

# HYPOTHETICAL BIAS IN PRIVATE VALUE AUCTIONS WITH COSTLY INFORMATION ACQUISITION\*

**Rolando M. Guzman<sup>+</sup>**  
**and**  
**Charles D. Kolstad<sup>++</sup>**

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## ABSTRACT

A number of recent papers in environmental economics have focused on the process of researching preferences – agents are uncertain about preferences but at a cost may narrow their uncertainty. This issue has arisen in formulating bids in contingent valuation as well as the debate over the divergence between WTP and WTA. In the context of contingent valuation, it has been hypothesized that the hypothetical nature of the preference elicitation biases responses. This paper provides both a theoretical model and experimental evidence to contribute to this debate. The model consists of a series of auctions where subjects compete for an object with private but unknown value – the Risk Neutral Rational Expectations model. The information regarding the value of the object is costly. Furthermore, at the end of the auction, a random number is drawn to determine if the whole process is hypothetical or not. The experiment tests this theoretical model of bidding equilibrium and analyzes the effects of variations in the parameters (hypotheticalness, information costs and number of agents) on the endogenous variables (such as the proportion of bidders who buy information and the winning bid). Experimental results are generally consistent with the theoretical model. In particular, an increase in the hypotheticalness of an auction tends to decrease the size of the winning bid and decrease the extent to which bidders pay for information on their valuation.

<sup>+</sup>Grupo de Consultoría Pareto, Dominican Republic.

<sup>++</sup>Department of Economics and Bren School of Environmental Science & Management, University of California at Santa Barbara, USA (Corresponding author; email: kolstad@econ.ucsb.edu).

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## **I. INTRODUCTION**

Imperfect information regarding valuation combined with costly information acquisition have become recognized as significant determinants in the formation of bids, both in auctions as well as contingent valuation studies. Rasmusen (2003) terms this “value discovery” in the context of auctions whereas Cummings and Harrison (1994), in the context of contingent valuation, term this “researching one’s preferences.” In either case, the problem is that agents must articulate a bid for a good when they are unsure of their own private valuation of that good. Furthermore, the extent to which they refine their knowledge of their own private value is endogenous – information can be acquired at a cost.

Costly information acquisition means that agents may choose to remain ignorant and to form bids in a state of ignorance, a process which can distort bids relative to the perfect information case. This phenomenon has been advanced as an explanation for a number of apparent anomalies in rational behavior. Kolstad and Guzman (1999) use such informational arguments to address the well-known question of the apparent divergence between willingness-to-accept compensation for losing an environmental good and willingness-to-pay for that good. Rasmusen (2003) and Compte and Jehiel (2004) argue that costly “value discovery” can explain a good deal of behavior in private value auctions. The concept of “researching one’s preferences” has emerged as an explanation for otherwise anomalous behavior in

forming contingent valuation “bids” for environmental goods.

A specific issue of concern in this paper is hypothetical bias in contingent valuation (CV). The fact that the hypothetical nature of CV reduces the incentive to acquire costly information may lead to a bias in bids, relative to the non-hypothetical case. In fact, a number of authors suggest that hypothetical bias is one of the most serious problems associated with CV.

How agents develop responses to environmental valuation queries is fundamentally a question of how agents formulate “bids” based on their underlying values. One approach to examining the influence of hypotheticalness in CV is to look at behavior over the continuum between pure non-hypothetical (a real market experience) and pure hypothetical (a CV experience). We are interested in limiting behavior as the probability of hypotheticalness goes to unity.

Although CV is not an auction, the analogy with a standard auction is clear. The primary issue is the formation and articulation of bids in an environment of uncertainty with costly information acquisition. Although the theory of bidding in this context has been somewhat developed, the connection to actual behavior by agents is more tenuous.

The primary issue explored in this paper regards how an agent’s bid is influenced by the *ex ante* likelihood of winning the auction or buying/selling an environmental good. We develop a model in which agents bid in an atmosphere of uncertainty – uncertainty which can be reduced at a cost. We parameterize the

likelihood of winning by hypothesizing that when bidding is done, a random number will be drawn to determine if the bidding was hypothetical or not. The question is how does an agent's bid and information acquisition change as the extent of hypotheticalness changes?"

We first develop a theoretical model of bidding in a first-price auction and show (among other things) that the more hypothetical an auction is (ex ante), the lower the proportion of bidders who choose to become informed and the lower the level of the winning bid. We also investigate the influence of the number of bidders and the magnitude of information costs.

We then proceed to construct an experiment to validate the theoretical predictions. The experiment consists of a series of auctions where the subjects compete for an object with a private but unknown value. The distinctive feature of the environment is that the information about the private values of the object is costly: a bidder only discovers his own valuation provided he pays some amount to become informed.

The structure of the paper is as follows. Section II provides a concise discussion of the literature on costly information acquisition and a motivation for this research. Section III presents the theoretical model in which our analysis is based, while section IV uses the model to develop theoretical predictions regarding the effect of hypotheticalness, information costs and the number of agents. In sections V and VI these theoretical predictions are tested in the context of a laboratory experiment.

Finally, the paper closes with some conclusions.

## II. BACKGROUND

The role of information in auctions has been extensively studied, both from theoretical and empirical standpoints. Indeed, a commonly stated motivation for the study of auctions is that they provide a convenient device for the analysis of the price system under informational asymmetries (Milgrom, 1981; Milgrom and Weber, 1982; Kagel et al., 1987). An unpleasant shortcoming of the conventional approach, however, is that the information level of the bidders is typically treated as an exogenous variable. That is, most models take a certain level of information as a given, and then consider the effects of arbitrary changes in that level. Of course, it would be more realistic to view the information possessed by each agent as a decision variable, whose level depends on the comparison between its acquisition cost and its expected benefit.

Surprisingly, models with endogenous information acquisition are relatively scarce. In a comprehensive survey by McAfee and McMillan (1987), for example, the case of costly information acquisition is exhausted by a single footnote.<sup>1</sup> A similar observation is valid for the survey by Wilson (1992), in which the topic merits a single reference.<sup>2</sup> Indeed, the number of papers in which the acquisition of information is

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<sup>1</sup> McAfee and McMillan (1987), p. 722, fn. 21.

<sup>2</sup> Wilson (1992), p. 241.

endogenous is quite short. Mathews (1984) and Lee (1985) were perhaps the first to set up auction models focused on that topic, though Schweizer and Ungern-Sternberg (1983) used simulation methods to analyze the role of costly information in a two-agent framework. Over the past decade, a few more authors have addressed this issue.<sup>3</sup> In particular, Rasmusen (2003) and Compte and Jehiel (2004) have developed models similar to that presented here, though these authors focus on sequential second-price auction settings.

The same pattern can be found in experimental research. Although there are a number of studies dealing with the effects of information, almost all are confined to the following topics: the effects of public provision of information on the revenue of the seller (Kagel and Levin, 1986), the effects of feedback information about previous outcomes in repeated auctions (Isaac and Walker, 1985; Cox et al., 1984), and the effects of uncertainty about the number of bidders (Dyer et al., 1989; Battalio et al., 1990). In an extensive survey of the empirical literature by Kagel (1995) no reference is made to any direct study of bidders' behavior under costly information acquisition. Rasmusen (2003) is an exception, in that he examines eBay and Amazon auctions in the context of costly information acquisition. He concludes that value discovery explains the last-minute bidding found in these auctions.

This paper attempts to fill a gap in the literature, proposing a theoretical

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<sup>3</sup> Persico (2000) provides a very general model in which she shows, among other things, that a first-price auction provides a greater incentive to acquire information than a second-price auction. See also Engelbrecht-Wiggans (1988, 2001) and Hausch and Li (1993).

model of information acquisition and an experiment in which the endogenous nature of information acquisition plays a major role. More specifically, the auctions in the experiment treat the acquisition of information as an endogenous variable, rather than an exogenous parameter.

