1. Here is a variation of the Monty Hall game. The contestant faces four doors. Behind one door is the prize. The other doors lead to empty rooms. The contestant chooses one door. The master of ceremonies then opens two doors that lead to empty rooms, but he does not open the chosen door. Now the contestant has an opportunity to switch her choice of doors.

   A. In words, why is it best for the contestant to switch doors?
   Answer: The master of ceremonies has information the contestant does not have. He knows where the prize is. In opening a door he avoids opening the one with the prize. Thus his action reveals something about the prize’s location.

   B. What is the probability of winning the prize if the contestant does not switch? What is the probability of winning the prize if the contestant does switch? Explain.
   Answer: The probability of a correct guess is 1/3. In that case the best policy is to switch. Therefore 1/3 is the probability of winning if the contestant switches. The probability of an incorrect guess is 2/3. In that case the master of ceremonies must open the door that is neither chosen nor hiding the prize. By assumption the prize is not behind the chosen door, and on of the other doors has been opened. Therefore the prize is behind the remaining door. The best policy is switch. Switch wins with probability 2/3. A good answer will show the tree for the game. That is much more effective than words, but diagrams are difficult in this text editor.

2. (Like 5.5 in the text) A bond with a semi-annual (twice a year) coupon payment is sold for $1160.584, which is above its par value of $1000. The bond is 12 years to maturity and the yield currently required by the market for such bonds is seven percent. Show how to solve for the coupon rate for the bond. Show that it is nine percent. (Hint: As usual, you may assume that the next coupon payment is due in exactly six months.)

   Answer: Payments from the bond are

   \[
   \begin{array}{cccc}
   t=0 & t=1 & t=2 & \ldots & t=24 \\
   \text{coupon} & 0 & 1000c/2 & 1000c/2 & 1000c/2 \\
   \text{face value} & 1000 & & & \\
   \end{array}
   \]

   \( (1) \)

   The value of the bond is given by the formula

   \[1160.584 = A_{24}^{0.035} \times 1000 \times \frac{c}{2} + \frac{1}{(1.035)^{24}} \times 1000\]

   Solve for \(c\). Everything else is known

   \[\frac{c}{2} = \frac{1160.584 - \frac{1}{(1.035)^{24}} \times 1000}{A_{24}^{0.035} \times 1000}\]

   \( (3) \)

3. (Like 4.37 in the text). On November 1, 2003, Mike White bought a BMW for $50,000. He paid $5,000 down and financed the balance with a five-year loan at an interest rate of 5 percent, compounded monthly (in this market, 5 percent per year means .05/12 = .41666667 percent per month). The first monthly payment was made exactly one month after the purchase. In October 2005 Mike inherited some money and decide to pay off the loan on November 1. The bank charges him a prepayment penalty amounting to 1% of the principal balance. How much does he pay the bank on November 1. (Hint: As usual, start with a time-line showing the timing of cash flows. This is a good problem to put in a spreadsheet. You may paste or tape an appropriate part of the spreadsheet to this page.)

   Answer: The answer is in a separate file.
1. Suppose that a tree can be planted for a cost of $140. Its value IF CUT at time $t$ is $-200 + 40t$. The land cannot be reused after harvest. The continuous rate of interest is 6%. Assuming that the tree is planted, when should the tree be cut? Should the tree be planted at all? What is the value in time-$t$ dollars of the partially-grown UNCut tree at time $t$? Explain everything and illustrate in a diagram.

Briefly, 

$$\text{maximize} \ ( -200 + 40t)e^{-0.06t}$$

which is the value in time zero dollars of the tree if it is cut at time $t$. Differentiate and solve for $t^*$ in

$$\frac{40}{-200 + 40t^*} = 0.06$$

Solution is: $\{[t^* = 21.667]\}$. Present value of the tree is $(\exp(-0.06 \times 21.666666)) \times (-200 + 40 \times 21.666666) = 181.69$. Planting is worthwhile because the net present value is positive, and in fact is $181.69 - 140 = 41.69$.

The tree is now an investment like any other. As such, it earns the market return of 6%. Thus the market value of the tree at time $t < t^*$ is

$$181.69 \times e^{0.06t}$$

Please forgive the rounding.

2. The formula for put-call parity is $S + P = Xe^{-r(T-t)} + C$. Define puts and calls and show that at expiration the put-call parity relation holds by definition. (Hint: for the second part, it helps to write the values of puts and calls in terms of max( , ) functions.) A call is the right but not the obligation to buy a share at a stated exercise price $X$ on or before a fixed expiration date $T$.

