Instructions about \( r \) have changed. For this problem set only, let \( r \) be equal to the last three digits of your registration number (PID, perm number), divided by 1000, BUT if your \( r \) is \( \leq .1 \), use \( r = .1 \). (I had trouble getting the sensitivity analysis to run for values of \( r \) less than .1.) All three items this week are concerned with the following problem.

Maximize \( x_1^r + \ln(x_2) + \ln(x_3) \)
subject to \( x_1 + x_2 + x_3 \leq 12 \)
and \( x_i \geq 0 \) for \( i = 1, 2, 3 \)

You should get very similar numerical answers in all three items. All answers should be neatly printed using excel (or word if you wish) or similar on two or three pages in portrait configuration with your name and PID number neatly printed using word, excel, or similar in the upper right-hand corner of each page.

1. Solve the problem using solver in excel or similar. Don’t forget to report the value of the multiplier. This answer should take much less than one page.

2. Solve the problem by writing the Karush-Kuhn-Tucker conditions for the problem – writing them in excel is slightly annoying but persevere – and solving. At a certain point you will have the condition

\[
\frac{1}{u^*} + \frac{1}{u^*} + \left( \frac{r}{u^*} \right)^{\frac{1}{r-1}} = 12
\]

Think of the function \( \frac{1}{u} + \frac{1}{u} + \left( \frac{r}{u} \right)^{\frac{1}{r-1}} - 12 \) as the derivative of a concave function, which it is. Solve using the bisection method in the finding-the-root form. (Why is that better than Newton’s method here?) Use \( ulow = .01 \) and \( uhigh = 5 \) as starting values and let precision be \( \varepsilon = 10^{-5} \). Solve for \( u \) and then complete the problem.

3. Solve the problem using dynamic programming. Type in the steps on your excel sheet. At a certain point you will have the condition

\[
f_i^r(12) = \max_{x_1} \ x_1^r + 2 \ln(12 - x_1) - 2 \ln(2)
\]

Find the maximum using the bisection method with starting values \( x_1low = .01 \) and \( x_1high = 15 \). Then finish the problem. The values of \( f_i^r(12) \) and of the \( x_i \) should agree with the values found in the previous two items.