Learning About Climate Sensitivity

From the Instrumental Near-Surface Temperature Record

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ABSTRACT

The debate over the magnitude of anthropogenically induced climate change has raged for over a century [Arrhenius, 1896; Callendar, 1938; Hansen and al., 1981; Houghton et al., 1996; Idso, 1980; Newell and Dopplick, 1979]. Today considerable uncertainty remains about the magnitude of greenhouse-gas-induced climate change, particularly the climate sensitivity – the equilibrium change in global-mean near-surface temperature per unit of radiative forcing. The rapidity at which uncertainty in the climate sensitivity is resolved has significant policy implications. If resolution is expected soon, deferring action until the picture is clearer may be prudent. If uncertainty is likely to be resolved only slowly, then action today on the basis of expected costs and damages may be the wisest course. Here we use a simple reduced-form model of global mean temperatures, the instrumental temperature record and historic emissions, and IPCC scenarios of future emissions of greenhouse gases and SO$_2$ to estimate the time required to reduce the uncertainty in the climate sensitivity. We find that more than half a century is required to be 95% confident that the true value of the climate sensitivity lies within $\pm 25\%$ of the estimated value. Further, accelerated control of greenhouse-gas emissions significantly slows this rate of learning, while control of SO$_2$ emissions accelerates it.

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I. INTRODUCTION

Although a great deal is known about climate change and the role of greenhouse gases in that change, there remains considerable uncertainty which is only gradually being resolved. It might seem that it is impossible to shed any light on the extent of future resolution of uncertainty; after all, who can know what future research on climate will yield? Yet we have over a century of data on the global-mean near-surface temperature and it has revealed much. The global-mean instrumental temperature record has been used as an important piece of evidence for concluding that a perceptible change in the global climate has occurred. For instance in its Second Assessment Report [Houghton et al., 1996], the IPCC concluded that "the balance of evidence suggests a discernible human influence on global climate." A major reason for reaching this conclusion was stated to be that "assessments of the statistical significance of the observed global mean surface air temperature trend over the last century ... have detected a significant change and show that the observed warming trend is unlikely to be entirely natural in origin." The IPCC went on to say that "our ability to quantify the human influence on global climate is currently limited because the expected signal is still emerging from the noise of natural variation" [p5, Houghton et al., 1996].

In statistical terms, the IPCC was saying that the effect of greenhouse gases on global-mean near-surface temperature is significantly different from zero, but that its magnitude is subject to significant uncertainty. Here we ask the natural follow-up question: "Given how noisy the instrumental temperature record is, how many more years of observations do we think will be necessary in order to narrow the uncertainty regarding climate sensitivity to some predetermined level?" We pose this question in terms of climate sensitivity, defined as the change in global-mean near-surface temperature per unit radiative forcing at the tropopause. The reason for doing so is that this is the most important parameter of the climate system for human-induced climate change, both in terms of the resulting damages and the cost of abatement to avoid those damages. This conclusion is supported by value-of-information studies which show that a large benefit accrues to
reducing the uncertainty in the climate sensitivity [Lempert and Schlesinger, 2000; Peck and Teisberg, 1993].

It may seem that the geographical distribution of near-surface temperature is required to estimate the climate sensitivity, rather than the global-mean near-surface temperature. To understand why this is not so, we write the near-surface temperature change as a function of geographical position x and t by

\[ T(x, t) = \bar{T}(t)P(x, t) + N(x, t) \]

where \( \bar{T}(t) \) is the amplitude of the forced temperature change, \( P(x, t) \) is the nondimensional geographical pattern of the forced temperature changes, and \( N(x, t) \) is any unforced temperature change; that is, noise. We now write \( \bar{T}(t) = \lambda F(t) + \delta T(t) \), where \( F(t) \) is the global-mean radiative forcing at time t, \( \lambda F(t) \) is the instantaneous equilibrium temperature change the climate system would have if it had zero thermal inertia, and \( \delta T(t) \) is the difference between \( \bar{T}(t) \) and \( \lambda F(t) \) due to the climate system's non-zero thermal inertia. Combining the two equations above yields

\[ T(x, t) = \lambda F(t) + \delta T(t)P(x, t) + N(x, t) \]