Answer: A put is the right but not the obligation to sell a share at a stated exercise price $X$ on or before a fixed expiration date $T$. Lowercase letters are values at expiration. At expiration, $p = \text{max}(0, X - s)$, $c = \text{max}(0, s - X)$. The put-call parity relation at expiration is $s + p = X + c$. Substituting gives $s + \text{max}(0, X - s) = X + \text{max}(0, s - X)$ which is the relation that needs to be verified. If $s > X$, the relation reduces to $s + 0 = X + (s - X)$. If $s < X$ the relation becomes $s + (X - s) = X + 0$, that is, $X = X$.

3. Suppose that at time $T$ a stock will have a value either of 54 or 48 with equal probability. The safe discount rate is zero. The current price of the stock is 50. What is the value of a call on the stock with exercise price 50? What is the value of a put with the same exercise price? Put call parity is not satisfied. Construct the riskless arbitrage that allows a trader to profit from the situation.

Answer: The call is worth $.5 \times 4 = 2$, and the put is worth $.5 \times 2 = 1$. To exploit the failure of put-call parity, recognize that the share is underpriced, i.e., $S - X < C - P$. Borrow 50 and buy the share. Also buy the put and sell the call, at which point you have a dollar in profit (= $50 - 50 + 2 - 1$). The remaining position is riskless. Here’s why: At expiration the debt $X = 50$ is paid by selling the share and either paying off the call if $s > X$, or exercising the put if $s < X$. In case $s = 54$ the holder of the call exercises it leaving the options trader with 54 - 4 = 50, and in case $s = 48$ the trader exercises the put and thus ends with $48 + 2 = 50$. That shows that the position is riskless.
1. A parent is saving for the college education of two children. The elder will begin college at $t = 10$ years from the present, which is $t = 0$. The younger begins college at $t = 13$. Each child will complete college in four years. The cost of college will be $24,000 per year payable at the beginning of each year. The parent will make equal annual deposits beginning at $t = 0$, and ending the year the younger child leaves college. The annual interest rate is five percent. How much money must the parent deposit in each year? Show in a spreadsheet how value builds and then declines to zero in the savings account. Explain in this space. Attach the spreadsheet:

Answer is on a separate spreadsheet entitled "First problem of set 3."

2. Compute the internal rates of return for the cash flows of the following two projects. Why does a person suspect that A might have two internal rates? Why does a person know that B has a single internal rate? Explain your steps.

<table>
<thead>
<tr>
<th>Time</th>
<th>Cash flows A</th>
<th>Cash flows B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-3000</td>
<td>-2000</td>
</tr>
<tr>
<td>1</td>
<td>3000</td>
<td>700</td>
</tr>
<tr>
<td>2</td>
<td>9000</td>
<td>1100</td>
</tr>
<tr>
<td>3</td>
<td>-9000</td>
<td>1300</td>
</tr>
</tbody>
</table>

Answers: The internal rate of return is a rate $r$ satisfying

$$0 = c_0 + \frac{1}{1+r}c_1 + \frac{1}{(1+r)^2}c_2 + ... + \frac{1}{(1+r)^T}c_T$$

(7)

Internal rates for project A are 0 and $\sqrt{3} - 1 = 0.73205$. One suspected there might be two because there are two sign changes in the stream of cash flows. The internal rate of return for project B satisfies

$$0 = -2000 + \frac{1}{1+r} \times 700 + \frac{1}{(1+r)^2} \times 1100 + \frac{1}{(1+r)^3} \times 1300$$

(8)

Solution is: $r = 0.22845$.

3. Suppose that you are offered $30K today in exchange for making some payments in the future. Your cash flows if you accept are as follows:

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>30K</td>
</tr>
<tr>
<td>1</td>
<td>-6K</td>
</tr>
<tr>
<td>2</td>
<td>-13K</td>
</tr>
<tr>
<td>3</td>
<td>-13K</td>
</tr>
<tr>
<td>4</td>
<td>-6K</td>
</tr>
</tbody>
</table>

What is the internal rate of return? Should you accept the offer if the appropriate discount rate is 6 percent? Should you accept it if the appropriate discount rate is 12 percent? Explain, of course

Answer: The internal rate of return is a rate $r$ satisfying

$$0 = c_0 + \frac{1}{1+r}c_1 + \frac{1}{(1+r)^2}c_2 + ... + \frac{1}{(1+r)^T}c_T$$

(9)

Here that becomes

$$0 = 30 - \frac{1}{1+r} \times 6 - \frac{1}{(1+r)^2} \times 13 - \frac{1}{(1+r)^3} \times 13 - \frac{1}{(1+r)^4} \times 6$$

(10)

The solution is: $r = 0.10096$. This is a "financing" project, or a borrowing from nature. Therefore it is accepted if its rate of return is less than the hurdle rate. Thus it should be rejected when the hurdle rate is 6% and accepted at 12%.
1. Office Stuff, Inc. must choose between two copiers, the XX40, which lasts 3 years, and the RH45, which lasts 5 years. Costs and maintenance expenses are expressed in real terms, i.e., in prices of time zero, as follows.