Our goal is to examine the strength of the economic incentives in a well-controlled environment. How rational are the bidders with respect to the acquisition of information? How does the additional complexity of the environment affect their ability to take optimal decisions? Can the outcome of the auctions be accurately predicted by some particular paradigm? How is the seller's revenue affected by the ignorance of the bidders about their valuations? In spite of their relevance, these interesting problems have never been submitted to close examination through experimental research. The only way to do it, of course, is to take into explicit account the possibility of costly information acquisition, but that has not been done in previous experiments.

Thus, although its structure is probably very specific, we feel that our experiment suggests a fertile direction for additional research on a topic usually neglected in the auction literature. Moreover, it represents an experimental bridge between the economics of search and the economics of bidding behavior.

### **III. THE THEORETICAL MODEL**

#### **A. The Environment**

Consider  $i = 1, 2, \dots, N$  risk neutral<sup>4</sup> bidders, who are trying to buy an object in a first-price sealed-bid auction.<sup>5</sup> Let us assume that the object has some particular value,  $V_i$ , for each particular bidder,  $i$ , but the value  $V_i$  is unknown, even to the agent  $i$  himself. In fact, the valuations of the object are random variables with identical density functions:

$$V_i \sim f(.) \quad i = 1, 2, \dots, N \quad (1)$$

where  $f$  is a continuous density function (denote the cumulative distribution function by  $F$ ) with support in the interval  $[V_l, V_u]$  and with expected value  $\bar{V}$ . This is a private values auction; even though all valuations are drawn from the same commonly known distribution, the draws are independent of one another.

An agent can follow one of two alternate strategies. First, he might spend an amount  $c_i$  in order to discover the true value that the object represents for himself. Then, based on his findings, he might form an optimal bid so that his expected profit is maximized conditioned to the new information he has received. On the other hand, this agent might decide to save  $c_i$  and just use the expectation of  $V_i$  as an estimate of his (unknown) true valuation. In this case, he would form an optimal bid conditioned to his beliefs and prior information.

The costs of information are also random variables with a commonly known distribution. Each agent knows his particular non-random cost but is unaware of the

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<sup>4</sup> Risk neutrality is a standard assumption in lotteries where payoffs are a small part of the participants endowment and even in some cases where the stakes are larger (Rabin, 2000).

<sup>5</sup> In a first-price sealed-bid auction, each bidder submits his bid in a private way, without observing the bids offered by his competitors. The auction is won by the bidder who presents the highest bid.

costs of other bidders. Specifically,

$$c_i \sim g(.) \quad i = 1, 2, \dots, N \quad (2)$$

where  $g$  is a continuous density function in the interval  $[C_l, C_u]$ ; thus costs for one individual are independent of costs for another individual.

In these circumstances, an agent should evaluate all the probable outcomes he could face in case he decides to be informed. Those outcomes depend, of course, on the behavior of his competitors and, in particular, on the information that other bidders have with respect to *their* valuations of the object.<sup>6</sup>

We start with each bidder assuming that a proportion  $p$  of bidders chooses not to be informed about their valuations and consequently, as we see below, are just using the common expectation,  $\bar{V}$ , as a bid.<sup>7</sup> Therefore, if a bidder pays  $c_i$  and then he discovers that his valuation is  $V_i$ , his optimal bidding strategy would have the form  $b_i = B(p, V_i)$ , and his expected gross profit -- given that particular valuation -- would be  $\pi = \pi(p, V_i)$ . Since *ex ante*  $V_i$  is a random variable,  $\pi(p, V_i)$  is also random, and the value of the information is given by its expected value. Define the expected profit as  $\Pi(p) = E(\pi(p, V))$ , where the expectation is over  $V$  and is based on the density function

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<sup>6</sup> A conventional way of determining the bid for each agent is to compute the Bayesian Nash equilibrium. Each player shares knowledge of a common distribution on  $(c, V)$  and knows her own  $c$ . Should the agent buy information, she will also know her own  $V$ . In any case, she will only have a distribution on the costs and values for the other bidders. Unfortunately, computing a Bayesian Nash equilibrium is extremely difficult, particularly with a continuum of possible costs and values and more than two bidders. Motivated by Grossman and Stiglitz (1980), we take a somewhat more tractable view of each agent's perception of the actions of other agents.

<sup>7</sup> We assume that bidders treat  $p$  as deterministic rather than as a random variable. Treating it as a random variable is perhaps more realistic but greatly complicates the analysis. In general, it would not appear that this is a very significant simplification, except perhaps when the total number of bidders is small.

f. This is the expected profit before information is acquired.

On the other hand, if the agent refuses to buy information, he will find it optimal to bid just  $\bar{V}$ , given his conjectures that  $pN - 1 > 0$  other bidders are behaving in an identical way.<sup>8</sup> As long as there are at least two uninformed bidders, no bids from uninformed bidders less than  $\bar{V}$  will be a Nash equilibrium.<sup>9</sup> In short, an uninformed bidder will offer  $\bar{V}$ , and his expected profit will be zero.<sup>10</sup>

Obviously, the decision of whether or not to acquire information depends simply on the relation between the value of the information --i.e., the positive amount of expected gross profit that information provides-- and the cost of information acquisition. For a particular  $p$ , the relation between  $\Pi(p)$  and  $c_i$  only depends on  $c_i$ : if  $c_i$  is too high, the agent  $i$  will refuse to be informed. A Risk-Neutral Rational Expectations Equilibrium (RNRE) is given by a value  $p^e$  such that the actual proportion of bidders for which  $\Pi(p^e) < c_i$  is (nearly) equal to  $p^e$  itself.

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<sup>8</sup> We assume away the pathological case in which there is only one uninformed bidder. If the bidder conjectures that he is the only uninformed bidder, he is not certain anymore that someone else has a valuation  $\bar{V}$ . Then, the bidder would find it optimal to offer less than  $\bar{V}$ ; in fact the bidder will behave as an informed bidder who knows his valuation is  $\bar{V}$ . A common-value, two-bidder auction in which only one is uninformed has been analyzed in Milgrom and Weber (1982b) and Engelbrecht-Wiggans et al. (1981). Those papers derive a mixed-strategy equilibrium and show that the expected profit of the uninformed bidder is zero. Rasmusen (2003) looks at a two player private values second-price auction with one uninformed player. In this case, not unexpectedly, it is optimal for an uninformed player to bid  $\bar{V}$ .

<sup>9</sup> Note that a symmetric equilibrium where all uninformed bidders offer less than  $\bar{V}$  is not possible, provided there are at least two uninformed bidders. Suppose, to the contrary, that all uninformed bidders offer  $\bar{V} - \delta$ , where  $\bar{V} \geq \delta > 0$ . In that case, any single bidder would find it profitable to offer  $\bar{V} - (\delta/2)$  since then he will have a positive expected profit, provided all other bidders maintain their previous strategies. Therefore, the original set of strategies could not be an equilibrium. This is the same argument that in a Bertrand equilibrium, price will equal marginal cost.

<sup>4</sup>A referee has pointed out that if the proportion of uninformed bidders,  $p$ , is viewed by each bidder as a random variable, then in fact no matter what bidders are assuming about  $p$ , there is a positive probability that an uninformed bidder will be the only bidder. This alone is sufficient to generate a bid from an uninformed bidder that is slightly lower than  $\bar{V}$ , and yield positive expected profit for the bidder. We do not treat  $p$  as a random variable.

