Real Business Cycles

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Calibration of the model

- **Calibration**: a procedure to find the parameters and functional forms of the model
- Three steps
  1. Restrict the **utility and production functions** to parametric classes that are consistent with the long run growth facts
  2. Constructs **measurements of the U.S. economy** that are consistent with the model
  3. Restrict the **parameters** of the model so that the model matches certain long-run facts about the U.S. economy

1. Utility and Production Functions

- Cancelling terms, we get

$$\max E_0 \sum_{t=0}^{\infty} \hat{\beta}^t \frac{[c_t^{1-\mu}(1-n_t)^{\mu}]^{1-\sigma}}{1-\sigma}$$

s.t.

$$c_t + (1+\eta)(1+\gamma)k_{t+1} = z_t {k_t}^{\alpha} {n_t}^{1-\alpha} + (1-\delta)k_t$$

where $\hat{\beta} = \beta(1+\eta)(1+\gamma)^{(1-\mu)(1-\sigma)}$.

- !! Need $\hat{\beta} < 1$

2. Measurement of the U.S. Economy

- Problems:
  1. Some NIPA categories are not in the model
     - inventories
     - net export
     - government sector
  2. Some NIPA categories are wrongly attributed
     - Consumption of durable goods: NIPA: consumption. Model: investment
  3. Some items are not included in NIPA and need to be imputed
     - flow of services from durable goods

2.1-2.2 Measurement of the U.S. Economy

	NIPA	Model
Personal Consumption Expenditures	C	
durable goods		I
nondurable goods		C
services		C
Gross Private Domestic Investment	I	I
Net Exports	NX	I
Government Purchases	G	
public consumption		C
public investment		I

2.3. Measurement of the U.S. Economy

- Three types of capital in the economy:
  - K : private capital
  - K_D : stock of durable goods
- To get a correct measurement of GDP, we need to impute incomes from K_D

$$Y = wL + (r+\delta)K + (r+\delta_D)K_D$$

  where w is the wage rate, r is the interest rate (the same for all sectors) and δ, δ_D are depreciation rates
- We know:
  - $GNP^{NIPA} = wL + (r+\delta)K$
  - K, K_D
  - δK, $\delta_D K_D$ (NIPA)
- We do not know:
  - r, δ, δ_D

2.3. Measurement of the U.S. Economy

- Computations:

a. Compute δ, δ_D directly from the data

b. use the income side of NIPA to compute the private capital share α

c. use α to compute the income from private capital $Y^K = (r + \delta)K = \alpha GNP^{NIPA}$ and the interest rate $r = \frac{Y^K - \delta K}{K}$

2.3. Measurement of the U.S. Economy

a. Depreciation rates

$$\delta = \frac{\delta K}{K}$$

$$\delta_D = \frac{\delta_D K_D}{K_D}$$

2.3. Measurement of the U.S. Economy

b. private capital share

$$
\begin{aligned}
GNP^{NIPA} &= wL + (r+\delta)K \\
&= wL + rK + \delta K \\
&= NNP^{NIPA} + \delta K
\end{aligned}
$$

Net National Product		
1.	Compensation of Employees	Labor
2.	Corporate Profits	Private Capital
3.	Rental Income	Private Capital
4.	Net Interest	Private Capital
5.	Proprietor's Income	Both
6.	IBT	Both

2.3. Measurement of the U.S. Economy

b. private capital share

$$
\begin{aligned}
Y^K &= \alpha GNP^{NIPA} \\
&= (2) + (3) + (4) + \alpha[(5) + (6)] + \delta K
\end{aligned}
$$

hence

$$
\alpha = \frac{(2) + (3) + (4) + \delta K}{GNP^{NIPA} - (5) - (6)}
$$

2.3. Measurement of the U.S. Economy

c. interest rate

$$
\begin{aligned}
(r+\delta)K &= \alpha GNP^{NIPA} = Y^K \\
rK &= Y^K - \delta K \\
r &= \frac{Y^K - \delta K}{K}
\end{aligned}
$$

Calibration of the model

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3. Restricting the Parameters of the Model

- Assume that

$$\ln z_t = \rho \ln z_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim N(0, \nu^2)$$

- Write the RBC model recursively:

$$V(k, z) = \max_{c,n,k'} \{ \frac{[c^{1-\mu}(1-n)^{\mu}]^{1-\sigma}}{1-\sigma} + \hat{\beta} E[V(k', z')|\varepsilon] \}$$

s.t.