This equation shows that \( T(x, t) \) contains three types of information: (1) the global-mean forced temperature change, \( \bar{T}(t) = \lambda F(t) + \delta T(t) \), which can be reproduced by a simple climate/ocean model to estimate the climate sensitivity \( \lambda \) [e.g., Schlesinger and Ramankutty, 1992; Schlesinger and Ramankutty, 1994]; (2) the nondimensional pattern of forced temperature change, \( P(x, t) \), which can be analyzed using fingerprint-detection methods [e.g., Barnett and Schlesinger, 1987; Santer et al., 1995] to learn which factors have been forcing the climate system, such as volcanoes and putative solar-irradiance changes [e.g., Hegerl et al., 1997; Tett et al., 1999]; and (3) the noise \( N(x, t) \), which can be analyzed to determine if there are unforced periodic or nearly periodic variations, and if so, where they occur [Schlesinger and Ramankutty, 1994].

Accordingly, our approach to answering the question posed above is to assume that the process of learning about climate sensitivity that has occurred during the last 100 years will continue and will be based on the instrumental temperature record. Furthermore, we assume that the instrumental temperature record will continue to expand as it has in the past. Given these
assumptions, we ask how long should we expect to have to wait to reach specific levels of confidence about climate sensitivity. Using a simplified model of learning we show that achieving some confidence regarding the value of climate sensitivity may take many decades. We do this by considering only human-induced radiative forcing, that is greenhouse gases (GHGs) and sulfur dioxide (SO$_2$), both in the past and in the future. The presence of non-human radiative-forcing factors, such as volcanoes and solar-irradiance changes, could accelerate our learning, if we knew that such forcing was at work on the climate system, or delay it if such forcings were misinterpreted as unforced climate variability. We show that accelerated control of GHGs and SO$_2$ emissions will have opposing effects on the rate at which we learn about climate sensitivity, the former slowing learning and the latter accelerating it. The policy implication is that if we wait until uncertainty is resolved before controlling emissions of greenhouse gases, we may wait a very long time.

II. A REDUCED-FORM GLOBAL TEMPERATURE MODEL

Define the climate sensitivity as the change in global mean temperature from a unit increase in atmospheric forcing: $\lambda = \partial T / \partial F$. A variant is the more familiar $T_{2x} = \lambda F_{2x}$ (with $F_{2x} = 3.71$ Wm$^{-2}$; Myhre et al., 1998): the equilibrium temperature rise from a doubling of GHG concentration from preindustrial levels. In either case, the key variable of interest is $\lambda$, climate sensitivity.

One approach to estimating the climate sensitivity, $\lambda$, is to use a simple physical model to represent the evolution of the instrumental temperature record. The parameters of this model can be estimated in a best-fit, maximum-likelihood sense using the instrumental record [Andronova and Schlesinger, 2000]. One such simple physical model is [Nordhaus, 1994; Schneider and Thompson, 1981]:

$$T_t = T_{t-1} + \frac{1}{\alpha} \left[ (F_t + F_{SO_2}) - \frac{T_{t-1} - \Gamma}{\lambda} \right] - K(T_{t-1} - O_{t-1})$$

(1a)
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\[ O_t = O_{t-1} + \alpha (T_{t-1} - O_{t-1}) \]  

