Suggested preparation for the first midterm
Economics 172A
Winter 2007

Some ground rules for home works and exams:

Write your homework answers on the sheets supplied. If necessary, you can get new sheets on the class web site.

All homework and examination items are answered with a limitation on space and a word count. Limitations on the homework items are typical of those on examination items. Write the best answer that you can in the space available. Writing that is clear and readable will be rewarded. As much as possible write in sentences and paragraphs. In many cases you will know more than can fit within the space and word allowance. You decide which parts are most important to write down.

You should outline the answer for yourself before writing it out for us. It also helps to connect the text to the diagrams and equations, which you do by labeling points in the diagrams (for instance by A, B, C, ...) or labeling equations (for instance, *, **, ***, ...) and then referring to the labels at the proper point in the text. For instance, you might write, "From the initial equilibrium of supply and demand, point A in the figure, the increase in demand leads to a new equilibrium at point B." You should practice this technique.

GENERAL COMMENTS

Your goal at this stage is to have a complete toolbox for solving small problems in linear programming. You need to know about the primal, the dual, the interpretation of the dual, the optimality theorem and its applications and the complementary slackness theorem and its applications. In my experience the hard parts are complementary slackness and interpretation of the dual. However, each student should take inventory of his or her present understanding and concentrate study on areas of weakness.

Later in the quarter we focus on problem formulation, but not now. Also, we return to game theory later but that won't be on this midterm.

I may ask a short question to verify your knowledge of excel. I could ask you to explain absolute and relative addressing or to describe the sumproduct function, or perhaps a combination of the two.

The midterm will be graded primarily on the strength of the explanations. Not much credit is given for having the correct numbers. In particular, in using the solution in the primal to solve the dual, or vice versa, you should write the words "Apply the complementary slackness theorem. Because the activities ... are non zero in the solution of the primal, the corresponding constraints, ... see below, are satisfied exactly. Because the constraints ... in the solution of the primal have positive slack, the corresponding variables in the dual ... are zero." Or words very much to that effect. A similar approach should be used in applying the optimality theorem.

I could ask you a general question like "What is the complementary slackness theorem and what are its applications?" Such a question does not necessarily require a proof, but giving a sketch of the proof would be a nice touch in the context of answering the question. The answer to the question would of necessity deal with applications.

SOME PROBLEMS TO TRY

Skip the game theory problem for now. In the other problems worked in class, you could try varying the parameters and reworking the problem. That is an excellent way to become comfortable with revenue maximization (Wyndor Glass) and cost minimization.

In chapter three of my edition of the Hillier and Lieberman text there is a problem on regional planning and an air pollution problem. Those are very nice problems for you to work. Although the text does not do this, you should write and solve the dual, using the solution to the primal and applying the complementary slackness theorem. The text does not help you with these tasks, but based on the problem sets you have turned in, I think you can succeed.

Also at the end of chapter three are four problems to which you can apply the whole toolbox. All four involve graphic solutions. There are two maximizations and two minimizations. In my edition the maximizations are numbered 3 and 4, and the minimizations are numbered 8 and 9.

You can also try the following problems, which I am borrowing from Gale's book.

1. Construct a standard maximum problem involving two inequalities and two unknowns which has more than one optimal solution although not every feasible solution is optimal.
2. Consider the problem below. Use the complementary slackness theorem to solve the dual and then show that the primal problem has optimal solution \(x_1^* = 1, x_2^* = 1, x_3^* = 0, \) and \(x_4^* = 1\).

Maximize \(x_1 + x_2 + x_3 + x_4\)  
subject to \(x_1 + x_2 \leq 3\)  
\(x_3 + x_4 \leq 1\)  
\(x_2 + x_3 \leq 1\)  
\(x_1 + x_3 \leq 1\)  
\(x_3 + x_4 \leq 3\)  
\(x_1, x_2, x_3, x_4 \geq 0\)

3. Write the duals of the following problems. Interpret the duals and the dual variables. Solve the primal by graphical means and then use complementary slackness to solve the duals. Explain, of course.

(a)  
Maximize \(2x_1 + x_2\)  
subject to \(x_2 \leq 10\)  
\(2x_1 + 5x_2 \leq 60\)  
\(x_1 + x_2 \leq 18\)  
\(3x_1 + x_2 \leq 44\)  
\(x_1, x_2 \geq 0\)

(b)  
Maximize \(10x_1 + 20x_2\)  
subject to \(-x_1 + 2x_2 \leq 15\)  
\(x_1 + x_2 \leq 12\)  
\(5x_1 + 3x_2 = 12\)  
\(x_1, x_2 \geq 0\)

4. Write the duals of the following problems. Interpret the duals and the dual variables. Solve the primal by graphical means and then use complementary slackness to solve the duals. Explain, of course. The unexpected directions of inequality in both problems and an equality constraint in part b create some interesting challenges for you when you come to solving the dual.

(a)  
Minimize \(15x_1 + 20x_2\)  
subject to \(x_1 + 2x_2 \geq 10\)  
\(2x_1 - 3x_2 \leq 6\)  
\(x_1 + x_2 \geq 6\)  
\(x_1, x_2 \geq 0\)

(b)  
Minimize \(3x_1 + 2x_2\)  
subject to \(x_1 + 2x_2 \leq 12\)  
\(2x_1 + 3x_2 = 12\)  
\(2x_1 + x_2 \geq 8\)  
\(x_1, x_2 \geq 0\)