Economics 204b Marek Kapicka
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Midterm Answers

1 A Dynastic Economy

Consider an economy with dynasties. A dynasty consists of an infinite sequence of agents. Each of the agents lives for one period, cares about the utility of his children, and makes decisions about his consumption c_t, number of children n_t, and next period assets a_{t+1}. To simplify matters, assume that the number of children n_t is a continuous nonnegative variable.

The representative dynasty (equivalently, the agent living in period zero) has preferences

$$U(\{c_t, n_t\}_{t=0}^{\infty}) = u(c_0) + \beta(n_0)u(c_1) + \beta(n_0)\beta(n_1)u(c_2) + \beta(n_0)\beta(n_1)\beta(n_2)u(c_3) + ...$$

The discount factor between period t and $t+1$, $\beta(n_t)$ thus depends on the number of children. It is assumed that $\beta(n)$ is increasing in n and that $\beta(n) < 1$ for all $n \geq 0$.

Raising children is time consuming. The time required to raise one child is exogenously given by x. The agents are endowed with one unit of time and spend the rest of their time $1 - xn_t$ by working. The rate of return on assets r and the wage rate w are both assumed exogenous and constant over time. The budget constraint is

$$c_t + a_{t+1} \leq w(1 - xn_t) + (1 + r)a_t.$$

Next period assets a_{t+1} are divided equally among all children. Initial assets a_0 are given.

a. Write down the Bellman equation (BE) corresponding to the sequence problem described above.

$$\begin{aligned} V(a) &= \max_{a',n,c}\{u(c) + \beta(n)V(\frac{a'}{n})\} \\ & \quad s.t. \\ c + a' &\leq w(1 - xn) + (1 + r)a \end{aligned}$$

The questions that follow involve modifications to (BE) in part (a). In each case, apply the indicated modification to the original Bellman equation (BE). That is, do not answer (c) by modifying the equation you have written in answer to (b).

b. Modify equation (BE) so as to incorporate the assumption that the wage rate w is stochastic, and can take value w_L with probability π and value w_H with probability $1-\pi$.

$$\begin{aligned} V(a,w) &= \max_{a',n,c}\{u(c)+\beta(n)[\pi V(\frac{a'}{n},w_L)+(1-\pi)V(\frac{a'}{n},w_H)\} \\ & \quad s.t. \\ c+a' &\leq w(1-xn)+(1+r)a \end{aligned}$$

c. Modify equation (BE) so as to incorporate the assumption that with probability $1-q$ one half of the children will die before the end of current period. Assume that the time costs of raising these children have been fully spent and that, if those children die, next period assets are divided equally among the surviving children.

$$\begin{aligned} V(a) &= \max_{a',n,c}\{u(c)+q\beta(n)V(\frac{a'}{n})+(1-q)\beta(\frac{n}{2})V(\frac{2a'}{n})\} \\ & \quad s.t. \\ c+a' &\leq w(1-xn)+(1+r)a \end{aligned}$$

d. Modify equation (BE) so as to incorporate the assumption that the time spent with each child x is chosen by the parents and affects human capital of the children. Specifically, if parents have human capital h_t then each child will have human capital $G(h,x)$, where G is increasing in both arguments. Human capital is useful because it increases earnings: an agent with human capital h earns wage wh.

$$\begin{aligned} V(a,h) &= \max_{a',n,c,x}\{u(c)+\beta(n)V(\frac{a'}{n},h')\} \\ & \quad s.t. \\ c+a' &\leq wh(1-xn)+(1+r)a \\ h' &= G(h,x) \end{aligned}$$

2 Asset Pricing with Habit Persistence

Consider an economy where the representative agent's preferences exhibit habit persistence: high consumption last period increases marginal utility of consumption today. The preferences of a representative consumer are given by

$$\sum_{t=0}^{\infty} \beta^t U[c_t(z^t) - \gamma c_{t-1}(z^{t-1})],$$

where $\gamma \geq 0$ measures habit persistence, z_t is deterministic output in period t, and $c_{-1}(z^{-1}) = c_{-1}$ is given.

a. Define the Arrow-Debreu Equilibrium in this economy.

   The Arrow-Debreu Equilibrium consist of prices $q = \{q_t(z^t)\}_{t=0}^{\infty}$ and quantities $c = \{c_t(z^t)\}_{t=0}^{\infty}$ such that the agents solve

$$\max_c \sum_{t=0}^{\infty} \beta^t U[c_t(z^t) - \gamma c_{t-1}(z^{t-1})]$$
$$s.t.$$
$$\sum_{t=0}^{\infty} q_t(z^t) c_t(z^t) \leq \sum_{t=0}^{\infty} q_t(z^t) z_t,$$

   and markets clear: $c_t(z^t) = z_t$.

b. Specify the competitive equilibrium price system. The formula for the price of a commodity should be in terms of the data that define the economy.