<table>
<thead>
<tr>
<th>Machine</th>
<th>t=0</th>
<th>t=1</th>
<th>t=2</th>
<th>t=3</th>
<th>t=4</th>
<th>t=5</th>
</tr>
</thead>
<tbody>
<tr>
<td>XX40</td>
<td>770</td>
<td>93.5</td>
<td>93.5</td>
<td>93.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RH45</td>
<td>990</td>
<td>121</td>
<td>121</td>
<td>121</td>
<td>121</td>
<td>121</td>
</tr>
</tbody>
</table>

The inflation rate is 5 percent and the nominal discount rate is 14 percent. Revenues are independent of the copier decision. Ignore taxes and depreciation. Which copier should the company choose? Explain why the real discount rate is .085714 and explain the steps for calculating the equivalent annuity for the XX40. Which copier should Office Stuff buy?

Answer:
The discount rate comes from

$$1 + R = \frac{(1 + r)}{(1 + i)}$$

that is

$$R = \frac{1.14}{1.05} - 1 = 0.085714286$$

The present value of the cost of XX40 = 1008.495544, and the PV of cost of RH45 = 1465.928382

Annuity factor for 3 years is $$A_3^{0.085714286} = 2.550754483$$
Annuity factor for 5 years is $$A_5^{0.085714286} = 3.933292411$$

The equivalent annuity, $$x$$, for the XX40 satisfies

$$x * A_3^{0.085714286} = 1008.495544$$

The solution is $$x = 395.3714679$$. Similarly, the equivalent annuity for the RH45 is $$y$$ satisfying

$$y * A_5^{0.085714286} = 1465.928382$$

for which the solution is $$y = 372.6975339$$.

The RH45 is the low cost machine and should be used under the assumptions of the equivalent annuity model. The equivalent annuity is one that has the same life as the machine and the same present value. The annual payment is c where $$c \cdot PVAF(r, T) = PV$$ (cost of machine).

Compare the c’s and choose the machine that has the lower one.
Consider a stock like the one discussed in class. The stock follows a binomial process for two periods, involving three distinct times, 0, 1, and 2. As in class, the stock starts at 50, branches to either 60 or 40, and then branches further to 70, 50, or 30. The safe rate of discount each period is still (unrealistically) equal to 0.1. The task is to value a call having exercise price 55 and expiration at time two. Do the exercise two ways: First, demonstrate it using replicating portfolios consisting of holding fractions of the stock and borrowing. Second, find the risk-neutral probabilities and value the call using them. Explain the steps carefully and concisely.

In this space, develop the replicating portfolios and find the value of the call at each node.

Answer: You can fill in the branches of the tree in the graph. The nodes are

<table>
<thead>
<tr>
<th>t=0</th>
<th>t=1</th>
<th>t=2</th>
</tr>
</thead>
<tbody>
<tr>
<td>S=70 C=15</td>
<td>S=60 C=10.909</td>
<td>S=50 C=5.4545 S=50 C=0 S=40 C=0</td>
</tr>
<tr>
<td>S=30 C=0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

At the 60 node the delta is swing of call over swing of stock = $\frac{3}{4}$. In the down state the stock is worth $\frac{3}{4} \times 50 = 37.5$. Therefore in the replicating portfolio borrow the sum $\frac{37.5}{1.1}$. In the down state the portfolio is worth zero, just like the call. In the up state the portfolio is worth 15, just like the call. The cost of this portfolio in dollars of time 1 is $\frac{3}{4} \times 60 - \frac{37.5}{1.1} = 10.909$.

Now move back to time zero at stock price of 50.

\[ \Delta = \frac{10.909}{20} \] (16)

that is, $\Delta = 0.54545$. In the down state the holding of stock is worth 0.54545 * 40 = 21.818. Therefore complete the replicating portfolio by borrowing $\frac{21.818}{1.1} = 19.835$. Then the portfolio is worth zero in the down state and 0.54545 * 60 - 19.835 = 10.909 in the up state. The call is replicated. The cost of the portfolio is the value of the call, that is 0.54545 * 50 - 19.835 = 7.4375.

In this space, derive the risk-neutral probabilities and show how to get the same answers using them. Note that the risk-neutral probabilities are the same as those derived in class, so you are graded here on the explanation, not on the numbers.

