The Impact of Individual Investment Behavior for Retirement Welfare:
Evidence from the United States and Germany

Thomas Post, Helmut Gründl, Joan T. Schmit, and Anja Zimmer

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Abstract Much of the industrialized world is undergoing a significant demographic shift, placing strain on public pension systems. Policymakers are responding with pension system reforms that put more weight on privately managed retirement funds. One concern with these changes is the effect on individual welfare if individuals invest suboptimally. Using micro level data from the United States and Germany, we compare the optimal expected lifetime utility computed using a realistically calibrated model with the actual utility as reflected in empirical asset allocation choices. Through this analysis, we are able to identify the population subgroups with relatively large welfare losses. Our results should be helpful to public policymakers in designing programs to improve the performance of privately organized retirement systems.

Keywords Asset Allocation, Retirement Welfare, Pension Reform

JEL-Classification D14, D91, G11, G28, I31

Helmut Gründl, Thomas Post (contact author), and Anja Zimmer are with the Humboldt-Universität zu Berlin, Dr. Wolfgang Schieren Chair for Insurance and Risk Management, sponsored by Allianz SE and Stifterverband für die Deutsche Wissenschaft, phone: +49-30-2093-9922; fax: +49-30-2093-5616; e-mail: tpost@wiwi.hu-berlin.de. Joan T. Schmit is the American Family Insurance Chair in Risk Management and Insurance at the University of Wisconsin-Madison, School of Business, phone: +1 608 262-4240, e-mail: jschmit@bus.wisc.edu.
1 Introduction

Much of the industrialized world is undergoing a significant demographic shift, with low birth rates and increasing longevity (see, e.g., United Nations, 2007). One consequence of the aging population is an increasing strain on public pension systems. Many such systems are pay-as-you-go plans, created under the expectation that inflow would be generated from current workers whose number exceeds the population of retirees. Today’s reality is a shrinking supply of workers relative to retirees. As a result of these changing demographics, concern is rising over the ability of government pay-as-you-go programs to remain viable.

Policymakers are responding with a variety of pension system reforms. The general trend is to put more weight on funded, individually organized retirement systems. Shifting the burden of old-age provision to individuals raises a central question: How well are various population subgroups prepared to make financial decisions with respect to their old-age provision? To answer this question, we use micro-level data and investigate the performance of individual retirement wealth accumulation. Within the wealth allocation process, we focus on asset allocation. Taking the existing government pension systems as given, we show the population subgroups for which public policy should implement improvement measures.

We further compare the performance of U.S. and German investors. This comparison is especially valuable because of the institutional differences in the respective retirement systems. The United States has a longer tradition for privately funded retirement systems, because government pensions historically have been less generous. Thus, we can investigate whether longer experience with individually
funded systems leads to better asset allocation results. This further helps to determine in which direction reforms should alter a pension system—that is, toward more individually managed funding (and income provision) or not.

To evaluate investment performance, we use a utility-based welfare benchmark. Alternative monetary-based benchmarks such as expected wealth (or quantiles of the distribution) at retirement (e.g., as in Poterba et al., 2007, or in Watson and Naughton, 2007) are not suitable because we assume heterogeneity in individual preferences and endowments. For example, for people with different degrees of risk aversion, it should be completely rational to follow asset allocation strategies with different types and amounts of risk and thus accept different levels in expected retirement wealth. Focusing only on expected retirement wealth, reforms may outweigh high-risk/high–expected wealth strategies.

To calculate our welfare measure, we employ a method similar to Dammon, Spatt, and Zhang (2004); Cocco, Gomes, and Maenhout (2005); and Yao and Zhang (2005). As such, we compare the expected lifetime utility an individual receives following an optimal asset allocation pattern with the expected lifetime utility received following the investment strategy observed in our data. We achieve this by taking the following steps: First, we analyze data from two large data sets, the U.S.-based Survey of Consumer Finances (SCF) and the German Income and Expenditure Survey (EVS). Using a regression model, empirical asset allocation policy is estimated as a function of individual characteristics (e.g., age, gender, education) and endowments (income and wealth). Next, we calculate the optimal—that is, benchmark—asset allocation policy and the resulting expected utility. For this we solve the dynamic optimization problem given by a realistically calibrated life-cycle consumption/saving/asset allocation model with stochastic uninsurable labor income, asset returns, and life span. Finally, we place the empirical asset allocation policy functions, instead of the optimal functions, into the expected utility model and compare the resulting utility with the optimal one.

Our results are highly relevant for policymakers in their deliberations about changes to public and private pension systems. We are able to identify the population

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Security Administration, 2006). In Germany, the government system (Gesetzliche Rentenversicherung) in 2003 provided around 66% of retirement income (Federal Ministry of Health and Social Affairs, 2005).
subgroups with relatively large welfare losses through suboptimal asset allocation decisions. Specifically, individuals in the United States with low financial wealth and in Germany with low income or medium wealth would benefit most from better asset allocations. Further analyses exhibit the impacts of gender, education, and age.

The paper is organized as follows: A review of the literature in section 2 is followed in section 3 by the definition and calibration of the normative benchmark model. Section 4 describes the data used for our empirical investigation, and results of our econometric analyses are presented in section 5. Building on the results of sections 4 and 5, a welfare analysis—the utility-based comparison of theoretically optimal and actual behavior—is done in section 6. We summarize our results and derive policy implications in section 7.

2 Relationship to Existing Literature

As summarized by John Y. Campbell in his 2006 presidential address to the American Finance Association (Campbell, 2006), there are two general approaches to research on household finance: positive research, which investigates the actual behavior of people, and normative research, which aims to derive how people should behave according to a set of rational criteria. Our contribution confronts the former with the latter by measuring differences in utility (measured using a normative model) for individuals whose asset portfolios we observe deviate from the normatively given optimal asset allocation.

2.1 Empirical Evidence – the Positive Research

Because of their fundamental role in decision making under uncertainty, risk attitudes are of significant interest to policymakers and economists. Variations in attitudes across demographic and socioeconomic characteristics are relevant because of the ultimate implications in setting public policy, particularly with regard to financial, occupational, and similar decisions. Empirical evidence suggests that gender, marital status, education, ethnicity, wealth, income, and age are among the relevant factors associated with risk-taking behavior.

Halek and Eisenhauer (2001) use the University of Michigan Health and Retirement Study to test the relationship between risk attitudes and a variety of demographic
characteristics, both as observed in life insurance purchases and in a speculative job choice option. They observe that women are more risk averse than men in both domains, and that risk aversion is U-shaped in age. Higher education appears to increase risk aversion with regard to pure risk; yet, regarding speculative risk, education tends to increase the willingness to accept it. Halek and Eisenhauer (2001) also observe that immigrants tend to be greater risk takers than nonimmigrants. They attribute this behavior to the self-selection aspect of immigrating in the first place.

In the pension domain, Bajtelsmit and VanDerhei (1997) use data on asset allocation choices in U.S. pension plans and find that investments in risky assets increase if the respondent is male, middle-aged, and wealthy. Hinz, McCarthy, and Turner (1997) find that women exhibit more risk aversion when making choices for assets in a U.S. pension fund. Employing a proprietary data set of U.S. 401 (k) pension plan choices, Agnew, Balduzzi, and Sunden (2003) find that men invest more in equities and trade more frequently than do women. They also find that being married and having higher income is associated with more aggressive investing behavior. Older age, however, leads to greater caution in asset allocations. Van Rooij, Kool, and Prast (2007) show that, in the Netherlands, men are generally less risk averse and would (if no restrictions forbade it) invest more in stocks than women and prefer a defined contribution plan to a defined benefit plan. For Australian investors, Watson and Naughton (2007) show that men choose riskier retirement plans.

In the individual investment domain, using data from the 1998 wave of the Survey of Consumer Finances (SCF), Sunden and Surette (1998) investigate probabilities to invest mostly in stocks or mostly in bonds. Their results highlight marital status as a driver for investment behavior. Single men are more likely than single women and married men to invest mostly in stocks; married women, however, do not differ

5 The overview of the psychological literature given in Byrnes, Miller, and Schafer (1999) summarizes broad evidence that women exhibit more risk aversion than men.

significantly from other groups with respect to the mostly stock probability. Single women are less likely than married women to choose mostly bonds. Using the same data set, Jianakoplos et al. (2003) find that not marital status per se but differences in financial endowments between married and nonmarried persons influence investment behavior.

Using a large (22,000 respondents) German survey data set, supplemented with field experiments, Dohmen et al. (2005) find that risk aversion is negatively related with age and being female, and that these results are robust to income differences. In an interesting addition, the German survey allows consideration of risk attitudes across domains of career selection, sports and leisure, car driving, health, and financial matters. Results of the study suggest that domain matters in that the level of risk aversion or risk loving changes across situations, and financial lotteries do not appear to be particularly good predictors of actual behavior. Women are more risk averse than men in each domain.