   Taking first order condition w.r.t. $c_t(z^t)$, we get

$$\begin{aligned} \lambda q_t(z^t) &= \beta^t [U'(c_t(z^t) - \gamma c_{t-1}(z^{t-1})) - \gamma \beta U'(c_{t+1}(z^{t+1}) - \gamma c_t(z^t))] \\ q_t(z^t) &= \beta^t \frac{U'(c_t(z^t) - \gamma c_{t-1}(z^{t-1})) - \gamma \beta U'(c_{t+1}(z^{t+1}) - \gamma c_t(z^t))}{U'(c_0(z^0) - c_{-1}) - \gamma \beta U'(c_1(z^1) - \gamma c_0(z^0)).} \end{aligned}$$

   Substituting in the market clearing condition, we get

$$q_t(z^t) = \beta^t \frac{U'(z_t - \gamma z_{t-1}) - \gamma \beta U'(z_{t+1} - \gamma z_t)}{U'(z_0 - c_{-1}) - \gamma \beta U'(z_1 - \gamma z_0).}$$

c. Now assume that $z_t = 1 - \epsilon$ if $t = \{0, 2, 4, ...\}$, $z_t = 1 + \epsilon$ if $t = \{1, 3, 5, ...\}$ and that $U(\cdot) = \ln(\cdot)$. Assume also that $c_{-1} = 1 + \epsilon$. Characterize the equilibrium price system as sharply as possible. (Hint: Guess that the equilibrium price system will be an alternating sequence $\{q_e, \beta q_o, \beta^2 q_e, \beta^3 q_o, ..\}$.) Does habit persistence increase or decrease the volatility of prices (as measured by $v = \frac{q_e}{q_o}$)? Provide an intuition for your result.

If t is even then

$$q_t(z^t) = \beta^t \frac{\frac{1}{1-\epsilon-\gamma(1+\epsilon)} - \frac{\gamma\beta}{1+\epsilon-\gamma(1-\epsilon)}}{\frac{1}{1-\epsilon-\gamma(1+\epsilon)} - \frac{\gamma\beta}{1+\epsilon-\gamma(1-\epsilon)}} = \beta^t.$$

thus, $q_e = 1$. If t is odd then

$$\begin{aligned} q_t(z^t) &= \beta^t \frac{\frac{1}{1+\epsilon-\gamma(1-\epsilon)} - \frac{\gamma\beta}{1-\epsilon-\gamma(1+\epsilon)}}{\frac{1}{1-\epsilon-\gamma(1+\epsilon)} - \frac{\gamma\beta}{1+\epsilon-\gamma(1-\epsilon)}} \\ &= \beta^t \frac{1-\epsilon-\gamma(1+\epsilon)-\gamma\beta[1+\epsilon-\gamma(1-\epsilon)]}{1+\epsilon-\gamma(1-\epsilon)-\gamma\beta[1-\epsilon-\gamma(1+\epsilon)]} \\ &= \beta^t \frac{1-\gamma-\epsilon(1+\gamma)-\gamma\beta[1-\gamma+\epsilon(1+\gamma)]}{1-\gamma+\epsilon(1+\gamma)-\gamma\beta[1-\gamma-\epsilon(1+\gamma)]} \\ &= \beta^t \frac{(1-\gamma)(1-\gamma\beta)-\epsilon(1+\gamma)(1-\gamma\beta)}{(1-\gamma)(1-\gamma\beta)+\epsilon(1+\gamma)(1+\gamma\beta)} \\ &= \beta^t \frac{1-\epsilon\frac{1+\gamma}{1-\gamma}}{1+\epsilon\frac{(1+\gamma)(1+\gamma\beta)}{(1-\gamma)(1-\gamma\beta)}} \end{aligned}$$

Therefore, $q_o = \frac{1-\epsilon\frac{1+\gamma}{1-\gamma}}{1+\epsilon\frac{(1+\gamma)(1+\gamma\beta)}{(1-\gamma)(1-\gamma\beta)}} < 1$.

The volatility of prices $v = \frac{1+\epsilon\frac{(1+\gamma)(1+\gamma\beta)}{(1-\gamma)(1-\gamma\beta)}}{1-\epsilon\frac{1+\gamma}{1-\gamma}}$ is increasing in the habit persistence γ. The intuition is that in bad (even) times the consumer has a habit of consuming a lot because previous period was good. This gives him extra incentives to increase consumption in bad times. In equilibrium, this is not possible, and the agents must be deterred by prices higher than in the absence of habit. Since prices in bad times are always higher, the volatility of prices increases.