1. Consider the function \( f(x) = -x(x - 1)(x - 2)(x - 4) \). Use the charting options of excel to find the global maximum, a secondary local maximum, and a local minimum. Submit your answers and an excel chart or charts of \( f(x) \) that show the situation clearly. The answer should occupy one sheet printed from excel on 8.5 by 11 paper in portrait format with your name in the upper right-hand corner. (As a midterm problem, you would have to describe the procedure or some part of it)

2. Consider the function \( f(x) = x_1x_2 - x_1^2 - 2x_2^2 \). Suppose that you are maximizing the function using gradient search starting at the point \((1, 1)\). Calculate and briefly explain your procedure for finding the next point. Hint: the next point is \((x_1, x_2) = (\frac{11}{16}, \frac{1}{16})\)

Answer: Check concavity. \( f_1 = x_2 - 2x_1, f_{12} = -2, f_2 = x_1 - 4x_2, f_{21} = 1, f_{22} = -4 \). Check \( f_{11} = -2 \leq 0 \) and \( f_{22} = -4 \leq 0 \). Okay. Check \( f_{11}f_{22} - f_{12}f_{21} = 8 - 1 \geq 0 \). The function is concave. Using partial differentiation and evaluating at point \((1, 1)\), the gradient at that point is

\[
\nabla f(1, 1) = \begin{pmatrix} -1 \\ -3 \end{pmatrix}
\]

That gives the direction of **steepest ascent**. Use \( t \) as a parameter for following that path up to its maximum. Here

\[
g(t) = f(1 - t, 1 - 3t) = (1 - t)(1 - 3t) - (1 - t)^2 - 2(1 - 3t)^2
\]

The \( g(t) \) is concave, as it must be since \( f \) is concave. The maximum of \( g(t) \) occurs where

\[
\begin{align*}
g'(t) &= 0 \\
-1(1 - 3t) - 3(1 - t) - 2(1 - t)(-1) - 4(1 - 3t)(-3) &= 0 \\
-1 + 3t - 3 + 3t + 2 - 2t + 12 - 36t &= 0 \\
10 - 32t &= 0 \\
t &= \frac{5}{16}
\end{align*}
\]

I think these numbers are right. You had better check them. Even if there is an error in them, you can finish successfully by explaining the steps briefly. At this point

\[
x_1 = 1 - \frac{5}{16} = \frac{11}{16}
\]

\[
x_2 = 1 - 3 \cdot \frac{5}{16} = \frac{1}{16}
\]

That is the starting point for the next step. A diagram would be nice.