Guiso, Haloassos, and Japelli (2002) contains studies on asset allocation choices in several countries. The results presented reveal similarities but also differences across national borders, giving weight to our consideration of both U.S. and German data. Again, age, income, wealth, and gender turn out to be important factors.

According to the literature, therefore, gender, marital status, education, ethnicity, wealth, income, and age all seem to affect risk-taking behavior.7

2.2 Optimal Asset Allocations and Individual Welfare Considerations – the Normative Analysis

2.2.1 Optimal Asset Allocations

During the 1960s and 1970s, researchers gave significant attention to the problem of optimal life-cycle allocation of resources between consumption and saving and across types of assets (Phelps, 1962; Yaari, 1965; Mossin, 1968; Hakansson, 1969, 1970).

7 For further survey results see Barsky et al. (1997) and Eisenhauer and Ventura (2003).
1970; Merton, 1969, 1971; Samuelson, 1969; Richard, 1975; Kotlikoff and Spivak, 1981). While optimal decision rules were economically logical—such as constant, age-invariant, proportional asset allocation for intertemporally separable constant relative risk aversion (CRRA) utility—they often failed to mirror actual behavior. The challenge posed to all researchers has since been to relax various rigorous assumptions that do not hold in reality, such as intertemporally separable utility, infinite time horizon, or perfect markets, where the individual can span all assets including human capital, or the absence of borrowing or short-selling restrictions.

A growing body of research now offers optimal decision rules with increasing relaxation of such assumptions. Based on realistically calibrated consumption-saving-asset allocation intertemporal optimization models, decisions depend on gender, education, age, human capital (i.e., the stock and riskiness of labor/pension income and pension age), wealth, taxation, transaction costs, and the likelihood of binding borrowing or short-selling restrictions.\(^8\) A detailed discussion of the specific impact of these factors based on the normative model used here is given in section 3.3.

### 2.2.2 Individual Welfare Considerations

Section 2.1 presented evidence that individual investment behavior differs according to various factors such as age, wealth, income, and gender. Not surprisingly, different investment strategies lead to different wealth outcomes—for example, wealth at the beginning of retirement. More conservative investment strategies result in lower expected final wealth; for example, the simulation by Watson and McNaughton (2007) shows for Australian investors that women, being more-risk averse, end up with lower expected retirement wealth.

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\(^{8}\) See Zeldes (1989); Deaton (1991); Carroll (1992, 1997); Hubbard, Skinner, and Zeldes (1994, 1995); Heaton and Lucas (1997, 2000); Laibson, Repetto, and Tobacman (1998); Viceira (2001); Campbell and Viceira (2002); Blake, Cairns, and Dowd (2003); Gomes and Michaelides (2003, 2005); Haliassos and Michaelides (2003); Dammon, Spatt, and Zhang (2004); Lachance (2004); Cocco (2005); Cocco, Gomes, and Maenhout (2005); Davis, Kubler, and Willen (2005); Yao and Zhang (2005); Horneff, Maurer, and Stamos (2006); Post, Gründl, and Schmeiser (2006); and Polkovnichenko (2007).
But, as section 2.2.1 pointed out, differences in investment behavior can be a completely rational reaction to differences in preferences or endowments. For example, a highly risk-averse individual may be completely happy with low expected final wealth, if the volatility of the wealth distribution is also sufficiently low.

Thus, a comparison of investment strategies should incorporate the rational contribution of preferences and endowments. For this, the literature uses the concept of lifetime utility. The actual behavior (in this study, the behavior is asset allocation) thus is compared with some benchmark behavior derived using the normative lifetime utility model.

Research on comparisons of investment strategies and behaviors (including this contribution) measures utility costs—that is, losses in welfare—associated with deviations from the normatively determined optimal behavior. Among the important works are Dammon, Spatt, and Zhang (2004); Cocco, Gomes, and Maenhout (2005); and Yao and Zhang (2005). Dammon, Spatt, and Zhang (2004) compare the expected lifetime utility of commonly observed investment choices with optimal choices. They undertake these measurements with respect to taxable and tax-deferred accounts. Cocco, Gomes, and Maenhout (2005) compare common investment advisers’ recommendations with the optimal choices and furthermore evaluate the effect of ignoring labor income or labor income risk while deriving the optimal choice. The welfare costs of not following an optimal renting versus owning a house strategy are calculated by Yao and Zhang (2005).  

3 Optimizing Asset Allocations and Wealth over the Life-Cycle – The Normative Benchmark Model
In this section, the normative benchmark model is defined and calibrated with empirical data. We derive the optimal asset allocation—depending on the

9 Poterba et al. (2006) also use a utility-based benchmark for investigating the performance of DB and DC plans, but restrict the measurement of utility to the wealth distribution at one single point in time (age 63), thus abstracting from life-cycle effects.
individual’s characteristics—over the life cycle. Based on this model, hypotheses for the econometric analyses are formulated in section 5.1.

3.1 The Individual’s Problem

For our normative analysis, we use the workhorse for solving intertemporal allocation problems, discounted utility. The individual maximizes the expected utility of consumption $C$ (all monetary variables are stated in nominal terms) over his or her stochastic life span. The intertemporally separable utility function $U(C)$ is defined as:

$$U(C) = \sum_{t=0}^{T-x} \delta^t \left( \prod_{i=0}^{t} p_i \right) U_t(C_t).$$

(1)

$T$ denotes the maximum life span, $x$ the current age, $\delta$ the subjective discount factor, and $p_t$ the probability of the individual to survive from period $t-1$ to $t$. We assume no bequest motives; thus, the one-period CRRA-utility function $U_t(C_t)$, with $\gamma$ as the coefficient of relative risk aversion, is given by:

$$U_t(C_t) = \begin{cases} \log \left( \frac{C_t}{(1+\pi)^t} \right), & \text{with } \gamma = 1 \\ \left( \frac{C_t}{(1+\pi)^t} \right)^{1-\gamma}, & \text{otherwise,} \end{cases}$$

(2)

as long as the individual lives and 0 otherwise. $C_t$ stands for nominal consumption at time $t$ and is adjusted for inflation at rate $\pi$.

At each point in time $t$, the individual decides how much to consume (implicitly determining savings) and how to allocate savings. Financial wealth at time $t$ is denoted by $W_t$, henceforth called “cash on hand” (Deaton, 1991). Savings $S_t$ are allocated to both a risk-free investment and a risky investment. The proportion of savings invested riskily each period, $\alpha_t$, earns the risky return $R_t$, whereas the rest $(1-\alpha_t)$ is compounded at the risk-free return $R_f$. We assume that the individual cannot borrow money or short-sell stocks. The individual earns stochastic labor income $L_t$. 
from age $x$ to age 64 at the end of each year $t$. In later periods, from age 65 to $T$, $L_t$ is replaced by a deterministic (government) pension income that stays constant in real terms. Thus, the retirement age is exogenously fixed at age 65.

The maximization problem is given by:

$$\max_{\alpha, \ell_t} E_0 \left( U(C) \right),$$

subject to consumption constraints:

$$C_0 = W_0 - S_0$$

$$C_t = S_{t-1} \left(1 - \alpha_{t-1}\right) R_f + S_{t-1} \alpha_{t-1} R_{1-t} + L_{t-1} - S_t \quad \forall \quad t \in \{1, 2, ..., T - x\},$$

subject to borrowing constraints:

$$0 \leq S_t \leq W_t,$$

and subject to no-short-sale constraints:

$$0 \leq \alpha_t \leq 1.$$

### 3.2 Calibration

In this section we calibrate our model for U.S. and for German individuals. We report the choice of our benchmark parameters, but also give alternative values that will be used for sensitivity analysis later in the paper. Table 1 summarizes the calibration.

The individual’s preferences are described by setting the constant of relative risk aversion $\gamma$ to 2 (alternatively to 1 or 3), the subjective discount factor $\delta$ to 0.97 (alternatively to 0.95 or 0.99), which are typical values found in intertemporal optimization models (see, e.g., Laibson, Repetto, and Tobacman, 1998).

For the U.S. survival probabilities, we use the United States Life Tables 2003 (see Arias, 2006); for German survival probabilities, we use the Life Table for Germany 2002/2004 from the German Federal Statistical Office (see Federal Statistical Office,
2003). Both tables reflect average population mortality, have a maximum age of 100 years, and are subdivided for males and females.

As proxy for the risky asset, we use the return of a broad-based stock market index. Stock market returns are assumed to be lognormal and i.i.d. (see, e.g., Hull, 2005). For the United States, we use data from 1926 to 2006 from Morningstar (see Morningstar, 2007). After deducting typical transaction costs of an index-investment fund of 0.7% per annum, the mean of $R_t$ is given by 1.1151, and the standard deviation is 0.1996. For Germany, we use 1955–2006 data provided by Professor R. Stehle, Ph.D., Chair of Banking and Stock Exchanges, Humboldt-Universität zu Berlin (Germany), which give mean 1.1264 and standard deviation 0.2792 for $R_t$ after assuming identical transaction costs.

