Let \( r \) be equal to the last three digits of your registration number (PID, perm number), divided by 1000. Write your answers neatly in the space provided.

1. In the search problem, suppose that the offers are uniformly distributed on the interval \([r, r + 1]\). Use dynamic programming to find the equation that generates the critical value function \( A_t \), where \( A_t \) is the least offer that will be accepted at when there are \( t - 1 \) potential offers remaining. (Hint: in the example given in class, the critical value was generated by the difference equation

\[
A_t = \frac{1}{2} + \frac{1}{2}A_{t-1}^2
\]

You are supposed to derive the counterpart equation for the new case.)

Answer: The value function is \( f_t^*(w_t) = \max\{w_t, A_t\} \) where \( A_t \) is the least acceptable offer at this time \( t \). \( A_t \) is the expectation of future offers assuming that they are managed optimally

\[
A_t = E_{w_{t-1}}\{f_{t-1}^*(w_{t-1})\}
\]

My \( r \) is .678. The \( w_t \) are independently drawn from the uniform distribution on \([.678, 1.678]\).

\[
A_0 = .678
\]

Not \( A_0 = 0 \), which could lead to the wrong successor values.

\[
A_t = \Pr\{w_{t-1} \leq A_{t-1}\}A_{t-1} + \Pr\{w_{t-1} \geq A_{t-1}\}E\{w_{t-1}|w_{t-1} \geq A_{t-1}\}
\]

\[
A_t = (A_{t-1} - .678)A_{t-1} + (1.678 - A_{t-1})\frac{1}{2}(1.678 + A_{t-1})
\]

\[
A_t = A_{t-1}^2 - .678A_{t-1} + \frac{1}{2}((1.678)^2 - A_{t-1}^2)
\]

\[
A_t = \frac{1}{2}(1.678)^2 - .678A_{t-1} + \frac{1}{2}A_{t-1}^2
\]

Checking the values,

\[
A_0 = .678
\]

\[
A_1 = \frac{1}{2}(1.678)^2 - .678 \cdot .678 + \frac{1}{2} \cdot .678^2 = 1.178
\]

\[
A_2 = \frac{1}{2}(1.678)^2 - .678 \cdot 1.178 + \frac{1}{2} \cdot 1.178^2 = 1.303
\]

An alternative answer is not as good: Recognize intuitively that the solution is the same as in class except that .678 is added to each of the values. Incidentally, that can be checked. Let the old values be denoted by \( B_t \) which satisfies

\[
B_0 = 0
\]

\[
B_t = \frac{1}{2} + \frac{1}{2}B_{t-1}^2
\]

Now \( B_t + .678 \) satisfies

\[
B_t + .678 = \frac{1}{2} + \frac{1}{2}(B_{t-1} + .678)^2 + .678
\]

\[
= \frac{1}{2}1.678^2 + \frac{1}{2}(B_{t-1} + .678)^2 + .678 - \frac{1}{2}(1.678^2 - 1) - \frac{1}{2}(1.678^2 + 2 \cdot .678B_{t-1})
\]

\[
= \frac{1}{2}1.678^2 + \frac{1}{2}(B_{t-1} + .678)^2 - .678B_{t-1} + .678 - \frac{1}{2}(1.678^2 + 2 \cdot .678B_{t-1})
\]

\[
= \frac{1}{2}1.678^2 + \frac{1}{2}(B_{t-1} + .678)^2 - .678(B_{t-1} + .678) + .678 - .678
\]

\[
= \frac{1}{2}1.678^2 - .678(B_{t-1} + .678) + \frac{1}{2}(B_{t-1} + .678)^2
\]

which confirms the relation. I don’t expect the confirmation in student answers.
2. Attached is a p.d.f. of the tree-cutting problem discussed in class and also the working part of the excel worksheet for the problem. Using the excel worksheet, redo the analysis using as a discount factor $\beta = .94 - r/10$ where $r$ is derived as usual. The solution should fit on a few sheets of 8.5 by 11 paper printed in portrait format with your name and registration number in the upper right-hand corners. Explain each important decision in a few words.

Answer: See the accompanying excel sheet.