## B. The Equilibrium

Let us first consider the economic problem of a bidder, say  $i$ , in case he decides to buy information and then discovers a value  $V_i$ . This agent conjectures that  $n < N$  other bidders are refusing to pay for information and are bidding just the expected value of  $V$ , while other  $[(N-n)-1]$  bidders have already discovered their respective valuations. If  $V_i$  is less than  $\bar{V}$ , this agent perceives that his probability of winning the auction is zero, because  $n$  other agents will be offering  $\bar{V}$ . If  $V_i$  is greater than  $\bar{V}$  we shall proceed by assuming that the optimal bid has the form  $b_i = B(p, V_i)$ ; i.e., it is a function of the observed  $V_i$  and conjectured  $p$ , where  $B$  is a continuous, strictly increasing function in  $V_i$ , which in equilibrium should be common to all agents.<sup>11</sup> Therefore, the maximum expected profit of the bidder is given by the expression:

$$\pi(p, V_i) = (V_i - b_i) [F(B^{-1}(b_i))]^{[(1-p)N-1]} \quad (3)$$

where  $p$  (the proportion of uninformed bidders) is defined as  $(n/N)$ ,  $B^{-1}$  is the inverse of  $B$  with respect to  $V$  and, of course,  $(1-p)N-1 \geq 0$ .

In the right-hand side of expression (3), the second factor represents the probability of winning the auction, i.e., the probability that each of the other  $[(1-p)N-1]$  bidders who are getting information discovers a valuation smaller than  $V_i$ .

To derive the function  $B$ , we use the envelope theorem which ensures that

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<sup>11</sup> The assumption that  $B$  is common to all agents follows from the symmetry of the situations faced by all bidders. The differences in information costs are relevant to determine whether or not to acquire information, but are totally irrelevant at the moment of forming an optimal bid after being informed. Once the bidder has acquired information, bygones are bygones, and the observed value,  $V_i$ , is all that matters.

$d\pi/dV_i = \partial\pi/\partial V_i$ . Therefore:

$$\frac{d\pi(p, V_i)}{dV_i} = [F(B^{-1}(b_i))]^{[(1-p)N-1]} \quad (4)$$

But, since  $b_i = B(V_i)$ , we end with:

$$\frac{d\pi(p, V_i)}{dV_i} = [F(V_i)]^{[(1-p)N-1]} \quad (5)$$

Now, this differential equation can be solved by simple integration, and by imposing the boundary condition that  $\lim_{V \Rightarrow E(V)} \pi(p, V) = 0$ . Hence, we obtain:

$$\pi(p, V_i) = \int_{\bar{V}}^{V_i} (F(\Theta))^{[(1-p)N-1]} d\Theta \quad (6)$$

where  $\pi(p, V_i)$  is a function of the random variable  $V_i$ .

Finally, substituting (6) into (3), and solving for  $b_i$ , we can get the following expression for the optimal bid:

$$B(p, V_i) = V_i - \frac{\int_{\bar{V}}^{V_i} F(\Theta)^{[(1-p)N-1]} d\Theta}{F(V_i)^{[(1-p)N-1]}} \quad (7)$$

It is trivial to check that  $\partial B(p, V_i)/\partial V_i > 0$ , so that the original assumption is satisfied. Standard arguments (Wilson, 1977) ensure that equation (7) is the unique symmetric equilibrium bidding strategy, among the class of differentiable, strictly increasing functions on  $[\bar{V}, V_u]$ . Note that, given the presence of uninformed bids, the auction can be thought as a conventional auction with  $\bar{V}$  as the reservation price of the seller.

With this interpretation, the equivalence between first-price and second-price auctions holds, and  $B(p, V_i)$  can be thought as the expectation of the second highest valuation among a fictitious seller and the informed bidders, conditioning on  $V_i$  being the highest valuation.

The value of the information is given by the expectation of  $\pi(p, V_i)$  over the relevant interval  $[\bar{V}, V_u]$ . Then,

$$\Pi(p) = \int_{\bar{V}}^{V_u} \left[ \int_{\bar{V}}^{\Phi} (F(\Theta))^{[(1-p)N-1]} d\Theta \right] f(\Phi) d\Phi \quad (8)$$

where  $\Pi(p)$  is the (*ex ante*) expected gross profit of a bidder, should he decide to buy information, conditioned on his beliefs that the other  $[(1-p)N-1]$  bidders are also becoming informed about their respective valuations. Those beliefs about the behavior of the competitors are summarized in  $p$ . Note that  $d\Pi/dp > 0$  for all relevant  $p$ . Consequently, we obtain the decision rule:

$$\begin{aligned} &\text{Buy information if } \Pi(p) \geq c_i \\ &\text{Otherwise, do not buy information} \end{aligned} \quad (9)$$

Finally, we impose the equilibrium condition that the expectations of the

bidders are rational. Hence, the value of  $p$  conjectured by all bidders when taking their decisions should be consistent with the true  $p$ <sup>12</sup>. Let  $G$  be the distribution function of information costs, and define

$$\Omega(x) \equiv 1 - G(x) \quad (10)$$

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<sup>12</sup> This idea depends on a somehow heuristic application of the Law of Large Numbers. Strictly speaking, this law would only apply if the number of bidders goes to infinity, so our reasoning should be seen as an approximation for a large  $N$ .

Thus,  $\Omega(x)$  is the proportion of bidders with information costs greater than  $x$ .

Obviously,  $d\Omega/dx \leq 0$ .

Since  $p$  can only take discrete values, a rational expectations equilibrium,  $p^e$ , will be defined by the condition:

$$\Omega(\Pi(p^e)) \geq p^e ; \quad p^e + \frac{1}{N} \geq \Omega(\Pi(p^e + \frac{1}{N})) \quad (11)$$

where

$$p^e \in \{\frac{i}{N} : i = 2, \dots, N-2\}$$

The last condition defines the feasible range of  $p$  consistent with the previous discussion. Recall that the model is not interesting for the case  $p = 0$  (i.e., all bidders are informed). That situation corresponds to the standard model where everybody knows his true valuation, and this has been extensively studied in auction literature. On the other hand, if  $p = 1$  (no one buys information), everybody would use the same strategy and their expected profits would be zero. This would be the case, if the minimum information cost,  $C_i$ , is very high with respect to the maximum valuation  $V_u$ . Finally, the extreme cases with only one informed or uninformed bidder has been analyzed elsewhere by other authors. Consequently, we also exclude that case here.

The following lemma results directly from the fact that  $\Omega$  declines as  $p$  increases.

**Lemma 1:** If  $\Omega(\Pi(1/N)) > (1/N)$  and  $\Omega(\Pi(1-(1/N))) < 1 - (1/N)$ , then there exists a unique rational expectations proportion of uninformed bidders,  $p^e$ , in the range  $(1/N,$

1-(1/N)).

Finally, we derive an expression for the expectation of the winning bid. That is,

$$\mathbf{E}_N(b) = \int_{\bar{V}}^{V_u} \left[ z - \int_{\bar{V}}^z \left( \frac{F(\Theta)}{F(z)} \right)^{[(1-p)N - 1]} d\Theta \right] \gamma_{N^*}(z) dz + \bar{V} \Gamma_{N^*}(\bar{V}) \quad (12)$$

where  $b$  represents the winning bid,  $\gamma_{N^*}$  is the density function of the maximum in a sample of  $[(1-p)N]$  draws on  $V$ , and  $\Gamma_{N^*}$  represents the corresponding distribution function.

