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<tr>
<td>2.0-2.6</td>
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</tr>
</tbody>
</table>
Supply Curve of Labor

\[ W = Z + R_a + cF(L) \]

1. Explain \( Z \)
2. Explain \( R_a \)
3. Explain \( cF(L) \)
4. Explain why upward sloping
Implications for Land Rent

\[ R_a + cF(L^*) \]
A Decrease in Commuting Costs

\[ W \quad L \quad R \quad d \]

\[ R_a \]
Review and Preview

- Office rent and distance from CBD
  - Office rent falls with distance from CBD
  - Building heights also fall with distance
  - Workers per acre fall with distance from CBD

- Today, density of residential population and distance from CBD
Chicago SMSA

Data Classes
- Persons/Sq Mile
- 0 - 7408
- 7482 - 16648
- 16718 - 28352
- 28488 - 50537
- 52917 - 90683

Features
- Major Road
- Street
- Stream/Waterbody

Items in gray text are not visible at this zoom level.

Approx. 65 miles across.
North Chicago

Data Classes
- Persons/Sq Mile
  - 0 - 7408
  - 7482 - 16648
  - 16718 - 28352
  - 28488 - 50537
  - 52917 - 90683

Features
- Major Road
- Street
- Stream/Waterbody
- Stream/Waterbody

Items in gray text are not visible at this zoom level

Approx. 20 miles across
San Francisco CMSA

Data Classes
Persons/Sq Mile
- 0 - 5457
- 5500 - 13738
- 13880 - 28072
- 28359 - 62014
- 72125 - 155171

Features
- Major Road
- Street
- Stream/Waterbody

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Approx. 65 miles across.
San Francisco CMSA

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Features
- Major Road
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Approx. 20 miles across.
Los Angeles CMSA

Data Classes
Persons/Sq Mile
- 0 - 6168
- 6189 - 14258
- 14292 - 26524
- 26620 - 51704
- 53458 - 92464

Features
- Major Road
- Street
- Stream/Waterbody

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Approx. 65 miles across.
Los Angeles

Data Classes

Persons/Sq Mile

- 0 - 6168
- 6189 - 14258
- 14292 - 26524
- 26620 - 51704
- 53458 - 92464

Features

- Major Road
- Street
- Stream/Waterbody
- Stream/Waterbody

Items in gray text are not visible at this zoom level.
From Toy Model

- Workers occupied $s$ units of land-fixed
- Commuting costs of $c$ per mile
- Distance from CBD is $d$
- $R(d)$ is rent for $s$ units of land as a function of distance
  \[ R(d) = R(0) - cd \]
Modify Toy Model

- People occupy housing (land and capital) not just land
- Amount of housing depends on price
Notational Confusions

- Distance from CBD
  - Toy Model: $d$
  - O’Sullivan: $u$

- Commuting cost per mile from CBD
  - Toy Model: $c$
  - O’Sullivan: $t_h$

- Go with O’Sullivan!
The Nature of Housing

- Think of square feet of living space
- Suppose everyone has exactly the same housing space: $H$
- Interior living space, not land
- Can produce more living space by building up
Housing Price Function

- Housing space is more valuable closer to CBD because less commuting
- Housing price as function of distance $P_h(u)$
How Much Higher?

- Offset commuting cost differences
- Commuting costs: $t_h u$
- Housing costs: $P_h(u)H$
- Sum must be constant:

  $P_h(u)H + t_h u = A$ (a constant)
Slope of Housing Price Function

\[ P_h(u)H + t_h u = A \]

Differentiate with respect to \( u \)

\[ P'_h(u)H + t_h = 0 \]

\[ P'_h(u) = -\frac{t_h}{H} \]
An Example

- H is 1,000 square feet
- $5 per mile per month
  - 20 miles per gallon
  - 20 work days per month
  - 2 miles travel per mile distance
  - $2.50 per gallon
- Change in rent
  \[ \Delta P_h 1000 = -$5 \]
  \[ \Delta P_h = -$5/1000 = $0.005 \]
Example Continued

- Monthly rent is $2,000 at CBD
- $2 per square foot
- Monthly rent per square foot at 10 miles: $2-10*$0.005=$1.95
- Monthly rent for 1000 square feet at 10 miles: $1,950
Graphically

$/sq.ft.

$2.00

$1.95

$P_h(u)$

$u$, miles from CBD
The Supply of Housing

- How much housing to build on an acre of land?
How much housing to build?

Marginal Cost

present value of rent per sq. ft.

$200

100,000 square feet
Higher Price, More Housing

$ per sq. ft. vs. square feet

- $100
- $200

40,000
100,000
Housing Density

rent per square foot of housing

square feet of housing per acre

x: distance from CBD

x: distance from CBD
Population Density

- Residential space per unit of land falls as we move away from CBD
- Population density falls
Law of Demand

- If price falls, demand more
- Price of housing falls with distance from CBD
- Households demand more housing with distance from CBD
- Housing demand
  \[ H(P_h) \]
Population Density

- Households demand more housing as distance from CBD increases
- Less square feet of housing per square foot of land as distance from CBD increases
- Population density falls as distance from CBD increases
Slope of Housing Price Function

With fixed housing, $H$

$$P'_h (u) = - \frac{t_h}{H}$$

Incorporate law of demand: $H(P_h)$

$$P'_h (u) = - \frac{t_h}{H(P_h(u))}$$
A Convex Price Function

sq. ft.  \[ H(P_h(u)) \]  $/sf  \[ P_h(u) \]

\[ u \]  \[ u \]
Residential Bid-Rent Functions

- How much are developers willing to pay for land for residences?
- $R_h(u)$ residential bid-rent function
- Higher rent for housing
  - >more housing for sq. ft. of land
  - >high value for land
- Same idea as for developers of commercial space
Residential Bid-Rent Function

$/sf \rightarrow P_h(u) \rightarrow R_h(u) / sf
Residential and Commercial

\[ R_c(u) \] for Residential

\[ R_h(u) \] for Commercial

$/sf$ axis

CBD

Residential
Conclusion

- Housing prices fall with distance from CBD because of commuting costs
- House price function is convex
- Density declines with distance because of both housing supply per acre and housing demand per person
Next Time

☐ People have different incomes
☐ Will higher income people live closer or farther away from CBD?
☐ Read two LA Times articles about regentrification