

# Real Business Cycles

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May 5, 2009

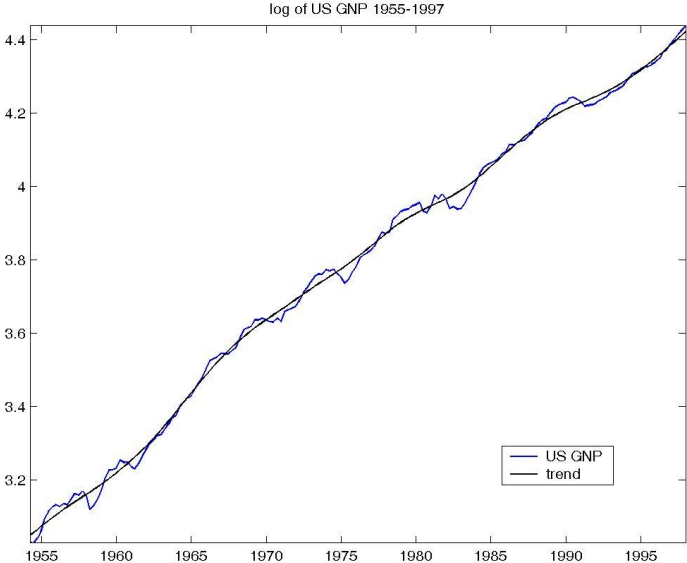
# Business Cycles

## Definition (Lucas)

Business Cycles are deviations of aggregate output from a trend

- ▶ Properties:
  - ▶ Recurrent, not periodic
  - ▶ Two to eight years
  - ▶ Comovement among variables
- ▶ We will use the stochastic growth model to study business cycles
- ▶ Read Cooley-Prescott (1995), "Economic Growth and Business Cycles" (on the web)

# US GNP



# Trend vs. Business Cycles

- ▶ How to extract the trend?
- ▶ Hodrick-Prescott filter: let
  - ▶  $\{y_t\}_{t=1}^T$  be a time series
  - ▶  $\{z_t\}_{t=1}^T$  be the trend component
  - ▶  $\{y_t - z_t\}_{t=1}^T$  be the business cycle component.
  - ▶  $\{z_t\}_{t=0}^T$  solves

$$\arg \min_{\{z_t\}_{t=0}^T} \sum_{t=1}^T (y_t - z_t)^2 + \lambda \sum_{t=1}^T [(z_{t+1} - z_t) - (z_t - z_{t-1})]^2$$

- ▶  $\lambda$  determines smoothness of the trend
  - ▶ for quarterly data, take  $\lambda = 1600$
  - ▶ for annual data,  $\lambda = 400$

# Business Cycle Facts

**Table 1.1**  
**Cyclical Behavior of the U.S. Economy: Deviations from Trend of Key Variables, 1954:I–1991:II**

Variable	SD%	Cross-Correlation of Output with:										
		$x(-5)$	$x(-4)$	$x(-3)$	$x(-2)$	$x(-1)$	$x$	$x(+1)$	$x(+2)$	$x(+3)$	$x(+4)$	$x(+5)$
Output component												
GNP	1.72	.02	.16	.38	.63	.85	1.0	.85	.63	.38	.16	-.02
Consumption expenditures												
CONS	1.27	.25	.42	.57	.72	.82	.83	.67	.46	.22	-.01	-.20
CNDS	0.86	.22	.40	.55	.68	.78	.77	.64	.47	.27	.06	-.11
CD	4.96	.24	.37	.49	.65	.75	.78	.61	.38	.11	-.13	-.31
Investment												
INV	8.24	.04	.19	.38	.59	.79	.91	.76	.50	.22	-.04	-.24
INVF	5.34	.08	.25	.43	.63	.82	.90	.81	.60	.35	.09	-.12
INVN	5.11	-.26	-.12	.05	.30	.57	.79	.88	.83	.60	.46	.24
INVR	10.7	.42	.55	.65	.72	.74	.63	.39	.11	-.14	-.33	-.43
Ch. INV	17.3	-.03	.07	.22	.38	.53	.67	.51	.27	.04	-.15	-.30
Government purchases												
GOVT	2.04	.03	-.01	-.03	-.01	-.01	.04	.08	.11	.16	.25	.32
Exports and imports												
EXP	5.53	-.48	-.42	-.29	-.10	.15	.37	.50	.54	.54	.52	.44
IMP	4.88	.11	.19	.31	.45	.62	.72	.71	.52	.28	.04	-.18
Labor input based on household survey												
HSHOURS	1.59	-.06	.09	.30	.53	.74	.86	.82	.69	.52	.32	.11
HSAVGHRS	0.63	.04	.16	.34	.48	.63	.62	.52	.37	.23	.09	-.05