For the risk-free asset return, the short-term money market is used as a proxy. Given the same sample periods, the $R_f$ is set to 1.0361 per annum for the United States (see Morningstar, 2007) and to 1.0472 for Germany (see IMF International Financial Statistics Online database, http://ifs.apdi.net/imf), again after assuming typical transaction costs of an index-based investment of 0.18%.

For inflation, we use the same sample period, resulting in a value of 0.031 for the United States (see Morningstar, 2007) and 0.0279 for Germany (see Federal Statistical Office, 2007).

The labor income process is calibrated to match empirically observed life-cycle income profiles. Ideally, we would like to have estimates that reflect the labor income process as given by data used also in the regression analysis later. But our data are cross-sectional and thus not well suited for this. Panel data that include both detailed longitudinal information on labor income and asset allocation for longer sample periods are, especially for Germany, not available. Thus, we decided to take income profiles from the literature that uses panel data.
For the United States, the mean real growth rates of income during the life cycle before retirement (until age 64) are taken from Cocco, Gomes, and Maenhout (2005). The profiles are age and education (low, middle, high), but not gender specific. For Germany, we use profiles based on Fitzenberger and Wunderlich (2002) and Behr et al. (2003). These profiles are age, education (low, middle, high) and gender specific. In both, the United States and Germany, real income profiles are in general hump-shaped in age; in nominal terms (applying the inflation rate defined above), they can be increasing in age. During retirement, labor income is exogenously replaced by (government) pension income by multiplying the income at age 64 with a replacement factor. For this we use “prospective” replacement factors, reflecting expected future replacement ratios. For the United States, we use a value of 35% (see Reno and Lavery, 2007), for Germany we use a value of 40% (see Börsch-Supan and Wilke, 2004).

To reflect the fact that labor income is risky, we modeled each period’s labor income to be lognormally distributed and subject to transitory shocks. The mean of $L_t$ is given by the current income at $t = -1$ with the growth rates and inflation described above. Until age 64, the standard deviation for U.S. individuals was set to $0.19 \cdot E(L_t)$ (see Carroll and Samwick, 1997). For German individuals, we do not have empirical estimates and used $0.05 \cdot E(L_t)$, which should reflect that in the German welfare state, income risks are comparatively lower than in the United States. From age 65—that is, during retirement—we assume no labor income uncertainty.

For our calculations we finally assume no taxes.

Table 1 summarizes the calibration of the model parameters.

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10 The literature contains much controversy about whether shocks to labor income are permanent or transitory. Newer empirical evidence gives mixed results (see, e.g., Guvenen, 2007). Because using transitory shocks makes the computational solution of the optimization problem much faster, we implemented only transitory shocks.
Table 1: Parameter Calibration for the Benchmark Model

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The optimization problem (3)–(6) is solved backward via stochastic dynamic programming. Further details are given in Appendix A.

3.3 Asset Allocations According to the Benchmark Model

In this section, we show how individual optimal asset allocations—that is, the percentage share of cash on hand invested into the risky asset—are influenced by the various input parameters. The results will serve as hypotheses for the empirical estimations in section 5 and build the basis for analyzing welfare effects in section 6. We describe the impact of risk aversion, the subjective discount factor, the survival probability, age, education, gender, the capital market environment, cash on hand, and expected labor income. We begin with an explanation of the effect of cash on hand and expected labor income, because some of the other effects depend on the ratio of expected labor income to cash on hand.

Whereas in a model without labor income the individual’s risky asset share is age, time, and cash on hand invariant, here, the risky asset share increases in the labor income–to–cash on hand ratio, because labor income serves partially as a risk-free asset (Viceira, 2001; Cocco, Gomes, and Maenhout, 2005) and due to diversification effects (Gollier, 2001). Note that the CRRA feature of the one-period utility function still can result in an asset allocation that is invariant with respect to some fixed value of the labor income–to–cash on hand ratio, if the cash on hand and labor income are perfectly scaled. This means that, for example, while doubling cash on hand and expected labor income, the asset allocation stays constant, if expected labor income is doubled in all future periods.¹¹

¹¹ Note that, if leaving, for example, the discounted value of expected labor income constant but changing the shape of the life-cycle income profile over the life cycle would, in general, result also in a change in asset allocation, because the likelihood of binding no-short-sale or borrowing constraints in future periods is changed.
A typical shape of the investment into the risky asset as a function in the labor income–to–cash on hand ratio is shown in Figure 1.

Figure 1: Investment into the risky asset as a function in the labor income–to–cash on hand ratio

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The impact of risk aversion on the risky asset share is straightforward. The risky asset share decreases with increasing values for $\gamma$. The impact of a change in the subjective discount factor $\delta$ depends on the individual’s expected labor income. Both increases and decreases of the risky asset share are possible. In general, increasing $\delta$ will result in higher savings, because the individual places more weight to future utility. By saving more, the individual’s expected labor income decreases relative to the higher saving–induced increase in cash on hand, thus resulting in a lower risky asset share. But putting more weight on future utility also implies that the individual’s value of future labor income is larger (less discounted), and thus the risky asset share can also increase. The overall effect in general is rather small.

Having a higher life expectancy—that is, higher one-period survival probabilities $p_t$ (like women compared to men)—is similar to an increase in $\delta$. Thus, the overall effect is small and ambivalent.

Increasing age changes the risky asset share via two channels. First, future labor income is less heavily discounted, leading to a higher risky asset share. Second, the amount of expected future labor income decreases, leading to a lower risky asset share. The overall effect depends on the individual’s saving decisions and the resulting cash on hand in later periods relative to expected labor income, but in general we can expect a decreasing effect of age on the risky asset share. This is illustrated in Figure 2. While holding the labor income–to–cash on hand ratio constant, the risky asset share follows the discounted value of future labor income.
(the kink around 65 results from the drop in income during retirement). Because, in general, the labor income–to–cash on hand ratio will decrease over the life cycle, the final risky asset share will be given by moving from higher to lower curves of the labor income–to–cash on hand ratio curves, as indicated by the downward-sloping arrow.

**Figure 2: Investment into the risky asset as a function in age**

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The effect of education on the risky asset share in our model is solely driven by differences in expected future labor income. Higher education is associated with higher growth rates of labor income during the life cycle, resulting in higher expected labor income (for a given current income), leading to a higher risky asset share.

Gender enters the benchmark model at several points. Women have a higher life expectancy and lower expected growth rates for future labor income than men (assuming for the moment no differential in relative risk aversion). Ignoring the rather small effect of higher survival probability of women, their risky asset share should be smaller than the share of men. Assuming further that women are more risk averse (higher $\gamma$), we further expect a smaller risky asset share.

The effect of capital market conditions is highlighted by comparing U.S. and German capital market data in Table 1. U.S. and German investors can expect a similar equity risk premium (nominal around 8%; in real terms around 5%), but at a considerably smaller standard deviation. Consequently, U.S. individuals should invest a higher share into risky assets. For example, an American investor with zero labor income (other parameters at base values) would invest 100%, and a German investor would invest around 61% into the risky asset.
After deriving these general theoretical results, we continue with a description of the data used in our regressions.

4 Data Description

4.1 U.S. Data: The Survey of Consumer Finances
For the United States, we use data from the 2004 wave of the Survey of Consumer Finances (SCF). The data set contains detailed information on 4,519 households, including household demographics, asset allocation and liabilities, income, and other characteristics.12 Data have been collected by a dual frame sample design. Data for about 3,000 households are drawn from a representative sample of households in the United States to reflect characteristics that are broadly distributed in the population, such as home and vehicle ownership. The other set of 1,500 survey cases are drawn from an oversampling of wealthy households (based on tax records) to represent characteristics such as investment behavior, which might be disproportional in wealth. Furthermore, missing values are systematically imputed by a multiple imputation technique, so that the data set includes 22,595 records (i.e., 4,519 cases times 5 implicates).

4.2 German Data: Income and Expenditure Survey
For Germany, we employ data from the 2003 wave of the Income and Expenditure Survey (EVS). The available data set for scientific use also includes numerous data on income, asset allocation, liabilities, and expenditures of 42,744 private households.13

4.3 Data Selection
Because we are particularly interested in individual differences in investment behavior, we only analyze data on persons that are neither married or live with a partner. This assures that the decision observed was made by, for example, either a

12 See Bucks, Kennickell, and Moore (2006) for an overview of the SCF.
13 In the 2003 survey, 53,432 households were originally interviewed. The data set for scientific use, though, was made anonymous, which has resulted in an exclusion of 20% of the household data.
woman or a man (among other characteristics of a respondent) and enables us to extract individual-specific characteristics from the data.\textsuperscript{14} We exclude all self-employed individuals from the analysis. For the United States, self-employed individuals represent 7.9\% (based on the weight given in the data set); for Germany, 4.3\% of individuals in the data set are self-employed. This exclusion was necessary, because, in the German data, the value of one’s own business—a major part of the asset allocation among those who are self-employed—is not reported. Furthermore, we excluded cases of income and net worth distribution below zero.