(1b)

where \( T_t \) is the annual global temperature (°C) difference of the upper ocean in year \( t \) from the 1961-1990 average temperature, taken to be synonymous with the near-surface temperature difference, and \( T_0 = O_0 = \Gamma \) is the temperature difference in some base year; \( O_t \) is the corresponding temperature difference for the deep ocean; \( F_t \) is the radiative forcing by greenhouse gases (GHGs), including tropospheric ozone; \( S_t \) is the emission rate of sulfur dioxide (SO\(_2\)), normalized by its value in 1990 (75 TgS/yr), which is converted to sulfate aerosol in the atmosphere; \( F_{SO_4} \) is the radiative forcing by sulfate aerosols in 1990; \( \alpha \) is the heat capacity of the upper ocean; and \( K \) and \( L \) equal the coefficient of heat transfer between the upper and deep ocean divided by their respective heat capacities. Equation (1) can be written as the following statistical model [Kelly and Kolstad, 1999],

\[ T_t = \frac{\Gamma}{\alpha \lambda} + \beta_1 T_{t-1} + \beta_2 F_t + \beta_3 S_t + K O_{t-1} + \varepsilon_t, \]  

(2a)

\[ O_t = O_{t-1} + L (T_{t-1} - O_{t-1}), \]  

(2b)

where \( \varepsilon_t \) is an error term of mean zero, which is assumed to be serially uncorrelated. Equation (2b) could also involve an error term but does not, for reasons which will become clear. Equation (2) could be estimated from observed records of \( T_t \) and \( O_t \), from which \( \lambda = \beta_2 / (1 - \beta_1 - K) \) and \( F_{SO_4} = \beta_3 / \beta_2 \). However, the absence of an observational record for the deep-ocean temperature, \( O_t \), makes it impossible to statistically estimate all of the coefficients. This temperature changes very slowly over time and thus it is unlikely that \( O_t \) has changed very much over our sample (1856-1995). Consequently, we make the assumption that \( O_t \) is constant and thus roll \( K O_{t-1} \) into the constant term. Equation (2a) thus reduces to:

\[ T_t = \beta_0 + \beta_1 T_{t-1} + \beta_2 F_t + \beta_3 S_t + \varepsilon_t = \beta X_t + \varepsilon_t, \]  

(3)
where \( \beta = (\beta_0, \beta_1, \beta_2, \beta_3) \) and \( X_i = (1, T_{t-1}, F_t, S_t)' \), where the prime denoted the transpose. Equation (3) can be estimated from the historical record. We conducted two tests on the nature of the error term in Eqn. 3 and are unable to reject the null hypothesis of serially uncorrelated errors.\(^1\) We thus assume each \( \varepsilon_t \) is independent and identically distributed (iid).

We perform ordinary least squares (OLS) estimation of Eq. (3), using the instrumental temperature record (1856–1995: 140 observations) \([\text{Nicholls et al., 1996}]\) and the historical GHG forcing and SO\(_2\) emissions (Fig. 1). Results are summarized in Table 1. Also shown in the table is the implied estimated value of \( \lambda \). Since \( \lambda \) is simply a function of \( \beta_1 \) and \( \beta_2 \), an estimate of \( \lambda \) can be obtained from the estimate of \( \beta \), which we denote \( b \). Since the estimate \( b \) is a random variable, it is straightforward to estimate the distribution of the estimate of \( \lambda \) induced by the distribution of \( b \), using a second-order Taylor series expansion of \( \lambda \) around \( b \). The standard error of the estimate of \( \lambda \), computed in this fashion, is found in Table 1. Since we were unable to estimate \( K \) in Eq.(2) due to the absence of deep-ocean temperature observations, we have used a value of \( K \) for computing the estimate of \( \lambda \), drawn from a more complex climate/ocean model \([\text{Schlesinger et al., 1997; Schlesinger and Jiang, 1991}]\).

The more familiar \( T_{2x} = \lambda F_{2x} \) (with \( F_{2x} = 3.71 \text{ Wm}^{-2} \)) is the equilibrium temperature rise from a doubling of GHG concentration from preindustrial levels (Table 1). The estimates in Table 1 imply \( T_{2x} = 1.99^\circ \text{C} \) with a standard error of 0.60\(^\circ\)C, which implies (assuming normality) a 95% confidence interval of ±60% (±1.2\(^\circ\)C). This central tendency, as well as the level of uncertainty, are not inconsistent with IPCC estimates, though perhaps a bit lower; however, as we shall see

\(^1\) Under the null hypothesis of serially uncorrelated errors in Equation (3) and the alternative hypothesis of AR(1) errors, the Durbin h-statistic has a value of 0.223 which does not permit us to reject the null at a 5% confidence level. We also conducted the Breusch-Godfrey test on Eqn. (3) under the null hypothesis of serially uncorrelated errors and two alternative hypotheses that errors are AR(p) or MA(p) for different values of p. We were unable to reject the null at the 5% level for any value of p between zero and twenty.
later, the time to reach equilibrium from a GHG shock, as implied by the coefficients in Table 1, is unrealistically rapid.

