregate, the elasticity of labor supply is infinite (utility is linear in leisure).

6.2. Variable Capacity Utilization

- Problem: In order to work, the basic RBC model requires large productivity shocks
  - King, Rebelo: The probability of technological regress is 18.6%
- In addition, the data show that capital is not used with the same intensity over the business cycle
- Idea: incorporate the fact that the capital can be utilized with varying intensity
  - with this extension, one needs smaller productivity shocks to generate realistic business cycles

6.2. Variable Capacity Utilization

The basic trade-off

- benefits of higher capacity utilization: more is produced
- costs of higher capacity utilization: capital depreciates faster
- the rate of capacity utilization λ_t :

$$\begin{aligned} y_t &= e^{\omega_t}(\lambda_t k_t)^\alpha n_t^{1-\alpha} \\ (1+\eta)(1+\gamma)k_{t+1} &= i_t + [1-\delta(\lambda_t)]k_t \end{aligned}$$

where $\delta(\lambda_t)$ is increasing and convex.

6.2. Variable Capacity Utilization

First order condition

- The resource constraint

$$c_t + (1+\eta)(1+\gamma)k_{t+1} = e^{\omega_t}(\lambda_t k_t)^\alpha n_t^{1-\alpha} + [1-\delta(\lambda_t)]k_t$$

- First order condition in λ_t :

$$\begin{aligned} \alpha e^{\omega_t}\lambda_t^{\alpha-1}k_t{}^\alpha n_t^{1-\alpha} &= \delta'(\lambda_t)k_t \\ \alpha y_t &= \lambda_t\delta'(\lambda_t)k_t \end{aligned}$$

  - λ_t co-moves positively with output (more capacity utilization in booms)

6.2. Variable Capacity Utilization

Remeasuring Productivity Shocks

- "True" Solow residual ω_t

$$y_t = e^{\omega_t} (\lambda_t k_t)^{\alpha} n_t^{1-\alpha}$$

- Measured Solow residual $\tilde{\omega}_t$

$$y_t = e^{\tilde{\omega}_t} {k_t}^{\alpha} n_t^{1-\alpha}$$

- Their relationship:

$$\begin{aligned} \omega_t &= \ln y_t - (1-\alpha) \ln n_t - \alpha \ln k_t - \alpha \ln \lambda_t \\ \tilde{\omega}_t &= \ln y_t - (1-\alpha) \ln n_t - \alpha \ln k_t \\ \tilde{\omega}_t &= \omega_t + \alpha \ln \lambda_t \end{aligned}$$

  - λ_t co-moves positively with ω_t. Hence the fluctuations in $\tilde{\omega}_t$ are larger than fluctuations in ω_t.

6.2. Variable Capacity Utilization

Results

$$\xi = \frac{\lambda \delta''(\lambda)}{\delta'(\lambda)}$$

Table 5
Sensitivity analysis to different ξ values

ξ value	Standard deviations						Persistence parameter (ρ)	Likelihood of technical regress
	Y	c	I	N	A	ε		
∞	1.36	1.01	2.62	0.90	0.79	0.0061	0.9783	0.1859
1	1.39	0.94	2.86	1.07	0.45	0.0034	0.9798	0.1106
$\frac{1}{2}$	1.40	0.92	2.94	1.13	0.34	0.0026	0.9822	0.0653
$\frac{1}{3}$	1.40	0.91	2.99	1.16	0.28	0.0021	0.9841	0.0352
$\frac{1}{5}$	1.41	0.91	3.05	1.20	0.22	0.0017	0.9866	0.0101
$\frac{1}{7}$	1.41	0.90	3.09	1.23	0.19	0.0014	0.9880	0.0050
$\frac{1}{10}$	1.42	0.89	3.15	1.26	0.15	0.0012	0.9892	0.0050

Figure:

6.3. Government Spending Shocks

- Problem: The correlation between hours worked and labor productivity is about zero in the data. In the basic real business cycle model they are highly correlated
- Idea: Introduce additional source of shocks: government spending
    - Government spending increases labor supply (negative wealth effect). as a consequence, labor productivity decreases.
- The setup

$$\begin{aligned} c_t + i_t + g_t &= y_t \\ \ln g_{t+1} &= (1-\theta)\ln \bar{g} + \theta \ln g_t + \zeta_t \end{aligned}$$

where ζ_t is uncorrelated with ω_t.

- Source: Hansen,Wright,"The Labor Market in RBC Theory", FRB of Minneapolis Quarterly Review (1992).

6.3. Government Spending Shocks

Data

Chart 1 The U.S. Data, 1947:1–1991:3
Based on the Household Survey

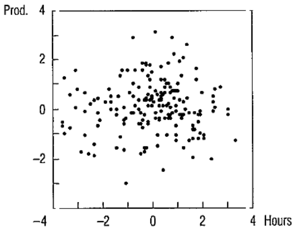

Figure:

6.3. Government Spending Shocks

Data

Chart 2 The Standard Model

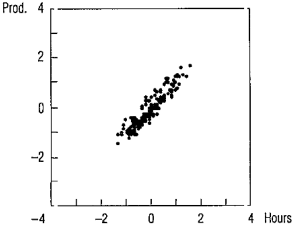

Chart 3 The Nonseparable Leisure Model

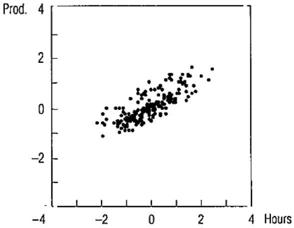

Chart 4 The Government Spending Model Without Technology Shocks . . .

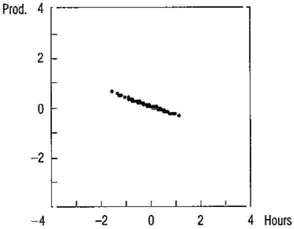

Chart 5 . . . And With Technology Shocks