Another restriction is given by the intertemporal optimization model of section 3, where these data and regression results will serve as input in section 6. The optimization model does—as a standard assumption—not allow for borrowing and short-selling. Deleting individuals having debt would exclude the majority of U.S. respondents, which is not desirable. Instead, we deducted debt from the amount of investments made into the risky asset—that is, we treated debt as a negative asset. Thus, finally we had to exclude individuals for whom the resulting risky asset share variable used in our optimization (see section 3.1) and regression (see section 5.2) models was below 0\% (18.8\% of weighted cases for the United States and 9.2\% for Germany).

Our final SCF data set reduces to a sample size of 871 cases (4,355 records divided by 5 implicates), and the EVS data set contains 10,316 cases.

\textbf{4.4 Descriptive Statistics}  
Table 2 shows country-specific descriptive statistics on demographic and financial characteristics of our sample. The variables finally used in our regression analyses are in italics. For the U.S. data, the statistics are based on the weight given in the data set to account for the oversampling of wealthy.

\textsuperscript{14} Using households with more than one person would make inferences about individual behavior almost impossible. Household decisions would reflect some kind of average preferences and would implicitly incorporate numerous not directly observable diversification effects (e.g., with respect to labor income risks or life span uncertainty).
Table 2: 2004 Survey of Consumer Finances (SCF) and 2003 Income and Expenditure Survey (EVS) (for definition of variables, see Appendix B)

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The descriptive statistics in Table 2 include two variables indicating the risky asset share. The first, risky 1, is a more traditional measure (e.g., as in Bertaut and Starr-McCluer, 2002, or Eymann and Börsch-Supan, 2002) including stocks and similar investments (see Appendix B). The second, more broadly defined measure, risky 2, includes the value of real estate and, as negative investments into risky assets, debt (assuming that debt is risky). Risky 2 will be used in our regressions later, and thus we concentrate on this measure in the following discussion.

Comparing the United States and Germany, we observe that Americans invest a higher share into the risky asset (58% vs. 39%). But we also see that the variables referring to financial wealth and income vary to a great extent in both countries, and, thus, as the benchmark model revealed, a regression model should control for that.

5 Econometric Analyses
5.1 Hypotheses about the Investment Process and Implications for Regression Models
In the literature (e.g., Bertaut and Starr-McCluer, 2002; Eymann and Börsch-Supan, 2002), two general hypotheses about the investment process are discussed. The first assumes that the choice to invest into risky assets is made simultaneously with the decision on the share of wealth invested. The second hypothesis assumes a two-stage investment process. At the first stage, the individual decides whether to invest in the risky asset; at the second stage, the share is independently derived. The rationale for this two-stage process is that, before investing in risky assets, costly information must be obtained (e.g., college education); if these costs are prohibitively high, no investment is made at all.
The normative benchmark model from section 3 is based on a simultaneous decision process. Consequently, we use an empirical estimation strategy that is compatible with such behavior, a tobit model (Tobin, 1958). This model takes into account that the dependent variable (percent risky 2) is censored at 0 and 1. The underlying economic interpretation is that individuals having the value of, for example, 0 for the risky asset share would actually invest negative shares, but are restricted from doing so.

5.2 Variable Selection and Expected Signs of Coefficients

The selection of variables is based on the input variables of the normative benchmark model of section 3. The dependent variable is the risky asset share of investments, “percent risky 2”. Our dependent variable includes the usual risky assets, but also real estate and with a negative sign debt. Of course, houses are not as liquid as stocks, but there is some evidence that individuals adjust their house size according to their individual situation—that is, they consider the possibility of trading in real estate (see Banks et al., 2007). The final variable percent risky 2 is obtained by dividing risky assets by net worth, thus considering debt in both the numerator and denominator implicitly.

As indicated in Table 2, the independent variables are those that are good candidates to be compatible with the input for the benchmark model. Thus, we included age, age squared, dummy variables for gender (female vs. male), education (low or high vs. middle), occupation (unemployed or retired vs. employed), the ratio of noncapital income to net worth (as a proxy for the labor income–to–cash on hand ratio), and,

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15 To include the housing (e.g., Cocco, 2005; Yao and Zhang, 2005) or debt (e.g., Davis, Kubler, and Willen, 2005) decision or both simultaneously (De Jong, Driessen, and Van Hemert, 2007) into a normative model is possible in general, but it would render our optimization approach computationally intractable, due to the large number of cases to be considered (see section 6).
finally, allowing for non-CRRA behavior, the logs of noncapital income and net worth.\footnote{For a CRRA investor, the ratio of noncapital income to net worth alone should be sufficient to determine the risky asset share.}

From the general results of the benchmark model in section 3.3, the expected sign of the combined effect of age and age squared is negative, although after controlling for income and cash on hand, in general, the outcome cannot be sufficiently modeled by a linear regression (precisely, the individual should move along a line in Figure 2). For the gender dummy variable, the expected sign could be negative due to risk-aversion differentials, but the opposite is also possible due to life-expectancy differentials. Due to expected labor income (growth) differentials according to education, the expected sign of low education is negative and of high education is positive. The expected sign of the occupation retired dummy variable is ambivalent, since this variable works like an age dummy variable, distinguishing whether one is in the first or second hump in the age-asset-allocation curve in Figure 2. The occupation unemployed dummy variable should have a negative sign, capturing lower expected future labor income effects. Noncapital income and net worth should both have zero coefficients, and the noncapital income/net worth should be positive if the individual acts according to the CRRA benchmark. Otherwise, the combined effect of the three variables should at least imply an upward-sloping curve as shown in Figure 1 for CRRA-similar behavior.

Finally, U.S. investors should (due to the better risk-return trade-off of the risky asset) for any values of the explanatory variables invest a higher share into the risky asset than German investors.

5.3 Regression Results and Discussion

Table 3 displays the results of our regression analyses. Variables with insignificant coefficients were removed from the regression equations in order to have the
simplest possible model as predictor for the risky asset share for the final model that measures the welfare effects.

**Table 3: Determinants for Share of Risky Assets (Tobit Regression); Dependent Variable: Percent Risky 2 (for definition of variables, see Appendix B)**

--- put Table 3 here ---

The age effect (age, age squared) for the United States is hump-shaped, peaking at an age of 53; for Germany, only the age coefficient is significant, implying a downward-sloping age effect. Both results are in general compatible with the predictions, but—considering behavior according to the benchmark model—again it can be doubted whether a linear regression is able to catch all effects for age.

The gender dummy variable has the expected sign for Germans: women invest 4 percentage points less into risky assets after controlling for other effects, indicating more risk-averse behavior. The U.S. results are surprising: women invest 6 percentage points more in risky assets. Thus, our analysis to this respect contradicts previous evidence. Similarly, the German education coefficients show the expected sign, but the opposite is true for the U.S. education coefficients. The occupation dummy variables were insignificant in all regressions.

The combined effect of labor income and cash on hand is clearly diametrical to the normative model for U.S. individuals. For any combinations of noncapital income and net worth, the risky asset share is a negative function in the noncapital income–to–net worth ratio (compare Figure 1). For German investors, the results are mixed. While varying the noncapital income–to–net worth ratio, the resulting function of the risky asset share is u-shaped; thus, only after reaching the minimum did it match the predictions of the benchmark model. Because both the coefficients for the logs of noncapital income and net worth are significant, the location of the minimum depends on the absolute values of the two variables and not only on their ratio. In
general, we need ratios of noncapital income to net worth—for example, 15 to 75—to reach the minimum (from the left).

The hypothesis that U.S. investors should invest a higher share into risky assets than Germans (due to the for CRRA utility better risk-return trade-off of the risky asset for) does not hold. The coefficients for high education, log(net worth), and noncapital Income/net worth allow for many combinations of the explanatory variables that result in a lower risky asset share for U.S. investors.

6 Welfare Analyses
In this section, we analyze the welfare consequences for the individuals in the SCF and EVS data sets that result from choosing different asset allocations than proposed by the normative model.

We utilize an equivalent wealth variation measure as, for example, in Brown (2001). The optimal expected utility as given by the value function for $t = 0$ from the benchmark model $V_0^*(W_0, L_0)$ is compared with the expected utility resulting from actual behavior $V_0^{act}(W_0, L_0)$. For each individual, equation (7) is solved for $\Delta W_0$ and then divided by $W_0$ to obtain a relative measure.

$$V_0^*(W_0, L_0) = V_0^{act}(W_0 + \Delta W_0, L_0). \quad (7)$$

The relative measure $\Delta W_0 / W_0$ has the advantage of enabling comparisons of U.S. and German individuals without having to consider differences in the purchasing power of dollars and euros and comparisons of individuals with different endowments (cash on hand and expected labor income). The economic interpretation of $\Delta W_0 / W_0$ is the answer to the following question: How much wealthier, on an expected utility basis, would an investor feel if he or she chose an asset allocation according to the benchmark model? Small values of $\Delta W_0 / W_0$ indicate that changing asset allocation toward the normative result would not enhance expected utility a lot.