#### IV. THEORETICAL PREDICTIONS

In the following discussion, we examine the effects of changes in key parameters on the winning bid and other relevant characteristics of the auction equilibrium, particularly the proportion of bidders choosing to become informed. In particular, we are interested in the effect of the number of bidders, specific changes in the distribution of information costs, and the hypotheticalness of the auction. Other comparative statics results may be found in Guzman and Kolstad (1997).

##### A. The Number of Bidders.

For any fixed  $p$ , a greater number of bidders implies a smaller value of information; that is,  $\partial \Pi(p, N) / \partial N < 0$ . This follows immediately from the fact that, given a particular  $p$ , a greater  $N$  implies a smaller value for the integral in brackets in expression (8), above. Therefore, an increase in the number of bidders represents

an upward shift in the graph of the function  $\Omega(\Pi(p, N))$  --as defined by expression (11) -- and, in turn, this ensures a higher equilibrium value for  $p$ .

More formally, consider the equilibrium condition (11) as an implicit function in  $p$  and  $N$ , and --just for the sake of exposition-- assume that the first inequality holds as equality. Then, the equilibrium condition is:

$$H(p, N) = 0 \quad (13)$$

where the function  $H$  is defined as<sup>13</sup>

$$H(p, N) \equiv \Omega(\Pi(p, N)) - p \quad (14)$$

Therefore,

$$\begin{aligned} \frac{dp}{dN} &= - \frac{\partial H/\partial N}{\partial H/\partial p} \\ &= - \frac{(\partial\Omega/\partial\Pi)(\partial\Pi/\partial N)}{(\partial\Omega/\partial\Pi)(\partial\Pi/\partial p) - 1} \geq 0 \end{aligned} \quad (15)$$

which validates the previous informal argument.

Now, since the equilibrium  $p$ ,  $p^e$ , increases with  $N$  (provided  $\Omega$  is not "flat"), it follows from the expressions (13)-(14) and the monotonicity of  $\Omega$  that the equilibrium value of  $\Pi(p^e, N)$  should be smaller the greater the value of  $N$ . Since the density function  $f$  has not changed, this is only possible through an increase of the exponent

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<sup>13</sup> Here we have made explicit the presence of  $N$  as an argument of  $\Pi$ .

[(1-p)N-1] in expression (8). These arguments lead to the following:

**Proposition 1.** A greater number of bidders implies no larger *proportion* of bidders buying information, though a greater *number* of well-informed bidders.

It is well known that, in the standard model of auction, an increase of the number of bidders increases the expectation of the winning bid. In our case, the effect of the increase of the total number of bidders on the expectation of the winning bid can be analyzed from expression (12) above. Heuristically, an increase in  $N$  causes both an increase in the expression in brackets, for any given  $z$ , and a shift of the mass of  $\gamma_{N^*}(z)$  to the right. Since  $B(p,z)$  is increasing in  $z$ , both effects act in the same direction. Thus the next proposition follows and is proved in the appendix.<sup>14</sup>

**Proposition 2.** An increase of the number of bidders implies an increase of the expected value of the winning bid.

## **B. Variations in the distribution of information costs.**

For this case, it is convenient to rewrite the equilibrium condition (13) as:

$$p^e = 1 - G(\Pi(p^e)) \tag{13'}$$

where  $G$  is the cumulative function corresponding to the density  $g$ . Next, let us begin by considering an arbitrary change in  $g$ . Since  $\Pi(p^e)$  is not affected by the changes in

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<sup>14</sup> An alternate proof can be based on the fact that the number of informed bidders increases with the total number of bidders. Thus, in accordance with a previous remark, the situation is equivalent to an increase of the number of bidders in a standard auction with reservation price  $\bar{V}$ .

g, it follows that changes in the initial equilibrium  $p^e$  depend only on changes of  $G(\Pi(p))$  when evaluated at that point. Formally, consider two different distribution functions,  $G_0$  and  $G_1$ , with  $p_0$  and  $p_1$  as the corresponding equilibrium proportions of informed bidders. Clearly

$$\Omega_1(\Pi(p_0)) \geq \Omega_0(\Pi(p_0)) \quad \text{iff} \quad G_0(\Pi(p_0)) \geq G_1(\Pi(p_0)) \quad (16)$$

where the subscript  $\{0,1\}$  identifies the pre and post-change functions. Therefore,  $p_1 \geq p_0$  iff  $G_1(\Pi(p_0)) \leq G_0(\Pi(p_0))$ .

The intuition behind this is obvious. For any value of  $p$ , including the equilibrium  $p^e$ , the value of information is independent of  $G$ . Thus, the original equilibrium can only be changed with changes in the proportion of bidders with information costs below or above the corresponding value of information.

To advance a further step, consider the case where changes in  $g$  induce an increase in  $p^e$  and, consequently, a decrease in  $[(1-p)N-1]$ . The latter effect acts in the same way as an exogenous decrease of  $N$ . Hence, it causes a decrease of the integral in the expression (Eqn. 12) for the expected winning bid,  $E(b)$ , and a simultaneous shift of mass of the density  $\gamma_{N^*}$  to the left. Of course, to have a *strict* change of  $p$ , the change in  $G$  must be big enough to overcome the discrete nature of  $p$ . This discussion can be summarized as follows:

**Proposition 3.** The effect of a change in the distribution of information costs depends on the change of  $G$ , evaluated at the initial equilibrium value of information. If  $G(\Pi(p^e))$  increases, then the proportion of informed bidders as well as the expectation of the winning bid will (weakly) increase; if  $G(\Pi(p^e))$  decreases, then both variables will (weakly) decrease.

Roughly speaking, an upward shift in the cumulative distribution function in the vicinity of the cut-off information cost (between costs low enough to warrant buying information and costs too high to buy information) increases the occurrence of lower information costs. This results in more bidders buying information which in turn results in an increase in the winning bid (at least in expectation).

### C Hypothetical Bias

We now introduce an additional condition into the auction. Suppose after bidding has concluded, a probability wheel is spun and with probability  $\alpha$  the winner pays her bid and receives her valuation. With probability  $(1-\alpha)$ , the auction is deemed hypothetical and no payments occur. In either case, any payments for information are real and are made in exchange for information on a bidder's true valuation. The probability  $\alpha$  is common knowledge.

Although this may seem like an artificial market design, it is closely related to the literature on the use of surveys to determine individual willingness-to-pay for public goods, particularly environmental goods and services – contingent valuation (see Carson et al, 2001). One of the primary controversies (see Hanemann, 1994; Diamond and Hausman, 1994) in that literature is the hypothetical nature of the

surveys – there is no actual monetary payment involved (List, 2001; List and Shogren, 2002). Although the question we examine here concerns hypotheticalness in an auction, the issue is not unrelated to the role of hypotheticalness in articulating a bid for a public good (though we do not formalize that connection here).

This addition of hypotheticalness is easy to deal with in the context of the model developed here. The following proposition is straightforward to prove (see appendix):

**Proposition 4.** Bidder behavior with information costs distributed as  $g_i(\cdot)$  and an *ex ante* probability of  $\alpha$  that the auction is real is equivalent in terms of bids and the proportion of informed bidders as an auction with 100% *ex ante* probability the auction is real and information costs distributed as  $\hat{g}_i(\cdot)$ , where  $\hat{g}_i(x) \equiv \alpha g_i(\alpha x)$ .

The interpretation of this proposition is straightforward. Diluting the payoff from the auction shifts the distribution of information costs to the right, effectively raising the cost of the information. Combining this result with Proposition 3 implies that, in a comparative statics sense, as  $\alpha$  is decreased from 1, the proportion of informed bidders declines as does the expected winning bid.

## V. THE DESIGN OF THE EXPERIMENT

The whole experiment consists of two separated *sessions*, each with a different set of sixteen subjects.<sup>15</sup> Each session, in turn, consisted of four *sets* of auctions.

