Real Business Cycles

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Real Business Cycles

4. Linear Quadratic Approximation ✓
5. The Results
6. Extensions of the basic model

5. Real Business Cycle Model - How it works

- Denote the interest rate and the wage rate

$$R = \alpha e^{\omega}(\frac{n}{k})^{1-\alpha} + 1 - \delta$$

$$W = (1-\alpha)e^{\omega}(\frac{n}{k})^{-\alpha}$$

The first order conditions imply

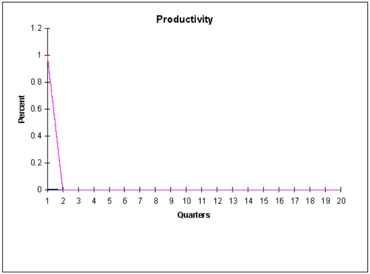
$$\beta E(\frac{c}{c'}R'|\omega) = 1 + \gamma$$

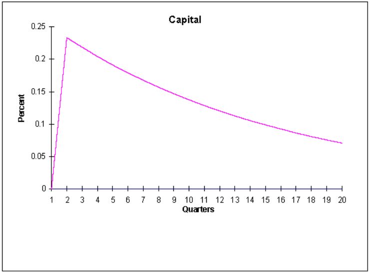
$$\beta E(\frac{1-n}{1-n'}\frac{W}{W'}R'|\omega) = 1 + \gamma$$

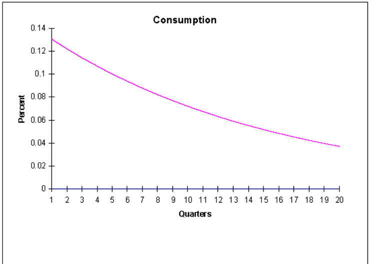
- Agents' current consumption is relatively low whenever the expected interest rate is high
- Agents' current labor supply is relatively high whenever the expected interest rate is high and/or current wage rate is relatively high
  - the importance of intertemporal effects on labor supply

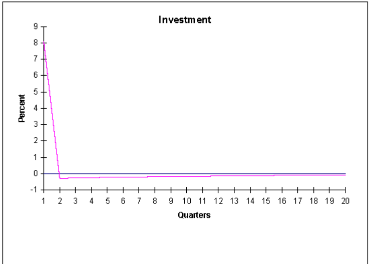
5. Real Business Cycle Model - How it works

Impulse Response to Temporary Shocks



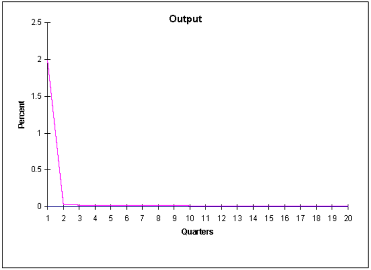


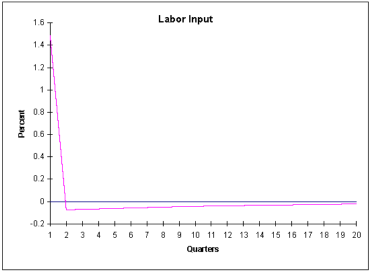


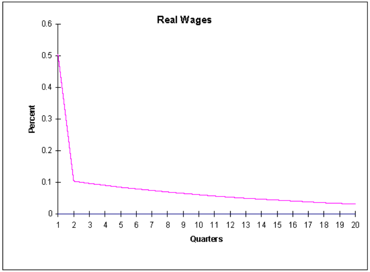


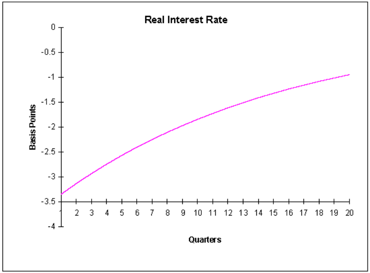
5. Real Business Cycle Model - How it works