17 See Appendix A.
whereas large values of $\Delta W_0 / W_0$ indicate that the individual could be considerably better off—that is, he or she gives away a lot of utility by not following the benchmark model. Thus, another interpretation of $\Delta W_0 / W_0$ is that it measures the welfare loss due to suboptimal behavior.

The specification of $V_0^{act}(W_0, L_0)$ includes the actual behavior with respect to asset allocation, as predicted by the regression equation of the tobit model. The decision with respect to consumption $C_t(W_t)$ (and saving) is assumed to follow the normative benchmark in each period. Thus, our results measure only the welfare effects that arise from choosing a suboptimal asset allocation.¹⁸

To calculate the welfare effects, we chose a certain subsample to avoid extremely long computation times (the SCF and EVS data sets contain 11,187 total cases). For this, we chose three ages: 30, 50, and 65. For each age group, cash on hand (net worth) is varied continuously,¹⁹ whereas for labor income (noncapital income), the 25%, 50%, and 75% age-specific quantiles as given by the data are assigned to the individuals.²⁰ Next, the welfare measure is calculated for these individuals assuming a relative risk-aversion parameter $\gamma$ of 2 and a subjective discount factor $\delta$ of 0.97 for different values of gender (male, female) and education (low, middle, high). Furthermore, the preference parameters $\gamma$ and $\delta$ are varied according to the calibrations shown in Table 1.

The analysis of our results begins with the outcome of the model calibrated with U.S. data. After this, we discuss the German results. Finally, we compare both countries and draw general conclusions. In general, U.S. and German individuals invest too little in risky assets. Thus, losses in welfare are significantly influenced by the size of

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¹⁸ Savings adequacy is addressed in Scholz, Seshadri, and Khitratrakun (2006). Furthermore, the SCF data set does not provide information on consumption and savings.

¹⁹ For this we can utilize the CRRA feature of the value function with respect to $W_t$.

²⁰ Precisely, we also included the neighbouring age groups (e.g., 29 and 31 in case of age 30) to calculate the quartiles in order to avoid too much distortion due to a low amount of cases in some age groups.
the (positive) gap between the optimal and the empirical investment share of the risky asset.

6.1 Results for U.S. Individuals

Figure 3-SCF shows how the loss in welfare is influenced by net worth, noncapital income, and gender:

Figure 3-SCF: Welfare Losses, $\Delta W_0 / W_0$, for U.S. (SCF) Data; Gender = 0 (male) or 1 (female), $\gamma = 2$, $\delta = 0.97$, Age = 50, Education = Middle

--- put Figure 3-SCF here ---

Figure 3-SCF shows that the loss in welfare is usually larger for higher levels of noncapital income. In general, the welfare loss due to variations in income is influenced by three factors. First, higher income relative to net worth is associated with lower savings (income crowds out savings). At higher levels of income, savings are often at or near zero. Thus, for those with high income, asset allocation sometimes does not matter at all (in case of zero savings) or does not matter much. This effect explains why the green curve is lower at low levels of net worth. Second, for those with higher income, the benchmark model predicts increasing shares of risky asset holdings. But the coefficient for noncapital income/net worth from the regression works in the opposite direction. Thus, the higher the income, the larger is the gap between optimal and empirical risky asset shares, and the larger is the loss in welfare. This effect explains why the green curve mostly lies above the blue, and the blue above the red curve. Third, the loss in welfare is measured in percent of current net worth. For low levels of net worth, most of the individual’s overall expected lifetime wealth derives from expected future labor income (future net worth). Thus, losses in welfare due to suboptimal asset allocation mainly stem from investing future labor income payments suboptimally (as opposed to current wealth for wealthy individuals). Relative to the considerably smaller level of current net worth, the losses thus are proportionally larger. The last effect also explains the horizontal
order of the curves and, for the red and blue line, the generally decreasing tendency in net worth.

Varying net worth (and always implicitly the ratio of noncapital income to net worth) produces a fourth effect that results in a decreasing shape of the curves. As stated above, for increasing net worth (i.e., for a decreasing noncapital income–to–net worth ratio), the benchmark model predicts lower risky asset shares (compare Figure 1). The regression coefficient for the log of net worth predicts exactly the opposite. Thus, for lower levels of net worth, the gap between optimal and empirical asset allocation becomes larger, and the welfare losses are also often larger.

Varying the gender variable from male to female increases the individual’s life expectancy, and increases the empirical risky asset share for U.S. investors (see Table 3). The variation in life expectancy yielded ambiguous results in the normative model and produced only minor differences in asset allocation. Thus, the optimal asset allocation for men and women according to the benchmark model is almost identical. But for women, the gap between optimal and empirical asset allocation (risky asset share) is smaller due to the positive coefficient for gender. Consequently, the welfare losses are smaller for women.

The impact of different assumptions for the coefficient of relative risk aversion $\gamma$ is shown in Figure 4-SCF.

**Figure 4-SCF: Welfare Losses, $\Delta W_0 / W_0$, for U.S. (SCF) Data; Gender = 0 (male), $\gamma = 1, 2, \text{ or } 3$, $\delta = 0.97$, Age = 50, Education = Middle**

--- put Figure 4-SCF here ---

21 The growth rates for income are assumed to be identical in our U.S. income profiles.
Higher risk aversion is associated with lower losses in welfare. The reason for this is that while increasing risk aversion, the empirically measured asset allocation stays constant, but the benchmark investment in the risky asset decreases. As a result, the gap between the optimal risky asset share and the empirical asset share decreases.

The influence of a variation in the subjective discount factor $\delta$ is shown in Figure 5-SCF.

**Figure 5-SCF: Welfare Losses, $\Delta W_0 / W_0$, for U.S. (SCF) Data; Gender = 0 (male), $\gamma = 2$, $\delta = 0.95, 0.97, \text{ or } 0.99$, Age = 50, Education = Middle**

--- put Figure 5-SCF here ---

The explanation for this effect is straightforward. A higher $\delta$ makes the individual more oriented toward the future; thus, savings, and the potential amount of money invested suboptimally, increases. Furthermore, with a higher $\delta$, all future losses are less heavily discounted.

Figure 6-SCF shows the impact of different education levels.

**Figure 6-SCF: Welfare Losses, $\Delta W_0 / W_0$, for U.S. (SCF) Data; Gender = 0 (male), $\gamma = 2$, $\delta = 0.97$, Age = 50, Education = Low, Middle, or High**

--- put Figure 6-SCF here ---

Higher education for U.S. investors generally leads to higher welfare losses. This is partially caused by the coefficients for the education dummy variables that imply adjustments to asset allocation in the opposite direction of the normative model.

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22 As can be seen from Figure 4-SCF, the impact of changes in income and net worth are confirmed. The gender effect is also confirmed. Because this is also valid for the following variations (except age), we will no longer refer to these effects (except for age).
predictions. Thus, the higher welfare loss for highly educated individuals can be explained by a larger gap between the optimal asset allocation and the empirical one. For the lower welfare loss for individuals with low education, another effect is responsible (the first would imply higher losses too). Having lower education is associated with lower expected income growth. Thus, for the same current income, the discounted value of expected labor income is lower for individuals with lower education. This leads to higher savings rates, making deviations in asset allocation more painful, due to the relatively larger amount saved and suboptimally invested. For those with higher education, this mitigates somehow the consequences of the first effect.

Finally, as the last within-country analysis for the U.S., we investigate age effects in Figure 7-SCF.

**Figure 7-SCF: Welfare Losses, \( \Delta W_0 / W_0 \), for U.S. (SCF) Data; Gender = 0 (male), \( \gamma = 2, \delta = 0.97 \), Age = 30, 50, or 65, Education = Middle**

--- put Figure 7-SCF here ---

Figure 7-SCF shows that the overall effect of the variations in noncapital income and net worth can be different according to the strength of the single effects. Here, we observe that for persons age 65, the welfare losses are larger the lower the income is. Comparing the absolute magnitude between ages is difficult. At different ages, the discounted value of the expected labor income stream changes following the curves depicted in Figure 2. Furthermore, the income quantiles are different. Finally, the time horizon—and thus the number of future welfare losses discounted—is different between different age groups. Thus, only some results can be clearly identified. For example, for individuals age 65, the welfare losses are the lowest, because the gap between optimal and empirical asset allocation is the smallest. Furthermore, there are fewer future periods with welfare losses. This also explains to some extent why younger age is associated with higher welfare losses.
6.2 Results for German Individuals

In this section, we repeat the analysis given in section 6.1 for the German EVS data.