Figure:

# Business Cycle Facts

## ▶ GDP Components

- ▶ Consumption of nondurables and services fluctuates much less than output and is procyclical
- ▶ Investment fluctuates much more than output and is procyclical
- ▶ Government expenditures are essentially uncorrelated with output
- ▶ Both imports and exports are procyclical and fluctuate more than output

# Business Cycle Facts

## ▶ **Factors of production**

- ▶ Total hours worked fluctuate about as much as output and are highly procyclical
- ▶ Most of the volatility in hours worked can be accounted for by volatility in employment rather than hours per worker
- ▶ hours worked are uncorrelated with productivity
- ▶ The capital stock fluctuates much less than output and is uncorrelated with output

## ▶ **Prices**

- ▶ Average hourly compensation is essentially uncorrelated with output

# The Real Business Cycle Model

- ▶ Essentially the stochastic growth model with some bells and whistles
  - ▶ Variable labor supply
  - ▶ Labor productivity growth  $\gamma$
  - ▶ Population growth  $\eta$
- ▶ Social Planner's problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t N_t U(\hat{c}_t, 1 - \hat{n}_t)$$

s.t.

$$\begin{aligned} N_t \hat{c}_t + N_{t+1} \hat{k}_{t+1} &= z_t f(N_t \hat{k}_t, A_t N_t \hat{n}_t) + (1 - \delta) N_t \hat{k}_t \\ A_{t+1} &= (1 + \gamma) A_t, \quad A_1 = 1 \\ N_{t+1} &= (1 + \eta) N_t, \quad N_1 = 1 \end{aligned}$$

where  $\hat{c}_t$  is per capita consumption,  $\hat{n}_t$  is per capita labor supply and  $\hat{k}_t$  is per capita capital stock.

# Changes in output per worker

- ▶ Secular Growth
  - ▶ Changes in capital :  $1/3$
  - ▶ Changes in labor:  $0$
  - ▶ Changes in productivity:  $2/3$
- ▶ Business Cycles
  - ▶ Changes in capital :  $0$
  - ▶ Changes in labor:  $2/3$
  - ▶ Changes in productivity:  $1/3$

# Calibration of the model

- ▶ **Calibration**: a procedure to find the parameters and functional forms of the model
- ▶ Three steps
  1. Restrict the **utility and production functions** to parametric classes that are consistent with the long run growth facts
  2. Constructs **measurements of the U.S. economy** that are consistent with the model
  3. Restrict the **parameters** of the model so that the model matches certain long-run facts about the U.S. economy

# 1. Utility and Production Functions

- ▶ **Fact No. 1:** The labor share  $1 - \alpha$

$$1 - \alpha = \frac{w_t L_t}{Y_t}$$

is approximately constant over time.

- ▶ **Implication:** The production function is Cobb-Douglas

$$Y_t = z_t K_t^\alpha L_t^{1-\alpha}$$

# 1. Utility and Production Functions

- ▶ **Fact No. 2:** After WWII, per capita leisure is constant over time (despite changes in incomes and wages) is approximately constant over time.
- ▶ **Implication:** The utility function is CRRA

$$U(c, l) = \frac{(c^{1-\mu} l^\mu)^{1-\sigma}}{1-\sigma}$$

# 1. Utility and Production Functions

## Normalizing the model

- ▶ Let  $n_t = \hat{n}_t$  and

$$c_t = \frac{\hat{c}_t}{A_t} = \frac{\hat{c}_t}{(1 + \gamma)^t}, \quad k_t = \frac{\hat{k}_t}{A_t} = \frac{\hat{k}_t}{(1 + \gamma)^t}$$

be consumption and capital stock per unit of effective labor.