Impulse Response to Temporary Shocks



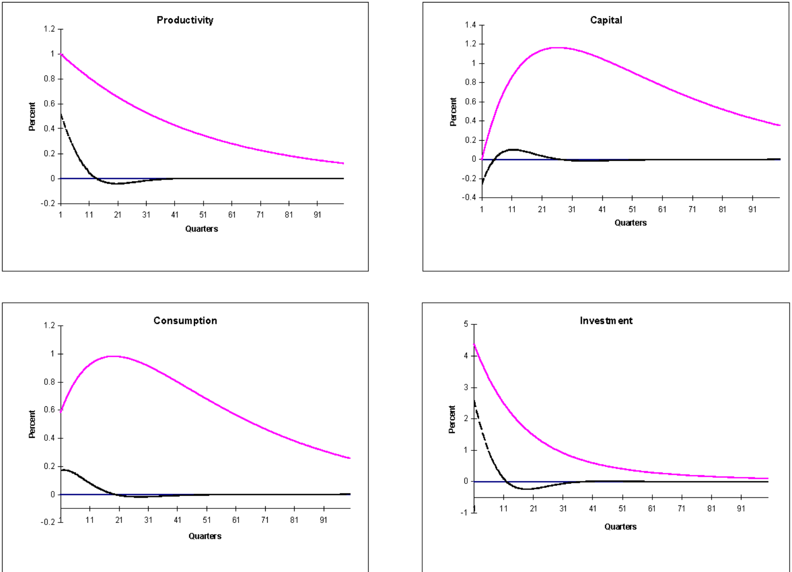






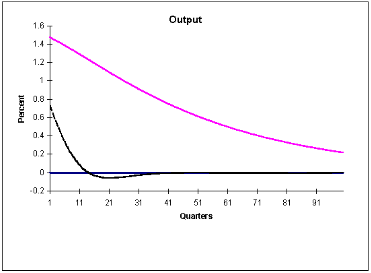
5. Real Business Cycle Model - How it works

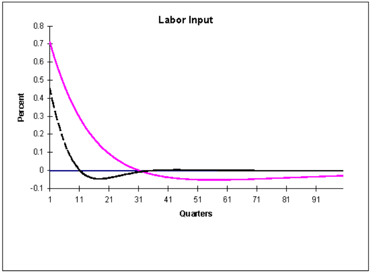
Impulse Response to Permanent Shocks

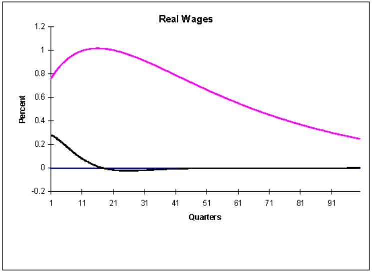


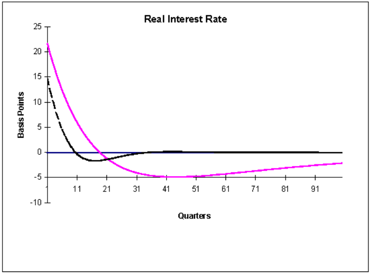
5. Real Business Cycle Model - How it works