Figure 3-EVS shows the effect of a variation in noncapital income, net worth, and gender for German individuals.

Figure 3-EVS: Welfare Losses, \( \Delta W_0 / W_0 \), for German (EVS) Data; Gender = 0 (male) or 1 (female), \( \gamma = 2 \), \( \delta = 0.97 \), Age = 50, Education = Middle

--- put Figure 3-EVS here ---

We first observe that a variation in income leads to results opposite those found for U.S. individuals. Lower income (comparing different curves for given levels of net worth) is associated with larger welfare losses. Here the effect that with higher income savings and thus potential deviations from optimal asset allocation are lower, is stronger than the other effects described for U.S. individuals. Although the—opposed to the normative benchmark model—negative sign of the coefficient for the log of noncapital income leads to greater deviations from the optimal asset allocation for high income individuals, their lower savings overcompensate for this. The combined effect of a change in savings and the change in the gap to the optimal asset allocation explains the hump-shaped welfare loss curves. First, due to the relatively large income, savings are zero and thus welfare losses are also zero. Then, with increasing net worth, savings increase, as does the impact of the asset allocation gap. For larger levels of net worth, the empirical asset allocation comes closer to the 100% optimal result of the normative benchmark (given certain levels of income and wealth), thus decreasing the welfare gap. For very large levels of net worth (not shown in Figure 3-EVS), this should reverse again, because optimal asset allocation goes below 100% risky assets, whereas the empirical asset allocation still increases.

In the German data, women generally experience higher welfare losses. First, for any given current income, the expected discounted value of future labor income is lower, due to the gender-specific German calibration of the life cycle income profiles.
Consequently, women save more, making deviations work on a larger amount invested. Additionally, for many parameter constellations, the optimal asset allocation is 100% risky. Thus, the negative coefficient for the gender dummy variable increases the gap in asset allocation for women, leading to a higher welfare loss.

The effect of a change in the assumption about relative risk aversion, shown in Figure 4-EVS, produces the same results as for the U.S. calibration.

**Figure 4-EVS: Welfare Losses, \( \Delta W_0 / W_0 \), for German (EVS) Data; Gender = 0 (male), \( \gamma = 1, 2, \text{ or } 3, \delta = 0.97, \text{ Age} = 50, \text{ Education} = \text{middle} \)**

--- put Figure 4-EVS here ---

An increase in the subjective discount factor \( \delta \) leads, as shown in Figure 5-EVS, to higher welfare losses, which also confirms U.S. results.

**Figure 5-EVS: Welfare Losses, \( \Delta W_0 / W_0 \), for German (EVS) Data; Gender = 0 (male), \( \gamma = 2, \delta = 0.95, 0.97, \text{ or } 0.99, \text{ Age} = 50, \text{ Education} = \text{Middle} \)**

--- put Figure 5-EVS here ---

The impact of a variation in education is shown in Figure 6-EVS.

**Figure 6-EVS: Welfare Losses, \( \Delta W_0 / W_0 \), for German (EVS) Data; Gender = 0 (male), \( \gamma = 2, \delta = 0.97, \text{ Age} = 50, \text{ Education} = \text{Low, Middle, or High} \)**

--- put Figure 6-EVS here ---

Although the regression coefficients for the low and high education dummy variables show the expected sign, individuals with lower education have higher welfare losses. Welfare Losses for individuals with higher education and middle education are
similar. The higher losses for those with lower education are driven by their larger savings (due to the lower expected income stream for a given current income) overcompensating the correct sign of the regression coefficient. The resulting larger basis, on which the deviation to the optimal risky asset share works, leads to larger welfare losses.

The final variation in age, depicted in Figure 7-EVS, shows similar results as for the United States, except for individuals age 65.

**Figure 7-EVS: Welfare Losses, $\Delta W_0 / W_0$, for German (EVS) Data, Gender = 0 (male), $\gamma = 2$, $\delta = 0.97$, age = 30, 50, or 65, Education = Middle**

--- put Figure 7-EVS here ---

For individuals age 65, the welfare losses are often higher than they are for younger individuals. Here, opposite the U.S. results, the steeply falling empirical age–risky-asset allocation profile leads to larger losses. The empirical risky asset share falls much faster in age than required by the benchmark model.

### 6.3 Comparison of the United States and Germany and Derivation of General Results

Comparing Figures 3-SCF to 7-SCF with Figures 3-EVS to 7-EVS, we observe that, for most combinations of parameters, Americans experience larger welfare losses than Germans due to suboptimal asset allocation. Exceptions can be found in some but not all parameter combinations for women, those with lower education, and individuals age 65. The major explanation for this finding is the larger gap between the U.S. risky asset share and the optimal one. For many parameter combinations, Americans invest a smaller share into the risky asset, although the benchmark model’s solution demands a higher risky asset share.

Women in the United States are better investors than their male counterparts; in Germany, it is the opposite, assuming identical relative risk aversion. Assuming
heterogeneity in relative risk aversion $\gamma$—for example, three for German women and two for German men or three for U.S. men and two for U.S. women (although the latter assumption runs counter to the bulk of empirical and experimental research)—can almost make gender-specific differences disappear.

With respect to net worth, being a good investor is associated in the United States generally with individuals in the top half of the wealth distribution. Only for older investors (age 65) lower wealth improves investment behavior. For Germans, the better investors can be found at the lower and upper part of the wealth distribution (again with an exception for older investors).

Except for older investors, being in higher income brackets generally leads to worse investment performance for U.S. investors, whereas in Germany the opposite holds true (for all age groups).

Education worsens investment performance for U.S. investors, whereas for Germans slight improvements can be identified.

Assuming that higher relative risk aversion leads to lower welfare losses, assuming a higher subjective discount factor increases welfare losses. Thus, to close the performance gap between the United States and Germany, one needed higher risk aversion and/or a lower subjective discount factor for U.S. investors.

7 Summary and Policy Implications
The analyses presented in this paper reveal the welfare consequences of suboptimal investment behavior for U.S. and German individuals. In general, Germans appear to be better investors in terms of welfare loss arising from suboptimal asset allocation. This is surprising, taking into account that stock market participation and the engagement in risky investments have a longer tradition in the United States than in Germany. Thus, reforming the German system toward more privately managed funds may not be a bad idea in general.
However, for many combinations of individual characteristics, our results show large welfare losses, reaching more than 100% in relation to wealth (net worth). Thus, there is considerable room for improvement. In addition, our model identifies those population subgroups that would benefit most from a better asset allocation. Because public policy resources are limited, our results can help target those groups for whom the return would be largest.

An example for such an analysis is given in Table 4. Combining the results of section 6 with the empirical population distribution allows us to see which individuals are located in the parts of the welfare loss distribution with relatively large losses. Here we focus on income and wealth, because section 6 revealed that they were the two main drivers of the magnitude of welfare loss. Table 4 shows which proportions of the SCF and EVS data sets are located in the multivariate income and wealth distribution. The shaded fields indicate those parts of the distribution with relatively large welfare losses, as given by Figures 3-SCF and 3-EVS.

Table 4: Distribution of Noncapital Income and Net Worth for the United States and Germany and Indication (Shaded Area) of Potentially Large Welfare Losses

--- put Table 4 here ---

Similar analyses can be performed across age, gender, and education domains.

An important topic for future research is determining which public policy measure or mix of measures—for example, investments in financial education$^{23}$ or advice for asset allocation default options in pension plans$^{24}$—should be implemented to achieve the maximum welfare gain. Combining knowledge about the welfare

$^{23}$ See footnote 3.

$^{24}$ The power of default options—the tendency that pension plan members stick to predefined asset allocations—is demonstrated in Beshears et al. (2006). Thus, one can reach desired asset allocation results while avoiding too much regulatory force.
contributions of various policy measures with the results on the location of welfare losses presented in this paper should help considerably to meet the demographic challenges ahead.

References


Federal Ministry of Health and Social Affairs (2005), *Old-age Pension Schemes in Germany 2003 (ASID '03), Summary of Survey Results*, Munich: Federal Ministry of Health and Social Affairs.


Appendix A: Solving Technique for the Normative Problem

The optimization problem (3)–(6) is solved backward via stochastic dynamic programming. The Bellman equation for this problem depends on three state variables: time $t$, cash on hand $W_t$, and the expected labor income path, represented by $L_t$. The Bellman equation (with $V$ denoting the value function) is given by $t = 0, 1, \ldots, T - x - 1$,

$$V_t(W_t, L_t) = \max_{\alpha_t, C_t} \left\{ U_t(C_t) + p_t \delta \mathbb{E}_t \left( V_{t+1}(W_{t+1}, L_{t+1}) \right) \right\}, \quad (A1)$$

subject to constraints (4) through (6). In the last period, the remaining wealth is consumed, and the value function is simply given by $U_{T-x}(W_{T-x})$. In general, for CRRA utility, the $L_t$-state can be reduced by dividing $W_t$ through $L_t$ (see Carroll, 2004). But, since our econometric results show that, in reality, consumers do not behave exactly according to CRRA, and thus in order to integrate empirical asset allocations into the model (which depends on both state variables, see section 5.3), the $L_t$ state should not be dropped. Nevertheless, problem (A1) is solved by referring explicitly only to the $W_t$ state. The $L_t$ state is considered implicitly, because equation (A1) is calculated for each individual separately, thus referring to each individual’s expected labor income path.