Answer: At the 60 node in time 1, the risk-neutral probability of the up state is the solution to

\[ 60 = \frac{1}{1.1}(p \times 70 + (1-p) \times 50) \] (17)

The solution is $p = 0.8$. At the 40 node in time 1, the risk-neutral probability of the up state (50) is the solution to

\[ 40 = \frac{1}{1.1}(p' \times 50 + (1-p') \times 30) \] (18)

to which the solution is $p' = 0.7$. And at the 50 node at time 0, the risk-neutral probability of the up state (60) is the solution to

\[ 50 = \frac{1}{1.1}(\bar{p} \times 60 + (1-\bar{p}) \times 40) \] (19)

that is, $\bar{p} = 0.75$. Assembling that information, the probabilities of the final nodes at 70, 50, and 30, are, respectively, .6, .375, and .075. The call is valuable only at the 70 node, in which case it is worth 15. Its present discounted value is then

\[ \frac{.6 \times 15}{1.1^2} \] (20)

: 7.438 which checks closely enough given the precision of this software.

A task to think about, but not turn in: In 200 words (count ‘em) and as many diagrams or tables as you like, what are the main problems with using internal rate of return as an investment criterion?
Instructions. Write your answer in the space provided or in an equal space elsewhere, or limit your answer to 220 words. Spill-overs and unreasonably tiny writing are penalized.

1. Here is a variation on the Monty Hall game. There are three doors. The prize is behind one door. The contestant chooses a door. Then the host chooses a door at random and opens it, regardless of whether it conceals the prize. Suppose the door opened by the host does not lead to the prize. Now the contestant has an opportunity to switch doors. Should he switch, or does it matter? Explain.

Answer: Intuitively, the host can play his role without knowing where the prize is. Therefore his actions reveal nothing. After the host opens one door, the contestant may switch or not. Either way the probability at that point in time of winning the prize is one-half. More analytically, build a three

Once the host opens a door and it does not have the prize behind it, the contestant knows only that he is in one of the top two states. Whether to switch or stay is immaterial. The chance of winning is 1/2 either way. Taken from the beginning, the chance of winning is 1/3 whether the policy is switch or stay.

2. This is a call valuation problem. Today is time zero, the call expires in three weeks. At that time the underlying share will be valued at either 68 or 72. The current price is 69. The safe discount rate is zero. Show that the value of the call with strike price $X = 70$ is .5. For full credit, find the value two ways.

Answer: The risk-neutral probability of 72 is .25. The call is worth 2 at that point. Thus its current value is .25 * 2 = .5. Or, the $\Delta$ is .5. In the down state, that share is worth 34. Thus borrow 34. Then the value of the portfolio in the up state is .5 * 72 – 34 = 2 which confirms that the portfolio replicates the call. The cost of the portfolio is $\frac{1}{2} * 69 – 34 = .5$

More slowly, the synthetic call:

$$\Delta = .5 \quad (21)$$

$$\begin{array}{c|cc}
\text{up} & \text{down} \\
\text{stock} & 36 & 34 \\
\text{borrow} & -34 & -34 \\
\text{target} & 2 & 0 \\
\end{array} \quad (22)$$

Buy .5 shares. Borrow 34. At expiration the value is either 2 or 0. Cost of the position is $.5 * 69 – 34 = .5$

Second, risk-neutral probabilities. Solve

$$69 = 72p + (1-p)68 \quad (23)$$

Result is

$$p = 1/4 \quad (24)$$
Value of call is
\[ 0.5 = \frac{1}{4} \times 2 + \frac{3}{4} \times 0 \] (25)

2A. Show that 1.5 is the price of the put with exercise price \( X = 70 \) and the same expiration as the call.

Answer: By put-call parity, \( S - X = C - P \). Filling in the values, \( 69 - 70 = 0.5 - P \). That solves out to \( P = 1.5 \). Alternatively, the put has a value of 2 in the down state, and that state has a risk-neutral probability of 0.75. Thus the put is worth \( 0.75 \times 2 = 1.5 \). On the exam some students constructed a replicating portfolio for the put, an excellent idea.

3. When should a tree be harvested? Supposing that the owner needs to sell it before harvest, how much can he expect to receive? Explain fairly carefully. Illustrate in a graph.

Answer: There is a homework problem just like this. The only special feature there is the linearity of the current value function \( V(t) \). A good answer will stress present values.
1. Active Radiation Inc. has designed a new drug to treat the common cold. If it markets the drug immediately there is a 50% chance of a successful launch, in which case the present value of the payoff is $1 billion, and there is a 50% chance of a present value of the payoff of $100 million. Alternatively, the firm could delay the launch by one year and in that time, at a cost of $200 million, it could test market the drug and thereby improve the probability of a success to 75%. The rate of discount is ten percent. Should the firm launch immediately or spend a year (and some money) in redesigning the product?

Answer: The expected present value of an immediate launch is $550. That’s already present value, and there are no remaining unsunk costs. The expected present value of a launch one year in the future is $775. Net present value is $775/1.1 − 200 = 504.55. Better value is attained by immediate launch.

Comment: The discount rate of .1 is rather low for such a risky undertaking. Perhaps the success of the cold medicine really is independent of the performance of the market portfolio.

