We recommend that you work in groups. After the group settles on a solution, each member should go apart and independently recreate the solution and format and turn in his or her unique product. Please, for your own benefit and in fairness to others, do not merely copy the group solution and turn it in. Neatly printed, well-organized answers will be rewarded. Do not turn in this sheet.

1. (Like the problem in class in the twelfth lecture, but be warned, the numbers are slightly different). This is the transport problem of a distribution firm. There are two supply sources $S_1$ and $S_2$, and three retail points $R_1, R_2,$ and $R_3$. The transport costs are given in the table. The requirements are given in the top row of the table, and the supply at $S_1$ is given in the left column. At $S_2$ the firm can buy as much as needed at a price of $60 per unit. The objective of the firm is to minimize the total cost of acquiring and distributing the required product to the three retail points.

<table>
<thead>
<tr>
<th>Supply</th>
<th>$R_1$</th>
<th>$R_2$</th>
<th>$R_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>700</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>$S_2$</td>
<td>765</td>
<td>8</td>
<td>2</td>
</tr>
</tbody>
</table>

Other firms are available to supply product at $R_1$ at some price. The question is this: how much should our firm be willing to pay to buy a unit of the product at $R_1$? You might solve this problem in various ways, but the requirement here is to solve it through linear programming. Answer in excel neatly printed in portrait configuration on one side of one sheet of 8.5 x 11 paper with your name in the upper right-hand corner. Explain carefully.

2. (Like 3.4 - 8 in Hillier and Lieberman, with minor modifications). Web Mercantile sells seasonal products. As it builds inventory in the slow months, it leases storage containers as needed. Requirements for storage containers in the next five months are

<table>
<thead>
<tr>
<th>Month</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Containers needed</td>
<td>30</td>
<td>20</td>
<td>40</td>
<td>10</td>
<td>50</td>
</tr>
</tbody>
</table>

The firm can lease containers month-to-month for a cost of $65 per container per month. In addition the firm can enter into leases of longer term that start and terminate at any of the five months. Monthly costs of leasing for one or more months are as follows:

<table>
<thead>
<tr>
<th>Leasing period</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost per container</td>
<td>65</td>
<td>100</td>
<td>135</td>
<td>160</td>
<td>190</td>
</tr>
</tbody>
</table>

The objective is to satisfy the storage requirements at minimum cost. Answer the following in excel neatly printed in portrait configuration on one side of one sheet of 8.5 x 11 paper with your name in the upper right-hand corner.

(a) Formulate as a linear programming problem. Explain the $x$ vector carefully. Identify the $b$, $c$, and $y$ vectors.

(b) Solve the primal and dual linear programming problems using solver in excel and, preferably, the templates developed in class.

(c) Interpret the dual variables with particular attention to those that are zero.

3. On one side of one sheet of 8.5 x 11 paper with your name in the upper right-hand corner, solve the following systems of equations using Gaussian elimination. Display all of the tableaux! Circle the pivot points. Show which vectors are in the basis at each step. Check the answers by matrix-vector multiplication.

(a)

\[
\begin{align*}
    x_1 + x_3 &= 10 \\
    x_2 + x_3 &= 3 \\
    x_1 + x_2 + x_3 &= 11
\end{align*}
\]
(b)

\[
\begin{align*}
x_1 + 2x_3 &= -2 & \quad (69) \\
x_1 + x_2 + x_3 &= 2 & \quad (70) \\
2x_1 + 3x_2 - x_3 &= 1 & \quad (71)
\end{align*}
\]