Impulse Response to Permanent Shocks



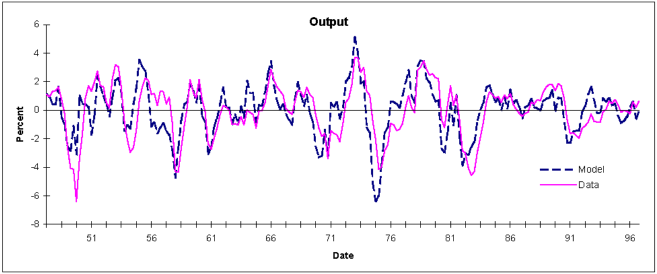


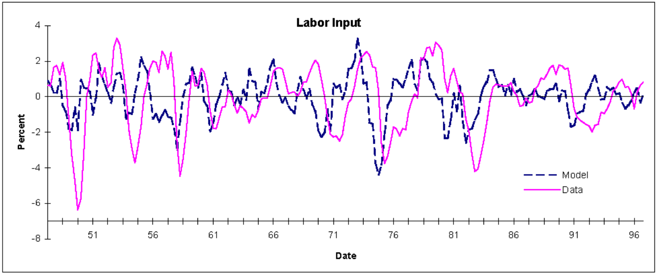




5. Real Business Cycle Model - Results

Time Series





5. Real Business Cycle Model - Results

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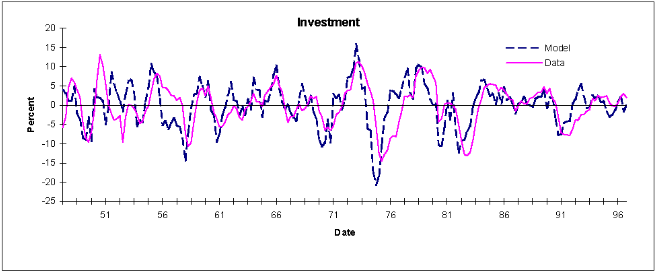

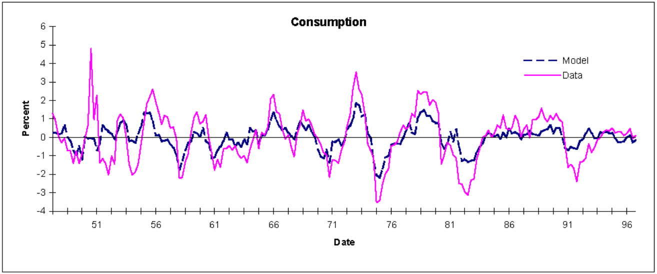

5. Real Business Cycle Model - Results

US data

Table 1
Business Cycle Statistics for the U.S. Economy

	Standard Deviation	Relative Standard Deviation	First Order Auto-correlation	Contemporaneous Correlation with Output
Y	1.81	1.00	0.84	1.00
C	1.35	0.74	0.80	0.88
I	5.30	2.93	0.87	0.80
N	1.79	0.99	0.88	0.88
Y/N	1.02	0.56	0.74	0.55
w	0.68	0.38	0.66	0.12
r	0.30	0.16	0.60	-0.35
A	0.98	0.54	0.74	0.78

Figure:

5. Real Business Cycle Model - Results

Model generated data

Table 3
Business Cycle Statistics for Basic RBC Model[35]

	Standard Deviation	Relative Standard Deviation	First Order Auto-correlation	Contemporaneous Correlation with Output
Y	1.39	1.00	0.72	1.00
C	0.61	0.44	0.79	0.94
I	4.09	2.95	0.71	0.99
N	0.67	0.48	0.71	0.97
Y/N	0.75	0.54	0.76	0.98
w	0.75	0.54	0.76	0.98
r	0.05	0.04	0.71	0.95
A	0.94	0.68	0.72	1.00

Figure:

5. Real Business Cycle Model - Results

- Overall, RBC model provides reasonable fit for the data
- It explains

$$\frac{1.39}{1.81} \approx 77\%$$

  of business cycles
- On several dimensions the model misses:
  1. the consumption is too smooth
  2. labor supply is too smooth
  3. interest rate is procyclical
  4. real wage is too volatile

5. Real Business Cycle Model - what drives the results

1. The productivity shocks must be **large**
2. The productivity shocks must be **persistent**
   - The model is very sensitive to changes in persistency of shocks
3. The Elasticity of Labor must be **large**

5. Real Business Cycle Model - what drives the results

Small Productivity Shocks

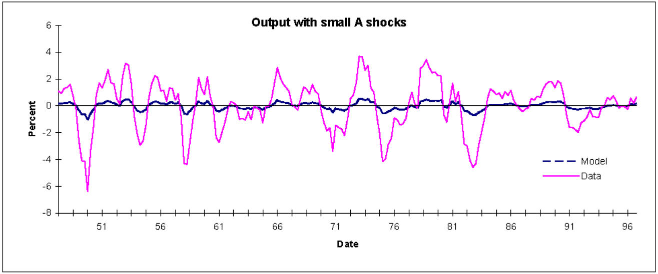

Figure:

5. Real Business Cycle Model - what drives the results

Small Elasticity of Labor

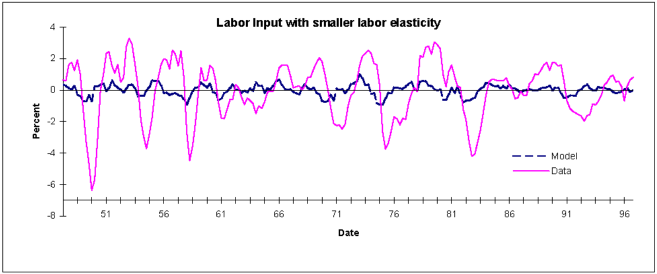

Figure:

6. Extensions of the basic RBC Model

Problems with labor markets

- The basic RBC model cannot capture the labor market dynamics very well
  - the labor supply in the model exhibits too little variability, compared to the data
- Increasing labor supply elasticity might help but is at odds with micro estimates
- In addition, most of the variations in aggregate labor supply is on the **extensive margin** (in/out of employment), rather than on the **intensive margin** (change in hours per person)
  - extensive margin: 2/3 of changes
  - intensive margin: 1/3 of changes

6. Extensions of the basic RBC Model

6.1. Indivisible labor

- Hansen (1985) and Rogerson (1988): incorporate the fact that labor supply is indivisible (fixed shifts)
  - This extension will both account for the extensive margin movement and make the aggregate labor supply more volatile
- Read Hansen, "Indivisible Labor and the Business Cycle" (on the reading list)

4.1. Indivisible Labor

The lottery

- fixed shifts: each agent can work either $n = H \in (0,1)$ or $n = 0$ hours
- $\{0, H\}$ is a nonconvex set: agents are better off by using lottery:
  - work with probability p, get wage w
  - stay at home with probability $1-p$, get nothing
- Expected utility

$$Eu = pU(\tilde{c}_1, 1-H) + (1-p)U(\tilde{c}_2, 1)$$

  - $\tilde{c}_1 = w + (r+\delta)k$: consumption if employed
  - $\tilde{c}_2 = (r+\delta)k$: consumption if unemployed
- With log-log, the expected utility becomes

$$Eu = p[\alpha \ln \tilde{c}_1 + (1-\alpha)\ln(1-H)] + (1-p)\alpha \ln \tilde{c}_2$$

6.1. Indivisible Labor

- Assume that there are insurance companies that can insure against consumption fluctuations
  - pay $w + (r + \delta)k$ if employed, $(r + \delta)k$ if unemployed
  - get c_1 if employed, c_2 if unemployed
- Zero profit condition implies

$$pc_1 + (1 - p)c_2 = pw + (r + \delta)k$$

- The agent's problem in a competitive equilibrium becomes

$$\max_{c_1, c_2, p} p[\alpha \ln c_1 + (1 - \alpha) \ln(1 - H)] + (1 - p)\alpha \ln c_2$$
$$s.t. \qquad pc_1 + (1 - p)c_2 = pw + (r + \delta)k$$

6.1. Indivisible Labor

Complete Consumption insurance

- with log-log utility, there is complete unemployment insurance:

$$c_1 = c_2 = c$$

- The agent's problem becomes

$$\max_{c,p} \alpha \ln c_1 + p(1-\alpha)\ln(1-H)]$$
$$s.t. \qquad c = pw + (r+\delta)k$$

6.1. Indivisible Labor

The Aggregation

- The expected (or aggregate) leisure is $L = 1 - pH$. The representative agent's problem can be written as

$$\max_{c,L} \alpha \ln c + \phi(L-1)$$
$$s.t. \qquad c = \hat{w}(1-L) + (r+\delta)k$$

where $\phi = -\frac{1-\alpha}{H}\ln(1-H) > 0$ and $\hat{w} = \frac{w}{H}$.

- **The representative agent has utility that is linear in leisure!**

6.1. Indivisible Labor

The implications

- At an individual level, the elasticity of labor supply is zero
- In the aggregate, the elasticity of labor supply is infinite (utility is linear in leisure).