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<sup>15</sup> Thus 32 different subjects participated in the experiment. After the experiment was completed, we noted that seven of the subjects appeared not to understand the experiment; these subjects placed bids in excess of their value and indicated in debriefing that they thought (incorrectly) they would be able to revise their bids. Consequently, we have eliminated these bidders from reported results in Table 1 and in the statistical analysis reported here.

Finally, a set of auctions was composed of four *auctions* with identical structure, except that some particular variable was changed from one auction to another.<sup>16</sup>

Auctions were conducted with each participant at an individual computer screen, and the computers were networked to a central auctioneer computer.

In each auction, the "item" posted for sale was a bundle of "tokens." The number of tokens in the bundle could be different from one bidder to another, and each individual was ignorant of the amount that he might receive as his eventual prize --hereinafter, his private value. However, each individual was given the opportunity to pay some amount of tokens --hereinafter, his information cost-- in order to receive that information.

For each auction, we selected in advance the intervals where all private values and information costs should lie, and then those intervals were publicly announced to the bidders. Next, a computer program randomly selected private values and costs for each subject; each subject then faced the decision of whether or not to buy information about their private valuations. In addition, the bidders were informed that, after the maximum bid was determined, we would spin a probability wheel to determine whether or not the transaction would actually be consummated. The probability of consummation of the sale was also publicly announced in advance. Parameter values

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<sup>16</sup> In addition, we carried on an initial set of four auctions with the objective of familiarizing the subjects with the general structure of the game, but the results of this benchmark set were not used in the statistical analysis of the data.

are summarized in Table 1, along with key results from the auctions.<sup>17</sup>

Summing up, a typical auction consisted of six steps: (1) the ranges for private values and information costs, and the probability of consummating the auction, were made public; (2) each bidder was privately informed of her private information cost; (3) bidders were asked if they wished to buy information and if they agreed, the bidder was secretly informed about her valuation, and her information cost was subtracted from her current balance; (4) bids were submitted and the winning bid of the auction was determined; (5) a probability wheel was spun (in some auctions) to determine whether the sale should be consummated; (6) the winning bidder received her prize.

The operations of the bidders were financed by a current account provided to each bidder. At the beginning of an experimental session, the balance of each current account was set to 1000 tokens and, as we already mentioned, these balances were updated after each auction. For the winning bidder, the following was added to his account:

$$\chi_v(V_i - b_i) - \chi_c c_i \tag{17}$$

where  $V_i$ ,  $b_i$ , and  $c_i$  represent his valuation, bid, and information cost, respectively;  $\chi_v$  is an indicator variable equal to 1 when the sale is consummated and 0 otherwise; and  $\chi_c$  is equal to 1 when the agent has bought information and 0 otherwise. For

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<sup>17</sup> Each of the sixteen auctions were repeated once. The data in table 1 for each auction represent the average of the results for the two repetitions.

other bidders, the updating of balances simply required subtracting the amount  $\chi_c c_i$ .<sup>18</sup>

After each set of auctions, the balance of the current account of any bidder was added to the previous balance of the bidders' *cumulative account*, while the new current balance was again set to 1000 tokens. The cumulative accounts could not be used to finance current operations of the bidders and their function was just to measure the amount accumulated for each bidder up to any particular moment. To motivate the subjects to take their decisions seriously, we announced that at the end of the experiment, a prize of 75 dollars would be awarded by lottery among all the participants, using for each a probability of winning proportional to his or her accumulated balance at the end of the experiment. The objective of this procedure was twofold. First, to ensure that the budget constraint was not binding for any set of auctions, since we were not interested in analyzing the effects of budget constraints; second, to guarantee that all participants were really motivated to accumulate as many tokens as they could over the whole experiment.<sup>19</sup>

The subjects were not allowed to communicate with each other. Thus, all public information was provided by a proctor, and any private information was sent to each bidder through his computer screen. Similarly, their decisions concerning whether or not to buy information, as well as the bids, were sent to the proctor through a computer system specifically designed for the experiment.

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<sup>18</sup> It should be emphasized that the subtraction of the information cost was certain and totally independent of whether or not the sale was ultimately consummated.

<sup>19</sup> A discussion of cash balance effects is found in Hansen and Lott (1991).

## VI. EXPERIMENTAL RESULTS

The goal of the experimental dimension of this research is to test several hypotheses suggested by the theory. A basic question concerns the extent to which laboratory behavior is predicted by the theory. A second set of questions concerns comparative statics theoretical results: the effect of changes in the number of bidders, the effect of changes in information costs, and the effect of the extent of hypotheticalness.

### A. Evaluating the Predictions of the Theoretical Model.

The predictive accuracy of the model is first tested through the comparison of the expected and actual values of the proportion of uninformed bidders,  $p$ , and the winning bid,  $B$  (Table 1).

An informal inspection of Table 1 suggests that both the actual proportion of uninformed bidders and the actual winning bid track very closely the movements of the theoretical equilibrium values. See Figures 1 and 2. In the case of the winning bid, however, the level of the sample values seems to be above the theoretical equilibrium: in all but one auction, the winning bid is higher than the prediction of the model (see Figure 2). Of course, this would imply that the (risk neutral) equilibrium persistently *underestimates* the actual winning bid.

A regression between sample and predicted values is a convenient way to formalize that observation. Specifically, we will use the regressions

$$p^s = a_0 + b_0 p^e \quad (18a)$$

and

$$B^s = a_1 + b_1 B^e \quad (18b)$$

as the basis for testing some appropriate hypotheses about the parameters  $a_i$  and  $b_i$ ,  $i = 0, 1$ , where the superscripts {s, e} denote the sample and theoretical equilibrium values, respectively.

The results of the regressions are given in Table 2. It can be seen that the "b" coefficient is slightly below 1 for the proportion of uninformed bidders (Eqn 18a), while slightly above 1 for the winning bid (Eqn 18b). Nevertheless, neither null hypothesis that  $a_i = 0$  or the null that  $b_i = 1$  can be rejected at reasonable levels of confidence, except for the case of  $b_0$ . However, a different result is obtained when we test the joint hypothesis

$$H_0 (i): a_i = 0, b_i = 1 \quad (19)$$

which amounts to test the equality of actual and equilibrium values. In this case, the hypothesis cannot be rejected for the proportion of uninformed bidders, but it is clearly rejected for the winning bid. Overall, this validates the heuristic caveat that the "movements" of the winning bid are reasonable predicted by the equilibrium values, but the "level" of the bid might be underestimated.

An alternate approach to measure the predictive power of the theoretical model is given by a goodness-of-fit test applied to the proportion of bidders acquiring information. The idea is simply to compare the number of bidders in each category (i.e., "buying" or "not buying" information) with the predicted numbers given by the

expressions  $N(1-p^e)$  and  $Np^e$ , respectively. The test is based in the statistic

$$\chi^2 = \frac{(Np^s - Np^e)^2}{Np^e} + \frac{(N(1-p^s) - N(1-p^e))^2}{N(1-p^e)} \quad (20)$$

where  $N$  denotes the size of the sample,  $p$  is the proportion of bidders not acquiring information and, as usual, the superscript distinguishes the sample and predicted values.<sup>20</sup> In only two cases – auctions 8 and 12 -- are the prediction of the model statistically different (at the 90% level) from the actual realization.<sup>21</sup>

As a counter to these conclusions, it might be argued that one problem with the goodness-of-fit test is that it allows the possibility that the "information acquisition errors" of some bidders are canceled out by the errors of others.<sup>22</sup> Thus, even if the observed numbers in each category agree with the predicted numbers, this could hide significant departures if we look at the behavior of each individual bidder. Therefore, we will follow an alternate approach that takes this into consideration.

