1. In the Miller “clienteles” model, the leverage of an individual firm is irrelevant to the value of the firm, with a few identifiable exceptions. Explain the result and the exceptions. Illustrate in an appropriate diagram.

2. At time zero investors trade assets in a complete set of financial markets. The states of the economy at time one are \( s = 1, 2, ..., S \), the prices of state claims are \( p_s \), and the probabilities of the states are \( \pi_s \).

   - By definition, the risk-free rate of return in this economy satisfies
     \[
     \frac{1}{1 + R_F} = \sum_S \pi_s p_s
     \]
     Explain why that makes sense.

   - What is the payoff to the market portfolio in this economy? Introduce the needed notation and explain why the market portfolio is important in the model.

3. Suppose that a tree can be planted at time zero for a cost of $350. It's value in time \( t \) dollars if cut at time \( t \) is \(-200 + 60t\). The land cannot be reused after harvest. The continuous rate of interest is 5%.

   - Assuming that the tree is planted, when should the tree be cut?
     \[\frac{V'(t)}{V(t)} = r \text{ when } \frac{60}{-200 + 60t} = .05, \text{ Solution is: } \{t = 23.333\} \]

   - At each time \( t \) there is a market in partially grown trees. What is the value in time-\( t \) dollars of the partially-grown tree at time \( t \)? Illustrate in a diagram
     The present value of the tree at time zero is \(-30000 + 1.1(1 - \frac{1}{1.1})^{7} \) = 13816. Its value at time \( t \) in time \( t \) dollars is 13816 * \( .05 + .6 \) * \( .05 \). As a check, look at the cutting date. Then the value is 13816 * \( .05 + .6 \) * \( .05 \) = 1200.0. Checking, \( \frac{60}{1200} = 0.05 \). Check succeeds

   - Should the tree be planted at all?
     Yes. The net present value of planting the tree is positive by a good margin

4. Explain beta, the theory behind beta, and the role of beta in the theory of the security market line?

5. A firm is developing a new type of garbage can lid which may or may not keep out flies. The development effort costs $5000. If the lid is a success, it gives rise at time one to a project that will cost $30,000 to start and then will yield a net cash flow of $9000 at each of times 2 through 8. If the lid fails, the project will be scrapped at no further cost. Obviously, if the firm doesn’t develop the new lid, it will never had the opportunity to market it. The discount rate is ten percent for projects that sell garbage can lids. The safe rate is two percent. The risk-neutral probability of success in developing the lid is .4. You might need the formula \( PVAF(T, r) = \frac{1}{r} \left( 1 - \frac{1}{(1+r)^T} \right) \).

   - Explain why the firm is purchasing a call option. What are the exercise price, maturity, and other variables.

   - Compute the value of the option and compare it to the cost.
     Suppose the research is a success. The value of the project in time one is \(-30000 + \frac{1}{r} \left( 1 - \left( \frac{1}{1.1} \right)^7 \right) \) * 9000 = 13816.
     Since the probabilities are risk neutral, the value of the option is \(.4 \cdot 13816 / (1.02) + .6 \cdot 0 / (1.02) = 5418.0 \).
     The option is worth more than it costs. The project has a positive net present value. It should be undertaken. Most likely variant answer: \(.4 \cdot 13816 + .6 \cdot 0 = 5526 \). Second most likely variant \(.4 \cdot 13816 / (1.1) + .6 \cdot 0 / (1.1) = 5024.0 \)