2. Suppose that you have invested in two stocks, A and B. You expect that returns on the stock will depend on the following three states of the economy, whose probabilities are given.

<table>
<thead>
<tr>
<th>State</th>
<th>Probability</th>
<th>Return on A</th>
<th>Return on B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bear</td>
<td>.4</td>
<td>6.3</td>
<td>-3.7</td>
</tr>
<tr>
<td>Normal</td>
<td>.3</td>
<td>10.5</td>
<td>6.4</td>
</tr>
<tr>
<td>Bull</td>
<td>.3</td>
<td>15.6</td>
<td>25.3</td>
</tr>
</tbody>
</table>

Calculate the mean, standard deviation, covariance, and correlation of the two stocks.

Answers

\[\begin{array}{c|c|c|c}
\text{State} & \text{Probability} & \text{Return on A} & \text{Return on B} \\
\hline
\text{Bear} & .4 & 6.3 & -3.7 \\
\text{Normal} & .3 & 10.5 & 6.4 \\
\text{Bull} & .3 & 15.6 & 25.3 \\
\end{array}\]

At a discount rate of .1, the value in the base case is

\[-4650 + \frac{1}{.1}(1 - \frac{1}{1.1^{10}}) * 750 = -41.575\]

For the option case one needs the present value of cash flows in case of success, namely

\[\frac{1}{.1}(1 - \frac{1}{1.1^{10}}) * 1500 = 9216.9\]
The decision to abandon is taken, if at all, at time one.

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>...</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>success</td>
<td>-4650</td>
<td>1500</td>
<td>1500</td>
<td>1500</td>
<td>...</td>
<td>1500</td>
</tr>
<tr>
<td>failure</td>
<td>-4650</td>
<td>250</td>
<td>0</td>
<td>0</td>
<td>...</td>
<td>0</td>
</tr>
<tr>
<td>Expected cash flow</td>
<td>-4650</td>
<td>750</td>
<td>750</td>
<td>750</td>
<td>...</td>
<td>750</td>
</tr>
</tbody>
</table>

Now the NPV is as before with the addition of \(0.5 \times \frac{250}{1.1} = 113.64\), for a total NPV of \(-41.575 + 113.64 = 72.065\). (I see from comparison to my excel spreadsheet that some rounding error is creeping in. That can’t be helped.) The value of the option to abandon is 72.065, not the whole expected value of 113.64. The reason is that without the option the project would not be undertaken. The payoff would be zero. With the option, the payoff is 72.065. I see from my notes that I may have graded the 113.64 number as correct. I’m sorry if that penalized you. The decision and event tree was my main objective in giving the problem.

My secondary objective was to recognize the option as a put. The option to abandon is a put option. It is the right but not the obligation to sell the marketing campaign for a stated exercise price, 250, on a specific date, \(t = 1\).
Data:
Pick a publicly traded firm. Go to "http://finance.yahoo.com" and look up the ticker symbol for your company. Under Quotes, click on "Historical Prices." Once there, select **weekly** data. Enter the dates **May 3, 2004** for a beginning date and **May 9, 2005** for an ending date. Download the adjusted closing prices to your spreadsheet. If these instructions don’t work perfectly or you prefer another source, okay, but get weekly data on adjusted prices for exactly the same period.

Get the S&P 500 index for the same period by going to http://finance.yahoo.com/. Look under “Today’s Market.” Click on the S&P 500, and its historical prices. Use the same dates as above and download the price data. Transfer both streams of price data to one spreadsheet and in both series, eliminate all price data except the adjusted closing price.

**Computations in the spreadsheet:**
The assigned computations should be done without using the built-in excel functions such as "var," "covar" and other statistical functions. You may use the built-in functions to check your work, but be aware of subtle differences among the available statistical functions. The regression section serves as another check.

1. Use adjusted closing prices for the firm because they count the dividend.
2. For the firm and the S&P 500, find the weekly rates of return.
3. Find the sample average rates of return in each case.
4. Find the deviations of returns and from them the sample variances.
5. Compute the "cross deviations" and from them the sample covariance.
6. Using the sample covariance, compute the sample correlation coefficient.
7. Compute the estimate of the beta for the firm. It should be the same as the beta you get by regression in the next part.

**Regression:**
1. In excel, under the "Tools" menu, click on "Data Analysis," if it is there, in which case go to 3. Otherwise, go to 2.
2. If "Data Analysis" does not appear in the "Tools" menu, click "Add-Ins" on that menu and activate the "Analysis Tool Pak." Then repeat step 1.
3. Click the regression option.
4. In other spreadsheets, find the linear regression package.
5. Using the regression package, regress the rate of return of your firm as the y variable on the rate of return of the S&P 500 as the x variable.
6. Interpret the coefficients estimated in the regression. Are the coefficients significantly different from zero? That is, is the estimated coefficient greater than twice the standard error of that coefficient?
7. Compare the regression estimate here to the result of your computations above. They should be exactly the same.
1. Suppose there are two stocks in the world, A and B. The expected returns of the two stocks are eleven percent and 19 percent, with standard deviations of five percent and fifteen percent. The correlation coefficient of the two stocks is zero. What is the expected return and standard deviation of a portfolio that is 30 percent A and 70 percent B? 90 percent A and 10 percent B? Would a risk-averse investor willingly hold a portfolio that is 100 percent invested in stock A? Explain, of course.