Define the variable  $J_i$  as the proportion of decision errors in auction  $i$ . The relevant question is whether or not this variable tends to be below some reasonably small cutoff. Since the selection of the threshold is necessarily arbitrary, we will

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<sup>20</sup> It is well known that the statistic follows a  $\chi^2$  distribution with one degree of freedom. An elementary exposition can be found in Senter (1969).

<sup>21</sup> In some cases, the application of the test requires some cautious because a very good approximation to the  $X^2$  distribution needs  $Np^e$  and  $N(1-p^e)$  to be greater or equal to 5. Consequently, this test was not applied to auctions 13-16 in which the number of observations was too small.

<sup>22</sup>By "information acquisition error" we refer to the decision of an agent who buys information when, given his information cost, the theory suggests not to buy it, or vice versa.

follow a conservative approach by testing the hypotheses:

$$\begin{aligned} H_0: J_i &\leq 0.20 \\ H_a: J_i &> 0.20 \end{aligned} \tag{21}$$

Of course, a rejection of the null would imply that decision errors are common and systematic, rather than purely random and negligible. The test is based on the comparison between  $J_i$  and the critical number corresponding to the 10% tail of a binomial distribution with parameters (25, 0.20, 0.80).<sup>23</sup> If, assuming the null hypothesis holds, the probability of drawing a number of errors equal to the actual number is below 0.20, then the null should be rejected. This is the case in only three (3) auctions (6-8); the null cannot be rejected in the remaining nine (9) auctions

An important implication of the standard private values model is that the equilibrium is efficient, in the sense that the object is won by the bidder with the highest valuation. This fact has been commonly verified in experimental research, giving support to the relevance of the symmetric Risk-Neutral Nash Equilibrium model (RNNE).<sup>24</sup> When we introduce costly information, of course, that is not true anymore: the bidder with the highest valuation might decide to remain uninformed, and then the auction can be won by another bidder. Indeed, the probability of an efficient outcome can be made very small, just by shifting the distribution of costs in

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<sup>23</sup> The use of the binomial distribution follows from the binary nature of the variable  $J$ . Here we consider a "correct decision" as a success, and a "wrong decision" as a failure.

<sup>24</sup> See, for example, Dyer et al. (1989, fn. 10) and the references there.

an appropriate way. This remains essentially true even if alternate criteria for efficiency are adopted -- as, e.g., that the auction is won by the bidder with a highest value  $V_i - c_i$ . In short, efficiency is not a testable implication of the Risk-Neutral Rational Expectations (RNRE) model.

Yet, the model still predicts that, *after* buying information, all bidders would use the same bidding function, increasing in valuation. This follows immediately from the fact that, after buying information, all bidders face the same situation, whatever their information costs might have been. Then, the bidder with the highest valuation among the informed bidders will also present the highest bid, and that does impose a constraint that our data should satisfy. Overwhelming evidence indicates that this is, in fact, the case: in all the auctions of the experiment, the highest bid among the informed bidders was presented by the bidder with the highest valuation.

## **B. Comparative Statics Results**

The theoretical model generates several comparative statics results, related to the level of the winning bid and the proportion of bidders who choose not to be informed. These theoretical results can be tested using our experimental results. The simplest way to test these hypotheses is to posit a relationship between the exogenous variables, that we are varying over the different treatments, and the endogenous variables ( $B$  and  $p$ ). In particular, we hypothesize the following relationships:

$$p^s = a_0 + b_0 N + c_0 C_U + d_0 \alpha + e_0 V_U \quad (22a)$$

$$B^s = a_1 + b_1 N + c_1 C_U + d_1 \alpha + e_1 V_U \quad (22b)$$

The theoretical predictions can be summarized in terms of the coefficients of Eqn. 22.

In particular, a review of Propositions 1-4 suggests that the proportion of bidders who choose not to be informed ( $p$ ) is:

- non-decreasing in  $N$  (Prop 1) [ $b_0 \geq 0$ ]
- non-decreasing in expected information costs (Prop 3) [ $c_0 \geq 0$ ]
- non-decreasing in the “hypotheticalness” ( $1-\alpha$ ) of the auction (Prop 4) [ $d_0 \leq 0$ ]

Further, the level of the winning bid ( $B$ ) is:

- non-decreasing in  $N$  (Prop. 2) [ $b_1 \geq 0$ ]
- non-increasing in expected information costs (Prop. 3) [ $c_1 \leq 0$ ]
- non-increasing in the “hypotheticalness” ( $1-\alpha$ ) of the auction (Prop 4) [ $d_1 \geq 0$ ]

The sign of  $e_i$  is not addressed in the theoretical part of this paper (see Guzman and Kolstad, 1997, for discussion of this issue).

Table 3 shows the results of an OLS estimation of Eqn. 22 on outcomes in experimental auctions 1-16. Signs are as theoretically predicted, except that only about half of the relevant coefficients are significant. One conclusion is that the experiments support the theoretical model of bidding with costly information acquisition.

The interpretation of the results on hypotheticalness are particularly telling. The theoretical and experimental results suggest that a more hypothetical environment will tend to decrease bids – the winning bid at least. Although these

results may not justify concluding that CV bids are biased downwards, this does suggest a plausible model that warrants further examination.

## VII. CONCLUSIONS

One of the central issues in environmental economics is how agents formulate estimates of willingness-to-pay or willingness-to-accept for environmental goods. This question is central in valuation, particularly contingent valuation, and is also an issue in the debate over the anomalous divergence between WTP and WTA. Information theoretic arguments have been put forward to explain this process of “bidding” in environmental valuation. This paper provides some of the first experimental validation of theoretical models on bid formation with costly information acquisition.

Thus the primary contribution of this paper is in validating a particular model of bid formation. To summarize our main conclusions:

1. The theoretical predictions of the Risk-Neutral Rational Expectations (RNRE) model about the proportion of bidders who acquire information are highly consistent with the results obtained in our experiment.
2. The theoretical predictions of the RNRE model concerning the winning bid are consistent with the movement in the actual winning bid, but they underestimate its actual level.
3. The theoretical predictions of the RNRE model with respect to comparative statics are also highly consistent with the experimental results, particularly in predicting the proportion of bidders who acquire information.
4. An increase in the hypothetical nature of an auction tends to increase the proportion of uninformed bidders and decrease the magnitude of the winning bid.

The second conclusion above coincides with a familiar result in experimental

studies. That is, the actual winning bid is above its equilibrium value. This result usually is found in experiments without endogenous information acquisition.<sup>25</sup> Since those controversial issues have not been convincingly solved in a simpler framework, we will not presume to solve them in our more general structure. A very brief comment will suffice here.

The fourth conclusion is perhaps the most intriguing of our conclusions. Although it is logical that hypotheticalness would tend to increase the proportion of uninformed bidders, it is interesting that increased hypotheticalness decreases the magnitude of winning bids. This is suggestive that hypotheticalness may lead to bias in CV. However, it should be pointed out that such a conclusion would be premature, since this paper focuses on bidding in auctions, at best only related to bidding in CV.

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<sup>25</sup> See Kagel (1995) for a discussion.

