Theory of Finance, 234B  
Problem Set #2, Due Tuesday, April 12, 2005  
Answers

1. Suppose that a tree can be planted for a cost of $140. Its value if cut at time $t$ is $-200 + 40t$. The land cannot be reused after harvest. The continuous rate of interest is 6%. Assuming that the tree is planted, when should the tree be cut? Should the tree be planted at all? What is the value in time-$t$ dollars of the partially-grown UNcut tree at time $t$? Explain everything and illustrate in a diagram.

Briefly, 

$$\maximize (-200 + 40t)e^{-0.06t}$$  \hspace{1cm} (1)

which is the value in time zero dollars of the tree if it is cut at time $t$. Differentiate and solve for $t^*$ in 

$$\frac{40}{-200 + 40t} = .06$$  \hspace{1cm} (2)

Solution is: $\{[t^* = 21.667]\}$ Present value of the tree is $(\exp(-.06 * 21.666666)) * (-200 + 40 * 21.666666) = 181.69$. Planting is worthwhile because the net present value is positive, and in fact is $181.69 - 140 = 41.69$.

The tree is now an investment like any other. As such, it earns the market return of $.06$. Thus the market value of the tree at time $t < t^*$ is 

$$181.69 * e^{0.06t}$$  \hspace{1cm} (3)

Please forgive the rounding.

2. The formula for put-call parity is $S + P = Xe^{-r(T-t)} + C$. Define puts and calls and show that at expiration the put-call parity relation holds by definition. (Hint: for the second part, it helps to write the values of puts and calls in terms of $\max( , )$ functions.) A call is the right but not the obligation to buy a share at a stated exercise price $X$ on or before a fixed expiration date $T$.

Answer: A put is the right but not the obligation to sell a share at a stated exercise price $X$ on or before a fixed expiration date $T$. Lowercase letters are values at expiration. At expiration, $p = \max(0, X - s)$, $c = \max(0, s - X)$. The put-call parity relation at expiration is $s + p = X + c$. Substituting gives $s + \max(0, X - s) = X + \max(0, s - X)$ which is the relation that needs to be verified. If $s > X$, the relation reduces to $s + 0 = X + (s - X)$. If $s < X$ the relation becomes $s + (X - s) = X + 0$, that is, $X = X$.

3. Suppose that at time $T$ a stock will have a value either of 54 or 48 with equal probability. The safe discount rate is zero. The current price of the stock is 50. What is the value of a call on the stock with exercise price 50? What is the value of a put with the same exercise price? Put call parity is not satisfied. Construct the riskless arbitrage that allows a trader to profit from the situation.

Answer: The call is worth $.5 * 4 = 2$, and the put is worth $.5 * 2 = 1$. To exploit the failure of put-call parity, recognize that the share is underpriced, i.e., $S - X < C - P$. Borrow 50 and buy the share. Also buy the put and sell the call, at which point you have a dollar in profit $(= 50 - 50 + 2 - 1)$. The remaining position is riskless. Here’s why: At expiration the debt $X = 50$ is paid by selling the share and either paying off the call if $s > X$, or exercising the put if $s < X$. In case $s = 54$ the holder of the call exercises it leaving the options trader with $54 - 4 = 50$, and in case $s = 48$ the trader exercises the put and thus ends with $48 + 2 = 50$. That shows that the position is riskless.