III. LEARNING ABOUT CLIMATE SENSITIVITY

We now define learning, the resolution of uncertainty. In general, learning is a very complex process. However, in keeping with our focus on the instrumental temperature record, we are defining learning in a fairly narrow fashion. Learning, as defined here, occurs as years pass, additional observations are made on the climate, greenhouse-gas concentrations and sulfur emissions, and better estimates are made of the parameters of Eqn. 3. Time expands the instrumental record and gives us better estimates of the key parameters of Eqn. 3 and thus of climate sensitivity.

For example, if in the early 1950's we had estimated the value of $T_{2x}$, using the instrumental record through 1950, we would have found the value to be 5.0°C with a standard error of 5.4°C, suggesting $T_{2x}$ is statistically indistinguishable from zero. Figure 2 shows how the estimates of $T_{2x}$ would have changed over time, from 1950 through 1995, as well as the 95% confidence interval ($\pm 1.98$ standard errors, assuming normality). Note that it is not until the 1970's that the value of $T_{2x}$ settles down to a clearly (statistically) non-zero value. Furthermore, one can see the size of the 95% confidence interval gradually shrinks over time, reaching [0.79, 3.19] in 1995. As noted earlier, the IPCC central estimate of $T_{2x}$ falls within this range.

What do we expect the instrumental record to reveal as more years are added to it? Clearly we cannot expect our estimates of the parameters of Eqn. 3 (b) to change; otherwise they would not currently be best linear unbiased estimates (which, statistically, we know they are). Similarly, we do not expect our estimate $s^2$ of the variance $\sigma^2$ of $\varepsilon$ (the noise in the equation) to change. But the variance of the estimates of $\beta$ is expected to decline as more and more observations are collected. Let $X_{(140+n)}$ be the $(140+n) \times 4$ matrix of observations on $X_t$ for the 140 years of our historic record plus n additional years. Let $b$ be the current estimate of $\beta$. And let $V_{(140+n)}$ be our
current expectations regarding the estimate of the variance of \( b \), after 140+n years of observations. Clearly, given \( b \) and \( V_{(140+n)} \), we can compute an estimate of climate sensitivity (\( \lambda \)) and the variance of that estimate. Furthermore, we can compute a confidence interval around the estimate of climate sensitivity. Keep in mind that these are expectations, based on the information we have today.

How can we compute the expected variance of \( b \) after 140+n observations, \( V_{(140+n)} \)? Consider first the simplification that the \( X_t \) in Eqn. 3 are all exogenous to the system. In this case, as is well known from simple linear regression theory, the expected variance of \( b \), after observing \( n \) more years of data is given by

\[
V_{(140+n)} = s^2 [X'_{(140+n)} X_{(140+n)}]^{-1}.
\]  

(4)

where \( X'_{(140+n)} \) is the transpose of \( X_{(140+n)} \). \( V_{(140+n)} \) should asymptotically approach zero as \( n \) approaches infinity. If we assume the \( X \) are known (a big assumption), then we can compute the variance of our estimate of \( b \) and thus the variance of the estimate of \( \lambda \).

A key assumption in Eqn. 4 is that the \( X_t \) are all exogenous. However, that is clearly not the case, as can be seen from inspecting Eqn. 3. Future greenhouse-gas and sulfur-dioxide emissions can be assumed exogenous and for these we use the IPCC estimates. However, the \( X_t \) contain lagged values of the endogenous variable \( T_t \). This means that a closed-form equation for the variance of \( b \) (such as Eqn. 4) is more difficult to obtain. However, it is straightforward to generate a numerical estimate of the variance of \( b \) and thus of the variance of climate sensitivity.