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## APPENDIX

A. Proof of Proposition 2. Let us define:

$$N^* \equiv (1-p)N \tag{A-1}$$

and

$$B(z, N^*) = z - \int_{\bar{V}}^z \left\{ \frac{F(\Theta)}{F(z)} \right\}^{[N^* - 1]} d\Theta \quad \text{if } z \geq \bar{V} \tag{A-2}$$

$$= \bar{V} \quad \text{if } z \leq \bar{V}$$

Clearly,

$$(i) \quad \frac{\partial B(z, N^*)}{\partial z} \geq 0 \tag{A-3a}$$

$$(ii) \quad \frac{\partial B(z, N^*)}{\partial N^*} \geq 0 \tag{A-3b}$$

and that the expectation of the winner bid can be written simply as:

$$(iii) \quad E(b) = \int_{V_l}^{V_u} B(z, N^*) \gamma_{N^*}(z) dz \tag{A-3c}$$

Now,

$$\frac{dE(b)}{dN} = \frac{dE(b)}{dN^*} \cdot \frac{dN^*}{dN} \tag{A-4}$$

From proposition 1 in the text, we already know that the second factor is positive. Thus, it suffices to prove that the first is non-negative. Differentiating (A-3c), we can say that:

$$\frac{dE(b)}{dN^*} = \int_{v_l}^{v_u} \frac{\partial B(z, N^*)}{\partial N^*} \gamma(z) dz + \int_{v_l}^{v_u} B(z, N^*) \frac{\partial \gamma(z)}{\partial N^*} dz \quad (A-5)$$

where, for notation convenience, the subscript of the density function  $\gamma_{N^*}$  has been dropped. It follows from (A-3b) that the first term in this expression is nonnegative. In turn, the second term can be written as:

$$\begin{aligned} \int_0^{v_u} B(z, N^*) \frac{\partial \gamma(z)}{\partial N^*} dz &\equiv \int_{v_l}^{v_u} B(z, N^*) \frac{\partial}{\partial N^*} \left( \frac{\partial \Gamma(z)}{\partial z} \right) dz \\ &\equiv \int_{v_l}^{v_u} B(z, N^*) \frac{\partial}{\partial z} \left( \frac{\partial \Gamma(z)}{\partial N^*} \right) dz \end{aligned} \quad (A-6)$$

which, by straightforward integration by part, yields:

$$\equiv B(z, N^*) \frac{\partial \Gamma(z)}{\partial N^*} \Big|_{v_l}^{v_u} - \int_{v_l}^{v_u} \frac{\partial B(z, N^*)}{\partial z} \frac{\partial \Gamma(z)}{\partial N^*} dz \quad (A-7)$$

Finally, we use the fact that, for all  $N^*$ ,  $\Gamma(v) = 0$  and  $\Gamma(V) = 1$ , so that its derivative at those points is zero. Therefore, we end with:

$$\int_{v_l}^{v_u} B(z, N^*) \frac{\partial \gamma(z)}{\partial N^*} dz = - \int_{v_l}^{v_u} \frac{\partial B(z, N^*)}{\partial z} \frac{\partial}{\partial N^*} (F(z)^{N^*}) dz \geq 0 \quad (A-8)$$

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(-)

and the result follows immediately.

#### B. Proof of Proposition 4.

The proof of this proposition hinges on showing the proportion of bidders choosing to become informed is the same in the two cases – the hypothetical case where the probability the auction is real is  $\alpha$  and the information costs are distributed as  $g(x)$  and the real case with the auction definitely real and information costs distributed as  $\hat{g}(x) \equiv \alpha g(\alpha x)$ . If the proportion of bidders is the same, then bids will be unaffected by the fact that after the auction a lottery is held to determine if the auction was real.

Eqn (11) defines the proportion of informed bidders in the hypothetical case, except that profits must be diluted by the probability that the auction is real,  $\alpha$ :

$$\Omega(\alpha\Pi(p^e)) \geq p^e ; \quad p^e + \frac{1}{N} \geq \Omega(\alpha\Pi(p^e + \frac{1}{N})) \quad (\text{A-9})$$

With the purely real auction, costs are distributed as  $\hat{g}(x) \equiv \alpha g(\alpha x)$  which implies  $\hat{\Omega}(x) \equiv \Omega(\alpha x)$ . It is easy to see that this implies that the conditions defining the proportion of informed bidders in the purely real auction are the same as in Eqn A-9. Thus the two auctions yield identical outcomes.

**TABLE 1**  
**STRUCTURE OF THE AUCTIONS**

<-----Parameter Assumptions-----> <---Theory---> <-----Results----->												
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Set	Auc #	$V_l$	$V_u$	$C_l$	$C_u$	$(1-\alpha)$	N	$p^e$	$B^e$	$p^s$	$B^s$	Info Err, %
I	1	0	100	0	60	0	15	0.87	55	0.80	97	0.20
I	2	0	200	0	60	0	15	0.81	118	0.88	158	0.16
I	3	0	300	0	60	0	15	0.76	188	0.76	276	0.28
I	4	0	500	0	60	0	15	0.71	330	0.64	486	0.16
II	5	0	300	0	130	0	15	0.84	169	0.84	200	0.16
II	6	0	300	0	30	0	15	0.69	202	0.72	292	0.32
II	7	0	300	0	15	0	15	0.59	219	0.52	282	0.40
II	8	0	300	0	5	0	15	0.40	241	0.64	239	0.48
III	9	0	300	0	30	10%	15	0.70	200	0.72	251	0.20
III	10	0	300	0	30	40%	15	0.76	188	0.96	246	0.16
III	11	0	300	0	30	60%	15	0.79	181	0.96	210	0.04
III	12	0	300	0	30	90%	15	0.91	154	0.84	293	0.12
IV	13	0	300	0	60	0	13	0.75	182	0.81	232	0.32
IV	14	0	300	0	60	0	9	0.70	174	0.71	249	0.29
IV	15	0	300	0	60	0	6	0.63	167	0.72	237	0.33
IV	16	0	300	0	60	0	3	0.50	156	0.25	222	0.20

Sets:

- I: Vary range for valuation ( $V_u$ )
- II: Vary range of information costs ( $C_u$ )
- III: Vary hypothetical level ( $\alpha$ )
- IV: Vary number of bidders (N)

Note: Results for each auction reflect the average of two repetitions of the auction from the two sessions (see text).

**Table 2**  
**Test of Predictability of Theoretical Model (Eqn 18)**

	$i=0$	$i=1$
$a_i$	0.0644 (0.175)	-12.2 (53.2)
$b_i$	0.944 (0.241)	1.36 (0.279)
$R^2$	0.48	0.60
F-statistic	0.287	6.60

**Note:** Model  $i=0$  involves regressing  $p^s$  on  $p^e$  (Eqn. 18a); Model  $i=1$  involves regressing  $B^s$  on  $B^e$  (Eqn. 18b). Figures in parenthesis are standard errors. The F-statistic is for the restriction in Eqn.19 that  $a = 0$  and  $b = 1$ . The critical value for the statistic at the 95% confidence level is 3.74.

**Table 3:**  
**Test of Comparative Statics Predictions of Theoretical Model**

	$i=0$	$i=1$
$a_i$ (constant)	0.679* (0.232)	-222.2* (95.16)
$b_i$ (coeff on N)	0.0260* (0.0090)	12.40* (3.71)
$c_i$ (coeff on $C_U$ )	0.0020* (0.0012)	-0.317 (0.497)
$d_i$ (coeff on $\alpha$ )	-0.2788* (0.1304)	24.57 (53.56)
$e_i$ (coeff on $V_U$ )	-0.00048 (0.0004)	0.9815* (0.1743)
Adj $R^2$	0.46	0.72

**Note:** Number of data points is 16; standard errors in parentheses; \* indicates coefficient is significantly different from zero at 90% confidence level.