The Bellman equation (A1) cannot be solved analytically, and hence a numerical technique is used. First, at each point in time $t$, the $W_t$-state space is discretized into $I \in \mathbb{N}$ points $W^i_t$, with $i = 1, 2, \ldots, I$. The upper and lower bounds of this $W^i_t$ grid were chosen to be nonbinding in all periods. The distributions of the risky return $R_t$ and the labor income $L_t$ were discretized using Gaussian quadrature methods. Since in the last period (i.e., at $t = T - x$), the value function $V_{T-x}(W_{T-x})$ is given by $U_{T-x}(W_{T-x})$, the numerical solution algorithm starts at the penultimate period (i.e., at $t = T - x - 1$). For each $W^i_t$, equation (A1) is solved with the MATHEMATICA® 6.0 implemented nonlinear optimizer NMaximize, yielding the optimal decisions $\alpha^i_t(W^i_t)$, $C^i_t(W^i_t)$, and the function value of $V_t(W^i_t)$. Next, a continuous function is fitted to the points $V_t(W^i_t)$, which delivers a continuous approximation of the value
function $V_t(W_t)$. Finally, the problem is rolled back to the preceding period.

25 The fitting algorithm used here guarantees that the relative risk aversion displayed by the optimal decisions $\alpha_t(W_t), C_t(W_t)$ is inherited to the continuous approximation of the value function $V_t(W_t)$. 
### Appendix B: Definition of Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Age</strong></td>
<td>Age of respondent</td>
</tr>
<tr>
<td><strong># Children</strong></td>
<td>Number of children in the household</td>
</tr>
<tr>
<td><strong>Gender</strong></td>
<td>Dummy for gender, with 0 = male and 1 = female</td>
</tr>
<tr>
<td><strong>Education - Low</strong></td>
<td>Dummy for education category = no degree, on the job training or no degree, still in school (EVS) and no high school diploma/GED (SCF)</td>
</tr>
<tr>
<td><strong>Education - Middle</strong></td>
<td>Holdout group for education category (= college degree (Meister, Berufs- und Fachakademie), apprenticeship (Lehre), still in apprenticeship or college (EVS) and high school diploma or GED or some college (SCF))</td>
</tr>
<tr>
<td><strong>Education - High</strong></td>
<td>Dummy for education category = applied science college degree (Fachhochschule) or university degree (EVS) and college degree (SCF)</td>
</tr>
<tr>
<td><strong>Occupation - Employed</strong></td>
<td>Holdout group for occupation = government employee (Beamter), work for someone else (Angestellter) or work for someone else (Arbeiter) (EVS) and work for someone else (SCF)</td>
</tr>
<tr>
<td><strong>Occupation - Unemployed</strong></td>
<td>Dummy for occupation = unemployed, student or other not employed (homemaker, pupil, ...) (EVS) and other groups not working (mainly those under 65 and out of the labor force) (SCF)</td>
</tr>
<tr>
<td><strong>Occupation - Retired</strong></td>
<td>Dummy for occupation = retired (Rentner) or retired (Pensionär) (EVS) and retired/disabled + (student/homemaker/misc. not working and age 65 or older) (SCF)</td>
</tr>
<tr>
<td><strong>Income</strong></td>
<td>Total amount of pre-tax income</td>
</tr>
<tr>
<td><strong>Noncapital Income</strong></td>
<td>Total amount of pre-tax income, excluding any income or withdrawals from investments (financial assets and real estate)</td>
</tr>
<tr>
<td><strong>Has risky 1</strong></td>
<td>Dummy for % risky 1 &gt; 0 vs. = 0</td>
</tr>
</tbody>
</table>
% risky 1  Risky financial assets (including directly-held stocks; risky share invested in investment funds; mortgage-backed bonds; corporate and foreign bonds; risky share invested in trusts, annuities and managed investment accounts, quasi-liquid retirement accounts; other financial assets (e.g., loans to someone else, future proceeds from lawsuits)) \( \text{divided by Assets} \)

has risky 2  Dummy for % risky 2 > 0 vs. = 0

% risky 2  Risky financial assets (risky 1) + House – Debt divided by Net worth

Own House  Dummy for house or real estate ownership

House  Total value of houses and real estate

Assets  Total value of assets, including financial assets and real estate, but excluding cars and other nonfinancial assets (e.g., paintings)

In Debt  Dummy for Debt > 0 vs. = 0

Debt  Total value of debt

Net worth  Assets – Debt
### Table 1: Parameter Calibration for the Benchmark Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value United States</th>
<th>Value Germany</th>
</tr>
</thead>
<tbody>
<tr>
<td>relative risk aversion $\gamma$</td>
<td>2 (1, 3)</td>
<td>2 (1, 3)</td>
</tr>
<tr>
<td>subjective discount factor $\delta$</td>
<td>0.97 (0.95, 0.99)</td>
<td>0.97 (0.95, 0.99)</td>
</tr>
<tr>
<td>survival probability $p_t$</td>
<td>United States Life Tables 2003</td>
<td>Germany 2002/2004</td>
</tr>
<tr>
<td>marginal tax rate</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Log-normal stock return $R_t$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>expected return $E(R_t)$</td>
<td>1.1151</td>
<td>1.1264</td>
</tr>
<tr>
<td>standard deviation of return $\text{Std}(R_t)$</td>
<td>0.1996</td>
<td>0.2792</td>
</tr>
<tr>
<td>risk-free return $R_f$</td>
<td>1.0361</td>
<td>1.0472</td>
</tr>
<tr>
<td>Inflation $\pi$</td>
<td>0.0310</td>
<td>0.0279</td>
</tr>
<tr>
<td>Log-normal labor income $L_t$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>expected growth rates during work life</td>
<td>life-cycle-income profile</td>
<td>life-cycle-income profile</td>
</tr>
<tr>
<td>expected real growth rates during retirement</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>replacement factor</td>
<td>35%</td>
<td>40%</td>
</tr>
<tr>
<td>standard deviation during work life</td>
<td>$0.19 \cdot E(L_t)$</td>
<td>$0.05 \cdot E(L_t)$</td>
</tr>
<tr>
<td>standard deviation during retirement</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 2: 2004 Survey of Consumer Finances (SCF) and 2003 Income and Expenditure Survey (EVS) (for definition of variables, see Appendix B)

<table>
<thead>
<tr>
<th></th>
<th>USA: SCF 2003</th>
<th>Germany: EVS 2004</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>full sample</td>
<td>individual has no risky 2</td>
</tr>
<tr>
<td></td>
<td>N = 871</td>
<td>N = 142</td>
</tr>
<tr>
<td></td>
<td>Mean (weighed)</td>
<td>Mean (weighed)</td>
</tr>
<tr>
<td>Demographics</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>55.76</td>
<td>51.24</td>
</tr>
<tr>
<td>Age^2</td>
<td>3.477</td>
<td>3.100</td>
</tr>
<tr>
<td>Gender</td>
<td>0.65</td>
<td>0.68</td>
</tr>
<tr>
<td># Children</td>
<td>0.40</td>
<td>0.51</td>
</tr>
<tr>
<td>Education</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low</td>
<td>0.18</td>
<td>0.30</td>
</tr>
<tr>
<td>Middle</td>
<td>0.51</td>
<td>0.60</td>
</tr>
<tr>
<td>High</td>
<td>0.31</td>
<td>0.10</td>
</tr>
<tr>
<td>Occupation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Employed</td>
<td>0.56</td>
<td>0.49</td>
</tr>
<tr>
<td>Unemployed</td>
<td>0.05</td>
<td>0.09</td>
</tr>
<tr>
<td>Retired</td>
<td>0.38</td>
<td>0.42</td>
</tr>
<tr>
<td>Income (local currency)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Income</td>
<td>35.497</td>
<td>16.286</td>
</tr>
<tr>
<td>Noncapital Income</td>
<td>31.146</td>
<td>15.344</td>
</tr>
<tr>
<td>Ln(Noncapital Income)</td>
<td>10.01</td>
<td>9.36</td>
</tr>
<tr>
<td>Wealth (local currency)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>has risky 1</td>
<td>0.50</td>
<td>0.001</td>
</tr>
<tr>
<td>% risky 1</td>
<td>0.14</td>
<td>0.001</td>
</tr>
<tr>
<td>has risky 2</td>
<td>0.85</td>
<td>0</td>
</tr>
<tr>
<td>% risky 2</td>
<td>0.58</td>
<td>0</td>
</tr>
<tr>
<td>Own Hose</td>
<td>0.75</td>
<td>0.023</td>
</tr>
<tr>
<td>House Value</td>
<td>146.346</td>
<td>1.035</td>
</tr>
<tr>
<td>Assets</td>
<td>252.132</td>
<td>7.371</td>
</tr>
<tr>
<td>In Debt</td>
<td>0.59</td>
<td>0.024</td>
</tr>
<tr>
<td>Debt</td>
<td>41.155</td>
<td>1.040</td>
</tr>
<tr>
<td>Net worth</td>
<td>222.205</td>
<td>9.053</td>
</tr>
<tr>
<td>Ln(Net worth)</td>
<td>10.99</td>
<td>7.85</td>
</tr>
<tr>
<td>Noncapital Income / Net worth</td>
<td>2.82</td>
<td>15.43</td>
</tr>
</tbody>
</table>
Table 3: Determinants for Share of Risky Assets (Tobit Regression); Dependent Variable: Percent Risky 2 (for definition of variables, see Appendix B)