Answer:

<table>
<thead>
<tr>
<th>wt on A</th>
<th>wt on B</th>
<th>portfolio mean</th>
<th>portfolio variance</th>
<th>portfolio st. dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.7</td>
<td>0.166</td>
<td>0.01125</td>
<td>0.106066017</td>
</tr>
<tr>
<td>0.9</td>
<td>0.1</td>
<td>0.118</td>
<td>0.00225</td>
<td>0.047434165</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>.11</td>
<td>.0025</td>
<td>.05</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>.19</td>
<td>.0225</td>
<td>.15</td>
</tr>
</tbody>
</table>

A risk averse investor would prefer the .9, .1 portfolio to a portfolio consisting of only asset A. The former has higher expected return and lower risk. The cause here is the diversification effect of adding another asset to the portfolio. The high-risk asset lowers the risk in the portfolio. This is an interesting possibility. Because of diversification effects, adding a little of a risky asset to a portfolio raises the risk by less than one would at first think.

2. The equity beta for Showy Sneakers Inc. is 1.3. It has a debt-to-equity ratio of .5. The expected return on the market is 15 percent, the risk-free rate is 7 percent, the cost of debt capital to Showy Sneakers is 8 percent, and the corporate tax rate is 34 percent. What is the required return to the equity of Showy Sneakers and what is its weighted average cost of capital? Explain, of course.

Answer:

\[
R_s = R_F + \left[ E[R_M] - R_F \right] \beta
\]

\[
R_s = .07 + .08 \times 1.3 = 0.174
\]

The weighted average cost of capital is

\[
R_{WACC} = \frac{S}{S + B} R_s + \frac{B}{S+B} R_B (1 - T_C)
\]

Here find the weights by using \( \frac{B}{S} = .5 \). That means \( \frac{S}{S+B} = \frac{2}{3} \) and \( \frac{B}{S+B} = \frac{1}{3} \). Thus

\[
R_{WACC} = \frac{2}{3} \times 0.174 + \frac{1}{3} \times .08 \times (1 - .34) = 0.1336
\]

3. Suppose that at time \( T \) a stock will have a value either of 54 or 44 with equal probability. The safe discount rate is zero. What are the risk neutral probabilities?

Answer: Duh. Bad question. Go with risk-neutral probabilities of .5, .5.

What is the value of a call on the stock with exercise price 50? What is the value of a put with the same exercise price?

Answer: The call is worth .5 \* 4 = 2. The put is worth .5 \* 6 = 3.

The current price of the asset is 50. Put call parity is not satisfied. Construct the riskless arbitrage that allows a trader to profit from the situation.

Answer: Look at \( S - X \) versus \( C - P \), the way options traders do. Here 50 - 50 = 2 - 3. The stock is over-priced, which we already know from the risk-neutral probabilities. Sell the stock (short), earning 50. Now you want to hold an asset that exactly hedges your short position in the stock. Looking at put-call parity again, use

\[
S = X + C - P
\]

This says that \( X + C - P \) with have the same performance at expiration as will the stock. That’s what the hedger wants, an imitation stock to cover his short position. Therefore he buys the bond for 50 (lends the 50 in other words), sells the put for 3 (short the put because of the minus sign) and buys the call for 2. He places the extra dollar in his pocket. The position is fully hedged. If the stock goes to 54. Exercise the call,
earning 4, and retrieve the loan, getting 50. Use that to buy the stock at 54 and close that short position. If the stock goes to 44, the trader still gets 50 from his bond and therefore he has exactly enough when the holder of the put exercises it, costing 6. This is a money pump. Repeat as often as desired or until put-call parity returns.

(Suppose the stock is at 48. It is underpriced. The arbitrageur buys the stock and then looks for a way to hedge. He wants the hedge to short the stock at expiration. The operative form of the put-call parity equation becomes

\[-S = -X - C + P\]  \hspace{1cm} (38)

It says that a short position in the stock is the same as shorting the bond and the call while selling (going long in) the put. Shorting the bond means borrowing 50. Shorting the call means selling it, for 2. Buying the put costs 3. Total income is 50 + 2 = 52. Total outgo is 48 + 3 = 51. The trader puts a dollar in his pocket and sits on a fully hedged position until expiration.
1. (Like 15.1) Gaucho and Triton Corp. are identical in every way except for their capital structures. They have the same (random-valued) EBIT. Neither firm pays taxes. There is no possibility of financial distress. Gaucho is an all-equity firm with 100,000 shares of stock outstanding. Each share of Gaucho sells for $20. Triton Corp. uses leverage in its capital structure. Its debt has a market value of $500,000.