To numerically compute \( V_{(140+n)} \), we take a series of draws from the distribution of \( b \), our OLS estimate of \( \beta \), and our estimated distribution of \( \varepsilon \) in the present (1995). For each draw on \( b \), we generate a century of draws, \( e \), on \( \varepsilon \): \( (b,e_i) \equiv (b,e_{(1995+1)},e_{(1995+2)},...,e_{(1995+100)}) \). Together with the assumed future \( F_t \) and \( S_t \), this is sufficient to generate from Eq. (3) a sample century (1996-2095) of temperatures (\( T_t \)). Let \( b_{(140+n)} \) be the OLS estimate of \( \beta \) based on data through \( n \)
years after 1995 for a particular trajectory i. The covariance matrix on $b_{(140+n)}$ is calculated by the Monte Carlo method (100 draws) from the distribution of $b_{(140+n)}$.

IV. RESULTS

Our results are presented in Figures 3-5. Shown in the figures are the standard errors on the estimates of $T_{2x}$, estimated using the instrumental record plus the generated temperature data through specific years in the 21st century, as described in the previous section. There are three different IPCC scenarios regarding greenhouse-gas emissions -- high, medium and low. Similarly, there are three IPCC scenarios regarding sulfur-dioxide emissions -- high, medium and low. Consequently, there are nine possible combinations of scenarios of sulfur-dioxide and greenhouse-gas emissions.

Figure 3 shows the evolution of the standard error on $T_{2x}$ assuming the greenhouse-gas and sulfur-dioxide emissions are both at the same level (high, medium or low). Shown in the figure is the standard error for the three scenarios on emissions. Keep in mind that if the estimate of $T_{2x}$ is normally distributed, then a 95% confidence interval that is $\pm 20\%$ of the estimated value of $T_{2x}$ (1.99) requires a standard error of approximately 0.2°C. We see from Figure 3 that it takes approximately 75 years for the standard error on $T_{2x}$ to drop to this level for the mid scenario on emissions (IS92A). Lower joint emissions accelerate the learning and higher joint emissions retard it. The High scenario has high levels of GHGs which increase the temperature signal, but also high $SO_2$ emissions which reduce the signal. The net effect is that the signal is weaker for “High” than for “Low.”

Figure 4 decouples the effects of GHG concentrations from sulfate aerosols by fixing $SO_2$ emissions at the levels associated with the mid-range emission scenario. Figure 4 shows that controlling GHG emissions dramatically slows the rate of learning about climate sensitivity. The most rapid learning occurs when we have the largest signal from GHGs: approximately 50 years to achieve a $\pm 20\%$ confidence band. Aggressive control makes the climate-change trend more
difficult to see within the noisy temperature record. The learning model has difficulty discerning between random warm and cold years and the warm years due to emission of GHGs.

Figure 5 shows the result for GHG concentrations set at the mid emissions scenario level, with varying SO$_2$ emission rates. Here aggressive control of SO$_2$ increases the upward trend in temperature and thus makes the climate change more visible within the noisy temperature record. A lack of control masks the signal.

Several conclusions emerge from these results. One is that it may take quite some time to achieve any sort of tight confidence interval on T$_{2x}$, perhaps the better part of a century. Secondly, the aggressive control of carbon slows down learning while the aggressive control of sulfur accelerates learning.