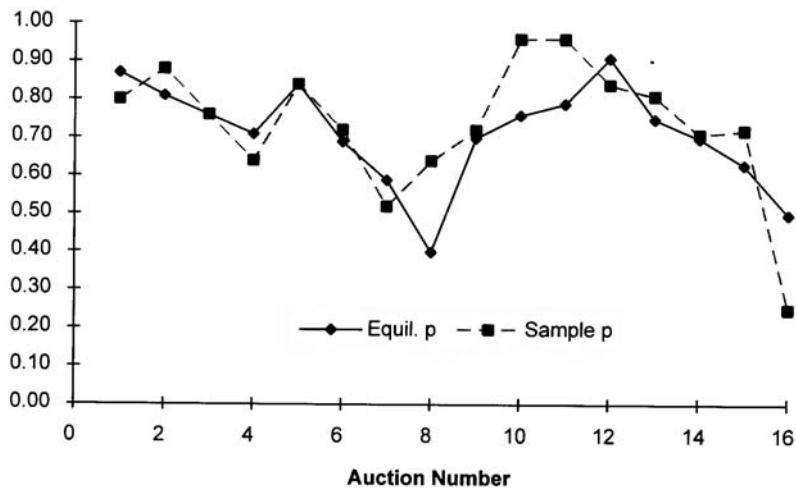


Fig 1a: Theoretically predicted proportion of uninformed bidders (equilibrium p) and experimental proportion (sample p) by auction

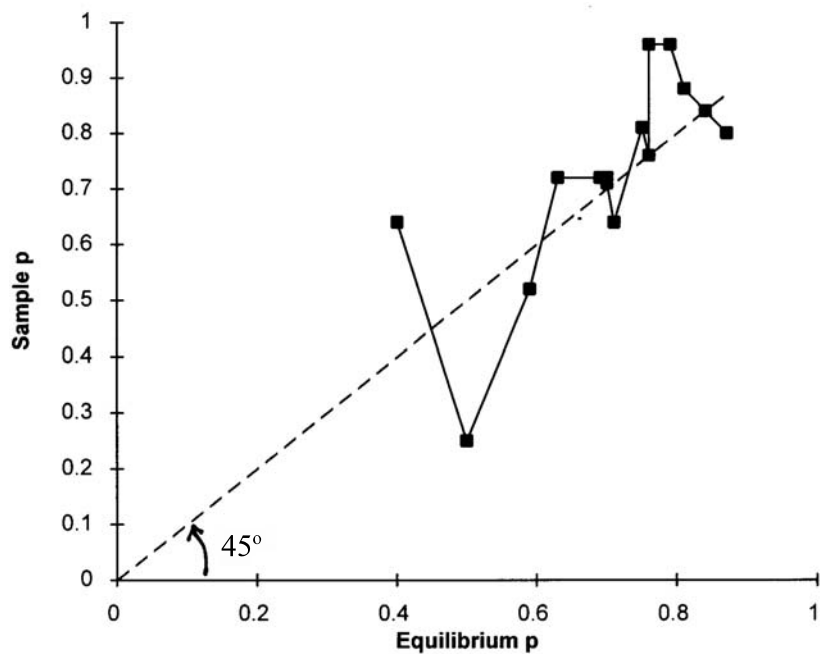


Fig 1b: Theoretically predicted proportion of uninformed bidders (equilibrium p) and experimental proportion (sample p). Each point represents an auction.

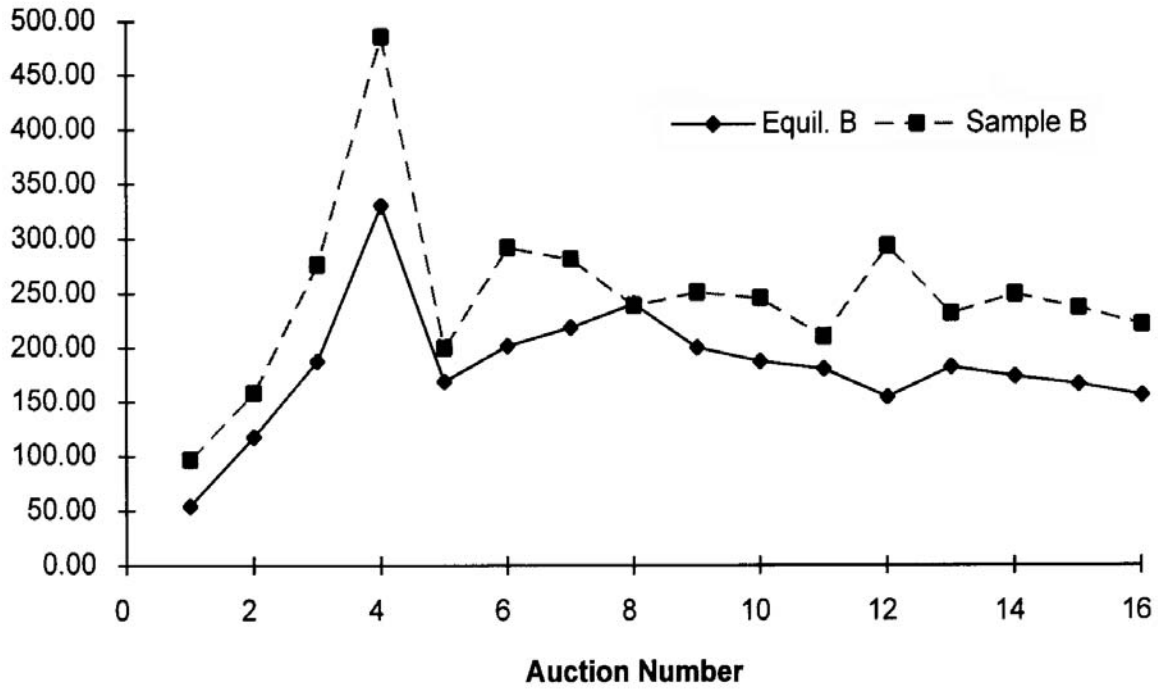


Fig. 2a: Theoretically predicted winning bid (Equilibrium B) and experimental winning bid (Sample B) by auction.

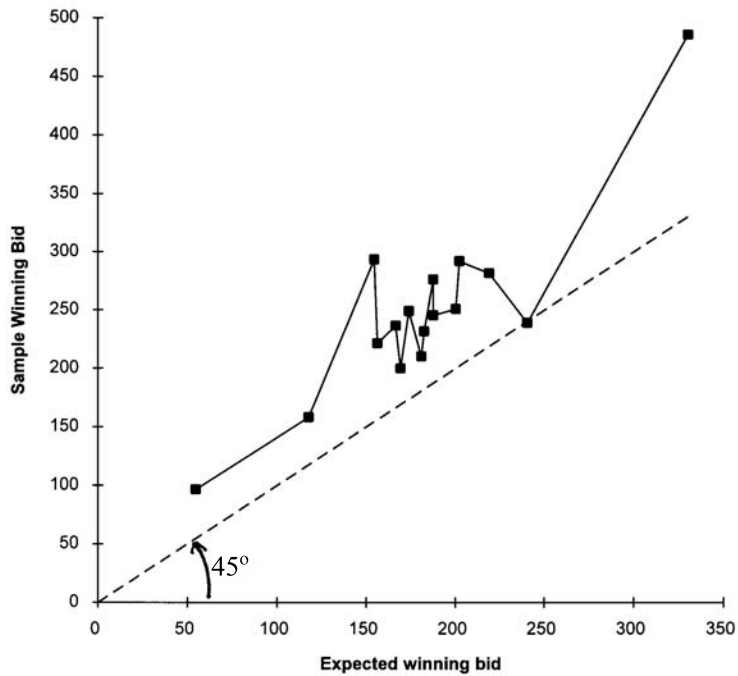


Fig. 2b: Theoretically predicted winning bid (Equilibrium B) vs. experimental winning bid (Sample B). Each data point represents one auction.