<table>
<thead>
<tr>
<th></th>
<th>SCF USA</th>
<th>EVS Germany</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>coef</td>
<td>std. err.</td>
</tr>
<tr>
<td>Age</td>
<td>0,0126</td>
<td>0,0039   ***</td>
</tr>
<tr>
<td>Age$^2$</td>
<td>-0,0001</td>
<td>0,00003  ***</td>
</tr>
<tr>
<td>Gender</td>
<td>0,0581</td>
<td>0,0244   **</td>
</tr>
<tr>
<td>Education Low</td>
<td>0,0902</td>
<td>0,0366   **</td>
</tr>
<tr>
<td>Education High</td>
<td>-0,0684</td>
<td>0,0284   **</td>
</tr>
<tr>
<td>Occupation Unemployed</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Occupation Retired</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Ln(Noncapital Income)</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Ln(Net worth)</td>
<td>0,0863</td>
<td>0,0094   ***</td>
</tr>
<tr>
<td>Noncapital Income / Net worth</td>
<td>-0,0429</td>
<td>0,0079   ***</td>
</tr>
<tr>
<td>Constant</td>
<td>-0,6890</td>
<td>0,1362   ***</td>
</tr>
<tr>
<td>Adj. R$^2$ or Pseudo R$^2$</td>
<td>0,4040</td>
<td>0,3710</td>
</tr>
</tbody>
</table>

* significant at 10%, ** significant at 5%, *** significant at 1% level, – not significant and removed from the regression equation
Table 4: Distribution of Noncapital Income and Net Worth for the United States and Germany and Indication (Shaded Area) of Potentially Large Welfare Losses*

<table>
<thead>
<tr>
<th>SCF USA</th>
<th>Net worth Quantile</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Noncapital Income Quantile</td>
<td>0-25</td>
<td>25-50</td>
<td>50-75</td>
<td>75-100</td>
</tr>
<tr>
<td>0-25</td>
<td>9%</td>
<td>7%</td>
<td>8%</td>
<td>1%</td>
</tr>
<tr>
<td>25-50</td>
<td>10%</td>
<td>3%</td>
<td>13%</td>
<td>1%</td>
</tr>
<tr>
<td>50-75</td>
<td>10%</td>
<td>8%</td>
<td>0%</td>
<td>7%</td>
</tr>
<tr>
<td>75-100</td>
<td>0%</td>
<td>1%</td>
<td>11%</td>
<td>12%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>EVS Germany</th>
<th>Net worth Quantile</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Noncapital Income Quantile</td>
<td>0-25</td>
<td>25-50</td>
<td>50-75</td>
<td>75-100</td>
</tr>
<tr>
<td>0-25</td>
<td>13%</td>
<td>5%</td>
<td>3%</td>
<td>4%</td>
</tr>
<tr>
<td>25-50</td>
<td>7%</td>
<td>8%</td>
<td>5%</td>
<td>6%</td>
</tr>
<tr>
<td>50-75</td>
<td>4%</td>
<td>6%</td>
<td>8%</td>
<td>7%</td>
</tr>
<tr>
<td>75-100</td>
<td>1%</td>
<td>6%</td>
<td>8%</td>
<td>10%</td>
</tr>
</tbody>
</table>

* Note, the distribution shown here refers to the subpopulation aged 50. Preference assumptions and indication of potentially large welfare losses are based on Figure 3-SCF and Figure 3-EVS.
Figures
Figure 1: Investment into the risky asset as a function in the labor income–to–cash on hand ratio

![Figure 1](image1.png)

Figure 2: Investment into the risky asset as a function in age

![Figure 2](image2.png)
Figure 3-SCF: Welfare Losses, $\Delta W_0 / W_0$, for U.S. (SCF) Data; Gender = 0 (male) or 1 (female), $\gamma = 2$, $\delta = 0.97$, Age = 50, Education = Middle

* The percentage values in parentheses refer to the age-specific quantiles for Net worth (dotted vertical lines) and Noncapital Income (colored curves)
Figure 4-SCF: Welfare Losses, $\Delta W_0 / W_0$, for U.S. (SCF) Data;
Gender = 0 (male), $\gamma = 1, 2, \text{ or } 3$, $\delta = 0.97$, Age = 50, Education = Middle

* The percentage values in parentheses refer to the age-specific quantiles for Net worth (dotted vertical lines) and Noncapital Income (colored curves)
Figure 5-SCF: Welfare Losses, $\Delta W_0 / W_0$, for U.S. (SCF) Data;
Gender = 0 (male), $\gamma = 2$, $\delta = 0.95, 0.97, \text{ or } 0.99$, Age = 50, Education = Middle

* The percentage values in parentheses refer to the age-specific quantiles for Net worth (dotted vertical lines) and Noncapital Income (colored curves)
Figure 6-SCF: Welfare Losses, $\Delta W_0 / W_0$, for U.S. (SCF) Data;
Gender = 0 (male), $\gamma = 2$, $\delta = 0.97$, Age = 50, Education = Low, Middle, or High

* The percentage values in parentheses refer to the age-specific quantiles for Net worth (dotted vertical lines) and Noncapital Income (colored curves)
Figure 7-SCF: Welfare Losses, $\Delta W_0 / W_0$, for U.S. (SCF) Data;

Gender = 0 (male), $\gamma = 2$, $\delta = 0.97$, Age = 30, 50, or 65, Education = Middle

* The percentage values in parentheses refer to the age-specific quantiles for Noncapital Income. For Net worth (solid vertical lines) the quantiles are given by the lines with the same color of the welfare gain curves.

<table>
<thead>
<tr>
<th>Noncapital Income</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
</tr>
</thead>
<tbody>
<tr>
<td>△ 19,011.3 △ 21,133.3 △ 10,873.3</td>
<td>(25%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>○ 27,480.0 ○ 36,180.0 ○ 22,687.0</td>
<td>(50%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>▽ 52,400.0 ▽ 50,666.7 ▽ 34,240.0</td>
<td>(75%)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 3-EVS: Welfare Losses, $\Delta W_0 / W_0$, for German (EVS) Data; Gender = 0 (male) or 1 (female), $\gamma = 2$, $\delta = 0.97$, Age = 50, Education = Middle.

* The percentage values in parentheses refer to the age-specific quantiles for Net worth (dotted vertical lines) and Noncapital Income (colored curves)
Figure 4-EVS: Welfare Losses, $\Delta W_0 / W_0$, for German (EVS) Data; Gender = 0 (male), $\gamma = 1, 2, \text{ or } 3$, $\delta = 0.97$, Age = 50, Education = middle$^*$

* The percentage values in parentheses refer to the age-specific quantiles for Net worth (dotted vertical lines) and Noncapital Income (colored curves)
Figure 5-EVS: Welfare Losses, $\Delta W_0 / W_0$, for German (EVS) Data; Gender = 0 (male), $\gamma = 2$, $\delta = 0.95$, 0.97, or 0.99, Age = 50, Education = Middle

* The percentage values in parentheses refer to the age-specific quantiles for Net worth (dotted vertical lines) and Noncapital Income (colored curves)
Figure 6-EVS: Welfare Losses, $\Delta W_0 / W_0$, for German (EVS) Data; Gender = 0 (male), $\gamma = 2$, $\delta = 0.97$, Age = 50, Education = Low, Middle, or High

*The percentage values in parentheses refer to the age-specific quantiles for Net worth (dotted vertical lines) and Noncapital Income (colored curves)
Figure 7-EVS: Welfare Losses, $\Delta W_0 / W_0$, for German (EVS) Data, Gender = 0 (male), $\gamma = 2$, $\delta = 0.97$, age = 30, 50, or 65, Education = Middle $^*$

* The percentage values in parentheses refer to the age-specific quantiles for Noncapital Income. For Net worth (solid vertical lines) the quantiles are given by the lines with the same color of the welfare gain curves.