1. Let \( r \) be equal to the last three digits of your registration number (PID, perm number), divided by 1000. That is, if the last three numbers are 678, \( r = .678 \). Consider the problem

Maximize \( 10 - (x_1 - 4)^2 - (x_2 - 6)^2 \)
subject to \( x_1 + x_2 = 5 - r \)

(a) Write the Lagrangian.

(b) Write the first order conditions.

(c) Why are the first-order conditions sufficient? (35 words)

(d) Fill in the blanks for the optimum values and show your derivation briefly below.
\[ x_1^* = \underline{\text{__________}}, \quad x_2^* = \underline{\text{__________}}, \quad \lambda^* = \underline{\text{__________}} \]

(e) What is the interpretation of \( \lambda^* \)? (50 words)
2. Using the same value for $r$ as in the first problem, look at the function

$$0.6 \ln (x_1 + 1) + 0.4 \ln (x_2 + 1) - \lambda (x_1 + 2x_2 - 7 - r)$$

(a) Taking $\lambda$ as fixed, maximize over $x_1$ and $x_2$ and write out the solution functions $x_1(\lambda)$, and $x_2(\lambda)$.

(b) Find the value of $\lambda$ that makes $x_1 + 2x_2 = 7 + r$. Use the numerical value of $r$ at this point.

(c) Explain why this problem makes sense. (50 words)