Economics 204b
Spring 2007

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Problem Set 5 Answers

Due Wednesday May 30

1 Monopoly and Wedges

Consider the following economy: agents maximize expected utility

$$E\{\sum_{t=0}^{\infty}\beta^t U(c_t, l_t)\}$$

subject to a budget constraint and a law of motion for capital:

$$\begin{aligned} c_t + x_t &\leq w_t l_t + r_t k_t + T_t \\ k_{t+1} &= (1-\delta)k_t + x_t. \end{aligned}$$

There are two types of goods: a final good Y_t which is produced from intermediate goods y_{it}, $i \in [0,1]$ according to a production function

$$Y_t = \left(\int_0^1 y_i^\sigma di\right)^{\frac{1}{\sigma}}$$

where $\sigma \in (0,1]$. The intermediate good producers hire capital and labor on a competitive market. They all have the same production function given by $f(k,l) = z_t k^\theta l^{1-\theta}$ where z_t is an i.i.d. random variable and is the same for all producers. All the intermediate good producers are monopolies. That is, they solve

$$\max_{y \leq f(k,l)} p_{it}(y)y - w_t l - r_t k$$

where $p_i(y)$ is an inverse demand function for the good of producer i. There is no population growth and no government.

1. Show that the above economy with monopolies is equivalent to a standard ('prototype') Real Business Cycle economy with efficiency, labor and investment wedges. Compute the wedges.

(a) In class, we have seen the in the prototype RBC model, the following conditions link the labor wedges $1 - \tau_{lt}$ and investment wedges $\frac{1}{1+\tau_{xt}}$ and quantities:

$$\frac{U_{c_t}}{U_{l_t}} = (1-\tau_{lt})(1-\theta)z_t(\frac{n_t}{k_t})^{-\theta} \quad (1)$$

$$U_{c_t}(1+\tau_{xt}) = \beta E_t[U_{c_{t+1}}(\theta\frac{y_{t+1}}{k_{t+1}} + (1-\delta)(1+\tau_{xt+1})] \quad (2)$$

In addition, the efficiency wedge z_t is given by:

$$y_t = z_t k^\theta l^{1-\theta}. \quad (3)$$

(b) We want to compute the wedges in the economy with unions. The first order condition for the final good producer is

$$Y_t^{1-\sigma} y_{it}^{\sigma-1} = p_{it}$$

and so the inverse demand curve is $p_{it}(y) = p(y, Y_t) = Y^{1-\sigma}y^{\sigma-1}$. The intermediate good producer solves

$$\max_{y \le f(k,l)} Y_t^{1-\sigma} y^\sigma - w_t l - r_t k$$

with first order conditions being

$$\sigma(1-\theta)Y_t^{1-\sigma}y_{it}^\sigma = w_t l_{it}$$
$$\sigma\theta Y_t^{1-\sigma}y_{it}^\sigma = r_t k_{it}$$

In equilibrium, all intermediate good producers are identical and so $Y_t = y_{it}$ for all $i \in [0, 1]$. Hence the aggregate output is given by

$$y_t = z_t k^\theta l^{1-\theta} \quad (4)$$

and aggregate labor demand and capital demand satisfies first order conditions

$$\sigma(1-\theta)y_t = w_t l_t$$
$$\sigma\theta y_t = r_t k_t.$$

Combining this with household's intratemporal first order condition, we get that the intratemporal first order condition is given by

$$\frac{U_{l_t}}{U_{c_t}} = w_t = \sigma(1-\theta)\frac{y_t}{l_t}. \tag{5}$$

The intertemporal first order condition in the economy with unions can be written as follows:

$$\begin{aligned} \frac{1}{\sigma}U_{c_t} &= \frac{1}{\sigma}\beta E_t[U_{c_{t+1}}(r_{t+1}+1-\delta)] \\ &= \beta E_t[U_{c_{t+1}}(\theta\frac{y_{t+1}}{k_{t+1}} + \frac{1-\delta}{\sigma})]. \end{aligned} \tag{6}$$

c. The efficiency wedge is z_t, as follows directly from (4) and (3). The labor wedge is σ, as can be seen by comparing (5) and (1). The investment wedge is also σ, as follows from comparing (6) and (2).