As mentioned earlier, one troubling feature of Eq. (3) as estimated with the instrumental record is that the response from an instantaneous and permanent increase in GHG forcing is surprisingly, and unrealistically, rapid. Since the effect on the temperature n years after a hypothetical shock occurs is $\beta_1^n T_0$, where $T_0$ is the temperature at the time the shock occurred, the closer $\beta_1$ is to unity, the slower the response to a forcing shock. Because of this, we have also estimated Eq. (3) assuming a value of $\beta_1$ that is consistent with a more complex physical model; in fact, we use a value obtained from a more detailed climate/ocean model [Schlesinger et al., 1997; Schlesinger and Jiang, 1991]. We obtain $\beta_1$ estimates by simulating the effects of a CO$_2$ doubling for 245 years (equal in time to 1856-2100), using the climate/ocean model, for three different assumed temperature sensitivities, T$_{2x}$, and then use the generated data to estimate the appropriate coefficient values in Eq. (1) with $O_t$ neglected. For T$_{2x}$ = 1.5°C, 2.5°C, and 4.5°C, the results imply $\beta_1 = 0.87941, 0.92277$ and 0.95167, respectively. Consequently, we have also estimated Eq. (3) assuming a fixed value for $\beta_1$ of 0.9, and then estimated the equation again assuming $\beta_1 = 0.94$. The qualitative results of this exercise are clear: increasing the value of $\beta_1$ only serves to slow down learning about climate sensitivity. This is as would be expected since fixing $\beta_1$ leads to a poorer fit of Eq. (3) to the instrumental record, and thus more error. This
logically increases the amount of time necessary to reduce the error in the estimate of climate sensitivity.

V. CONCLUSIONS

In conclusion, using a simplified model of learning we have shown that achieving some confidence regarding the value of climate sensitivity may take many decades. Although additional factors could be included in our model (such as volcanoes and the sun), the results suggest a very slow resolution to the question of the magnitude of climate sensitivity. This notwithstanding, we have shown that accelerated control of GHGs and SO$_2$ emissions will have opposing effects on the rate at which we learn about climate sensitivity, the former slowing learning and the latter accelerating it. The policy implication is that if we wait until uncertainty is resolved before controlling emissions of greenhouse gases, we may wait a very long time.
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REFERENCES


Figure Captions

**Figure 1.** (a) Global average temperature relative to 1961-1990 (°C); (b) Radiative forcing due to greenhouse gases (GHGs) (22), and (c) SO₂ emission (22) normalized by its 1990 value of 75 TgS/yr.

**Figure 2** Estimated equilibrium temperature response to doubling of greenhouse gases, T_{2x}. Variable is estimated using the instrumental temperature record through the year indicated on the horizontal axis. Error bars indicate 95% confidence intervals.

**Figure 3.** Projected standard error on T_{2x}, assuming future Carbon and Sulfur emissions consistent with the same IPCC forecast, IS92A, IS92C or IS92E.

**Figure 4.** Standard Error on T_{2x} with assumed future Sulfur (S) emissions at Medium Level, IS92A, and future Carbon (C) emissions at different IPCC forecast levels.

**Figure 5.** Standard Error on T_{2x} with assumed future Carbon (C) emissions at Medium Level, IS92A, and future Sulfur (S) emissions at different IPCC forecast levels.
Table 1. Ordinary Least Squares Estimates of Parameters in Equation (3)

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Estimate</th>
<th>Standard Error</th>
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<tr>
<td>$\beta_0$</td>
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<td>0.0427</td>
</tr>
<tr>
<td>$\beta_1$</td>
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<td>0.0709</td>
</tr>
<tr>
<td>$\beta_2$</td>
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<td>0.0755</td>
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<td>$\beta_3$</td>
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</tr>
<tr>
<td>$\lambda$</td>
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<td>0.1611</td>
</tr>
<tr>
<td>$T_{2x}$</td>
<td>1.9852</td>
<td>0.5977</td>
</tr>
</tbody>
</table>

NB: 1. $\lambda \equiv \beta_2 / (1 - \beta_1 - K)$ computed using $K = 0.012$ from a more complex climate/ocean model [Schlesinger et al., 1997; Schlesinger and Jiang, 1991].

2. $T_{2x} \equiv \lambda F_{2x}$, with $F_{2x} = 3.7\, \text{W m}^{-2}$ [Myhre et al., 1998].
Fig. 1
Estimated Using Data Through Year

Figure 2
Figure 3
Figure 4
Figure 5