- ▶ The model becomes

$$\max E_0 \sum_{t=0}^{\infty} \beta^t (1 + \eta)^t \frac{[(1 + \gamma)^{(1-\mu)t} c_t^{1-\mu} (1 - n_t)^\mu]^{1-\sigma}}{1 - \sigma}$$

s.t.

$$\begin{aligned} & (1 + \eta)^t (1 + \gamma)^t c_t + (1 + \eta)^{t+1} (1 + \gamma)^{t+1} k_{t+1} \\ &= z_t [(1 + \eta)^t (1 + \gamma)^t k_t]^\alpha [(1 + \eta)^t (1 + \gamma)^t n_t]^{1-\alpha} \\ &+ (1 - \delta) (1 + \eta)^t (1 + \gamma)^t k_t \end{aligned}$$

# 1. Utility and Production Functions

## Normalizing the model

- ▶ Cancelling terms, we get

$$\max E_0 \sum_{t=0}^{\infty} \hat{\beta}^t \frac{[c_t^{1-\mu} (1 - n_t)^\mu]^{1-\sigma}}{1 - \sigma}$$

s.t.

$$c_t + (1 + \eta)(1 + \gamma)k_{t+1} = z_t k_t^\alpha n_t^{1-\alpha} + (1 - \delta)k_t$$

where  $\hat{\beta} = \beta(1 + \eta)(1 + \gamma)^{(1-\mu)(1-\sigma)}$ .

- ▶ !! Need  $\hat{\beta} < 1$

## 2. Measurement of the U.S. Economy

### ▶ Problems:

1. Some NIPA categories are not in the model
  - ▶ inventories
  - ▶ net export
  - ▶ government sector
2. Some NIPA categories are wrongly attributed
  - ▶ Consumption of durable goods: NIPA: consumption. Model: investment
3. Some items are not included in NIPA and need to be imputed
  - ▶ flow of services from durable goods

## 2.1-2.2 Measurement of the U.S. Economy

	NIPA	Model
Personal Consumption Expenditures	C	
durable goods		I
nondurable goods		C
services		C
Gross Private Domestic Investment	I	I
Net Exports	NX	I
Government Purchases	G	
public consumption		C
public investment		I

## 2.3. Measurement of the U.S. Economy

- ▶ Three types of capital in the economy:
  - ▶  $K$  : private capital
  - ▶  $K_D$  : stock of durable goods
- ▶ To get a correct measurement of GDP, we need to impute incomes from  $K_D$

$$Y = wL + (r + \delta)K + (r + \delta_D)K_D$$

where  $w$  is the wage rate,  $r$  is the interest rate (the same for all sectors) and  $\delta$ ,  $\delta_D$  are depreciation rates

- ▶ We know:
  - ▶  $GNP^{NIPA} = wL + (r + \delta)K$
  - ▶  $K, K_D$
  - ▶  $\delta K, \delta_D K_D$  (NIPA)
- ▶ We do not know:
  - ▶  $r, \delta, \delta_D$

## 2.3. Measurement of the U.S. Economy

► Computations:

- a. Compute  $\delta, \delta_D$  directly from the data
- b. use the income side of NIPA to compute the private capital share  $\alpha$
- c. use  $\alpha$  to compute the income from private capital

$$Y^K = (r + \delta)K = \alpha GNP^{NIPA} \text{ and the interest rate}$$

$$r = \frac{Y^K - \delta K}{K}$$

## 2.3. Measurement of the U.S. Economy

### a. Depreciation rates

$$\delta = \frac{\delta K}{K}$$
$$\delta_D = \frac{\delta_D K_D}{K_D}$$

## 2.3. Measurement of the U.S. Economy

### b. private capital share

$$\begin{aligned} GNP^{NIPA} &= wL + (r + \delta)K \\ &= wL + rK + \delta K \\ &= NNP^{NIPA} + \delta K \end{aligned}$$

#### **Net National Product**

- |    |                           |                 |
|----|---------------------------|-----------------|
| 1. | Compensation of Employees | Labor           |
| 2. | Corporate Profits         | Private Capital |
| 3. | Rental Income             | Private Capital |
| 4. | Net Interest              | Private Capital |
| 5. | Proprietor's Income       | Both            |
| 6. | IBT                       | Both            |

## 2.3. Measurement of the U.S. Economy

### b. private capital share

$$\begin{aligned} Y^K &= \alpha GNP^{NIPA} \\ &= (2) + (3) + (4) + \alpha[(5) + (6)] + \delta K \end{aligned}$$

hence

$$\alpha = \frac{(2) + (3) + (4) + \delta K}{GNP^{NIPA} - (5) - (6)}$$

## 2.3. Measurement of the U.S. Economy

c. interest rate

$$\begin{aligned}(r + \delta)K &= \alpha GNP^{NIPA} = Y^K \\ rK &= Y^K - \delta K \\ r &= \frac{Y^K - \delta K}{K}\end{aligned}$$