Suppose that you can borrow or lend money at the same rate as Triton can. Then answer and explain the following.

a. What is the value of Gaucho?

b. What is the value of Triton and why? What is the value of the equity of Triton?

c. Suppose that an investor is thinking of buying 1 percent of Triton’s equity. Show how she can use homemade leverage to get the same cost and the same pattern of returns by buying Gaucho equity and making one other transaction.

d. Suppose that an investor is thinking of buying 1 percent of Gaucho’s equity. Show how she can use homemade leverage to get the same cost and the same pattern of returns by buying Triton equity and making one other transaction.

Answers:

a. The value of Gaucho is $20 \times 100,000 = $2,000,000.

b. By the Modigliani-Miller theorem with no taxes and no financial distress, \( V_U = V_L \) where \( U \) is Gaucho, the unlevered firm, and \( L \) is Triton, the levered firm.

Value of Triton = $2,000,000, of which $1,500,000 is the value of equity and $500,000 is the value of debt.

c. The investor borrows (sells a bond for) $5,000 and uses that, plus another $15,000 to buy 1% of Gaucho. The cost is $15,000, which is the same as the cost of 1% of Triton’s equity. The cash flow is that of 1% of Gaucho less payments of interest on 1% of Triton debt. That is the same as the cash flow as from the equity of Triton. That is homemade leverage.

d. The investor buys 1% of Triton debt and 1% of its equity. Buying the debt is the same as lending $5,000 to any other borrower. The cost of the portfolio is $20,000, which is the same cost as buying 1% of Gaucho. The cash flow of the portfolio is that of 1% of Triton equity plus interest on 1% of Triton debt. But the cash flow of Triton equity is that of Gaucho less the interest on the debt. Thus the cash flow of the portfolio is the same as that on 1% of Gaucho.

2. An all-equity firm is subject to a 34 percent corporate tax rate. The firm’s initial market value is $4,000,000 and there are 200,000 shares outstanding. The firm issues $1,000,000 worth of bonds using the proceeds to repurchase its common stock. The firm is in no danger of financial distress. Personal taxes may be ignored. By the Modigliani-Miller theory, show that the new market value of the firm is $4,340,000 and the new value of the equity is $3,340,000. Explain briefly.

Answer: By Modigliani-Miller for corporate taxes and no threat of financial distress, \( V_L = V_U + TC_B \).

The increase in value stems from the tax shielding of earnings paid as interest. Numerically,

\[
V_L = 4,000,000 + .34 \times 1,000,000 = 4,340,000
\] (39)

By definition of value, \( V_L = S_L + B \) and the values of \( V_L \) and \( B \) are known. Therefore, \( S_L = 3,340,000 \). As usual, old equity gets the increased value.

The equity holders require a 20 percent rate of return on the all-equity firm. The bond rate is 10 percent. What rate of return do they require on the levered firm? (Hint: Modigliani-Miller II with taxes)

Answer: Required return \( r_S \) rises with the debt equity ratio because the stock is becoming more risky as leverage increases. The increase in risk is due to the subtraction from the whole cash flow of the firm to make a safe cash flow to debt. Under the assumption of no financial distress, the required return is given by Modigliani-Miller II, that is

\[
r_S = r_0 + (r_0 - r_B)(1 - TC_B)(\frac{B}{S_L})
\] (40)

\[
= .2 + (.2 - .1) \times (1 - .34)(1,000,000/3,340,000) = 0.219760479
\] (41)
3. In about 100 words, explain why the managers of a nearly bankrupt firm are likely to approve bad projects. (Please, be analytical and concise. Don’t write "They’re gambling with somebody else’s money.").

Hints: Use diagrams, algebra or numerical examples. Don’t try to write the whole answer in the first sentence.

3. Suggestion for exam preparation. In 200 words (count ’em) and a few diagrams, explain the Miller model of debt and equity with personal and corporate taxes. What are the effects of a rise in the rate of taxation of capital gains?
Instructions. This exam is closed book and closed note. Scratch pages are not allowed. Write your answer in the space provided or in an equal space elsewhere. Spill-overs, illegibility, and unreasonably tiny writing are penalized.

1. (a) The formula for put-call parity is \( S + P = Xe^{-r(T-t)} + C \). Define puts and calls and show that at expiration the put-call parity relation holds by definition.

A call is the right but not the obligation to buy a share at a stated exercise price \( X \) on or before a fixed expiration date \( T \).