$$\begin{aligned} c + (1+\eta)(1+\gamma)k' &= zk^{\alpha} n^{1-\alpha} + (1-\delta)k \\ \ln z' &= \rho \ln z + \varepsilon \qquad \varepsilon \sim N(0, \nu^2) \end{aligned}$$

where $\hat{\beta} = \beta(1+\eta)(1+\gamma)^{(1-\mu)(1-\sigma)}$

- Parameters to select: $\sigma, \mu, \eta, \gamma, \beta, \alpha, \rho, \nu^2$

3. Restricting the Parameters of the Model

- **Main idea**: assign parameters to match
  1. certain micro-level estimates
     - Set $\sigma = 1$.
  2. some long-run (growth) observations
     - Set $\mu, \eta, \gamma, \beta, \delta, \alpha$
  3. Behavior of the Solow Residual
     - Set ρ, ν^2

3.2. Matching some long-run observations

Facts

- The following facts are approximately true about the U.S. economy (1954-1992)
  1. Annual rates of growth of GDP per person is 1.56%
  2. Annual rate of growth of population is 1.2%
  3. The capital share is constant over time and is 0.4
  4. Investment/capital ratio is trendless and is 7.6% on average
  5. Time spent on market activities is approximately constant over time and constitutes 31% of total disposable time
  6. Capital/output ratio is trendless and is 3.32 on average

$$\text{6 facts} \;\Leftrightarrow\; \text{6 parameters } \mu, \eta, \gamma, \beta, \delta, \alpha$$

3.2. Matching some long-run observations

How to match these facts?

- Compute a nonstochastic version of the model (i.e. $z = 1$ in all periods) and choose the parameters $\mu, \eta, \gamma, \beta, \delta, \alpha$ to match these facts in **steady state**

$$\hat{V}(k) = \max_{c,n,k'} \{(1-\mu)\ln c + \mu \ln(1-n) + \beta(1+\eta)\hat{V}(k')\}$$

s.t.

$$c + (1+\eta)(1+\gamma)k' = k^{\alpha} n^{1-\alpha} + (1-\delta)k$$

- First order conditions

$$\begin{aligned}
\frac{1-\mu}{c} &= \lambda \\
\beta(1+\eta)\hat{V}_k(k') &= \lambda(1+\eta)(1+\gamma) \\
\frac{\mu}{1-n} &= \lambda(1-\alpha)(\frac{n}{k})^{-\alpha}
\end{aligned}$$

- Envelope condition

$$\hat{V}_k(k) = \lambda(\alpha(\frac{n}{k})^{1-\alpha} + 1 - \delta)$$

3.2. Matching some long-run observations

How to match these facts?

- In steady state,
  - c, k are constant over time
  - population grows at rate η
  - output per person grows at rate γ

1. Use fact 1 to set

$$\boxed{\gamma = 0.0156 \text{ (annual)}}$$

2. Use fact 2 to set

$$\boxed{\eta = 0.012 \text{ (annual)}}$$

3. Use fact 3 to set

$$\boxed{\alpha = 0.4}$$

3.2. Matching some long-run observations

How to match these facts?

4. From the law of motion for capital

$$\delta = \frac{i}{k} + 1 - (1+\eta)(1+\gamma)$$

Use fact 4 ($\frac{i}{k} = 0.076$) and this equation to set

$$\boxed{\delta = 0.048 \text{ (annual)}}$$

5. Combine the first order conditions in c, n to get

$$\frac{n}{1-n} = (1-\alpha)\frac{1-\mu}{\mu}\frac{y}{c}$$

Use fact 5 ($n = 0.31$) and facts 4,6 ($\frac{y}{c} = 1.33$) to set

$$\boxed{\mu = 0.64}$$

3.2. Matching some long-run observations

How to match these facts?

6. Use first order condition in k', envelope condition and resource constraint to get

$$\frac{1+\gamma}{\beta} + \delta - 1 = \theta \frac{y}{k}$$

Use fact 6 ($\frac{k}{y} = 3.32$) to get

$$\boxed{\beta = 0.947 \text{ (annual)}}$$

3.3. Using Solow Residual to determine the stochastic process for z

- From the production function

$$\begin{aligned} Y_t &= z_t K_t^{\alpha} N_t^{1-\alpha} \\ \ln z_t &= \ln Y_t - \alpha \ln K_t - (1-\alpha) \ln N_t \end{aligned}$$

- Compute the time series for $\ln z_t$ and estimate its autocorrelation ($\rightarrow \rho$) and variance ($\rightarrow \nu$).
- Results

$\rho = 0.95$
$\nu = 0.007$