A put is the right but not the obligation to sell a share at a stated exercise price \( X \) on or before a fixed expiration date \( T \).

Lowercase letters are values at expiration. At expiration, \( p = \max(0, X - s) \), \( c = \max(0, s - X) \). The put-call parity relation at expiration is \( s + p = X + c \). Substituting gives \( s + \max(0, X - s) = X + \max(0, s - X) \) which is the relation that needs to be verified. If \( s > X \), the relation reduces to \( s = s \). If \( s < X \) the relation becomes \( X = X \).

(b) Suppose that at time \( T \) a stock will have a value either of 54 or 48 with equal probability. The safe discount rate is zero. The current price of the asset is 50. What is the value of a call on the stock with exercise price 50? What is the value of a put with the same exercise price? Put call parity is not satisfied. Construct the riskless arbitrage that allows a trader to profit from the situation.

Answer: The call is worth \( 0.5 \times 4 = 2 \), and the put is worth \( 0.5 \times 2 = 1 \). To exploit the failure of put-call parity, recognize that the share is underpriced, i.e., \( S - X < C - P \). Borrow 50 and buy the share. Also buy the put and sell the call, at which point you have a dollar in profit. The remaining position is riskless. At expiration the debt \( X = 50 \) is paid by selling the share, paying off the call if \( s > X \), and exercising the put if \( s < X \).

2. Describe and illustrate the principle of separation and explain its significance for financial management.

This is Harry Hernandez. Read it in the book or review the homework problem.

3. A parent is saving for the college education of a child who will begin college at \( t = 10 \) years from the present, which is \( t = 0 \). The child will complete college in four years. The cost of college will be \$20,000 per year payable at the beginning of each year. The parent will make equal annual deposits beginning at \( t = 0 \), and ending the year the child leaves college. The annual interest rate is six percent. How much money must the parent deposit in each year? Show how value builds in the savings account for the first two years.

Plan: Find the present value of the future payments. Then find the savings amount that has the same present value.

\[
\text{PDV of cost} = \frac{1}{1.06^9} \times 0.06 \left(1 - \frac{1}{1.06^9}\right) \times 20 = 41.01981376
\]

Now solve \( c + c \times 0.06 \left(1 - \frac{1}{1.06^9}\right) = 41.01981376 \). The solution is: \( c = 4.163314086 \), in thousands, of course.
Or

<table>
<thead>
<tr>
<th>Time</th>
<th>payment total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.16314086</td>
</tr>
<tr>
<td>1</td>
<td>4.16314086</td>
</tr>
<tr>
<td>2</td>
<td>4.16314086</td>
</tr>
</tbody>
</table>

4. Answer the following:

(a) In the context of capital budgeting, explain the role of depreciation.
   Depreciation is not in itself a cash flow. Taxes shielded by depreciation allowances are cash flows
   and must be counted as such. One procedure is to calculate the tax shield separately and add it
to the other cash flows. The other procedure is to subtract depreciation from revenue etc. when
calculating taxable profit and then add the depreciation to after tax profits afterwards.

(b) Describe zero-coupon bonds and give some reasons why zeroes of different maturities have different
   yields.
   A zero is a U.S. government bond that has been stripped of its coupons which are sold separately.
The bond has a payment only at maturity. The yield $y_T$ on such a bond satisfies (assuming for
simplicity annual compounding)

   \[
   \text{current price of bond} = \frac{1000}{(1 + y_T)^T}
   \]

   The required yield rises with $T$ under normal circumstances. The reason for the rise is increased
risk of inflation and other hazards in the time before maturity. Exceptions can occur for very
short maturities or very long ones.

(c) Here is a variation on the Monty Hall game. There are three doors. The prize is behind one door.
The contestant chooses a door. Then the host chooses a door at random and opens it, regardless
of whether it conceals the prize. Suppose the door opened by the host does not lead to the prize.
Now the contestant has an opportunity to switch doors. Should he switch, or does it matter.

   Answer: Whether the contestant switches or not makes no difference. The state in which the
contestant chooses correctly the first time and the host has opened one of the two empty doors
occurs with probably $\frac{1}{3}$. In that case it is best to stay. The state in which the contestant
chooses incorrectly and the host has by chance opened the one empty door occurs with probability
$\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$, in which case it is best to switch. The states are equally probable and therefore the
decision to switch is not better than the decision not to switch. Either way, the chance of choosing
the correct one of the two unopened doors is 50-50. The third possible state is the one in which
the contestant chooses incorrectly and the host opens the prize-containing door. That occurs with
probability $\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$ but is of no concern to the decision considered in this problem. More briefly,
the host can play his role with no knowledge of the location of the prize. Therefore his actions
reveal nothing – nothing, that is, beyond reducing the number of possible locations of the prize
from three to two.

_________________ DO NOT WRITE BELOW THIS LINE   ________________