

# POPULATION GROWTH AND TECHNOLOGICAL CHANGE: A RE-EXAMINATION

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## Abstract

The long-term effects of population growth are of obvious importance. Previous studies, notably Kremer (1993), have provided evidence of positive effects of population growth on technological change. This paper reexamines existing results, introduces previously untested predictions from Kremer's model, and considers alternative global and regional data. The results show that conclusions based on samples that include modern data are typically reversed when recent observations are excluded. Causal inferences drawn from a *natural experiment*, the melting of the ice caps, are also reversed when appropriate controls are taken into account.

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# 1 Introduction

Population growth divides social scientists between Malthusians, who see the pressure of population against available resources as the cause of problems to do with economic development, and Boserupians, who see population pressure as the driving force behind technological change. This is an important debate that comes to mind when discussing, for example, long-term environmental concerns and the emergence of below-replacement fertility in many developed countries.

The most prominent assessment of the long-term effects of population growth has been made by Kremer (1993). Kremer (1993) proposed and tested a model that synthesizes Malthusian and Boserupian views. Using long time series data for world population and a *natural experiment*, the geographic isolation of populations after the last glaciation, Kremer (1993) argued that the long-run history of population growth is consistent with a view in which population growth spurs technological change. Kremer's (1993) results have become part of the conventional wisdom in economics.

To purpose of this paper is to further the discussion initiated by Kremer (1993). I revisit existing analyses, examine several previously untested predictions from Kremer's model, and consider alternative global and regional data. The findings differ substantially from those of Kremer (1993). For example, one prediction of the theory is a positive association between population growth and size. In samples that end after the 1970s, population growth and size are strongly positively associated, even in regional data. Regression diagnostics, however, show that point estimates are strongly influenced by recent observations. If recent observations are omitted, data as recent as post-1900 observations, the results become insignificant and sometimes even yield a "wrong" (e.g., negative) and significant sign. All the other predictions of the theory experience similar reversals when recent observations are excluded.

I also argue that existing causal inferences based on the melting of the ice caps are likely to be biased. Existing comparisons consider the following *natural experiment*. After the last glaciation, the New and the Old World became isolated. As in Kremer (1993), suppose populations were randomly allocated among the Old and the New World, and

assume that population densities were the same in both regions some 12 thousand years ago (KYA henceforth). If population spurs technological change, the Old World's larger area (which implies a larger population size since densities are equal) would generate a technological advantage for the Old World.

This conclusion seems reasonable. The previous comparison, however, fails to take into account differences in factor endowments and geography, among other omitted variables. Factor endowments and geography were not equal in the different areas, as stressed by Diamond's (1997) assessment of this natural experiment. The importance assigned to population size could be in part attributed to advantages in biogeography or differences in orientation which, according to Diamond (1997), favored the origin of agriculture and the diffusion of post-agricultural technologies such as metallurgy and weaponry. Once these influences are "controlled for," either through a careful selection of a control group or through the inclusion of appropriate control variables in an econometric specification, differences in initial population size appear no longer relevant to explain differences in economic or demographic conditions at the time these regions reunited.

The findings question the strength of existing results and suggest a sharp divide between pre-modern and modern regimes. In pre-modern samples, for example, population exhibits *mean reversion*. This finding is consistent with a stationary Malthusian equilibrium in population size. Further, the influence of arable land and biogeography on population growth changes signs when recent observations are excluded. This reversal also contradicts a view of continuity in economic and demographic conditions assumed by the theory. Instead, the evidence suggests that the Industrial Revolution and the modern demographic transition represent "radical" departures from a pre-modern epoch. These "radical" departures do not appear to have been triggered by population pressures, as suggested by a Boserupian view.

The temporal dimension in the paper is not as ambitious as in Kremer (1993). However, I extend the analysis to regional data in order to test predictions that cannot be examined with global data. I also point out that some of the existing estimates of population data in the very long run such as Deevey (1960), Kremer's (1993) source, are biased

toward accepting the theory’s hypotheses. Instead, I use data from McEvedy and Jones (1985) and Biraben (1979). There is no direct estimate of pre-modern population sizes across world regions but McEvedy and Jones (1985) and Biraben (1979) are independent assessments. McEvedy and Jones (1985) is the source economists typically use to study pre-modern conditions; see, e.g., Acemoglu et al. (2002), Ashraf and Galor (2011), Comin et al. (2010), and Klasen and Nestmann (2006).

**Related literature.** The growth literature has moved away from the “scale effects” that motivated Kremer (1993). Population growth, however, is central for technological change in ideas-based growth models such as Jones (1995), Kortum (1997), and Segerstrom (1998). These models consider “market size effects” where continuous population growth is needed to offset diminishing returns to knowledge. Given the reduced-form nature of the analysis, the results are not only informative about scale effects but also about market size effects related to population growth.<sup>1</sup>

With the renewed interest in global environmental change, population externalities have again become an important topic of discussion; see, e.g., Acemoglu et al. (2012), Bohn and Stuart (2011), Bongarts (1992), and Kelly and Kolstad (2001). The *positive population externality* emphasized by ideas-based growth models contrasts with *negative population externalities* due to market imperfections and natural-resource base constraints; see, e.g., Baland and Robinson (2002), Cohen (1995), Dasgupta (2000), Lee and Miller (1990), and Ehrlich et al. (1993). There is no consensus about the empirical importance of positive and negative externalities connected with population growth. Judging by the correlation between population growth and size, the findings presented here suggest positive externalities only during the modern epoch.<sup>2</sup>

The paper also sheds light on the nature of the transition between the pre-modern

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<sup>1</sup>Kuznets (1960) and Simon (1977) emphasized but did not quantify positive population externalities. Johnson (2000) and Jones (2001) discussed external economies of population in the production of knowledge. Lee (1988) is a previous synthesis of Malthus and Boserup. A positive relationship between population growth and population size was first studied in the 1960s in terms of “doomsday” models; see, e.g., Foerster et al. (1960) and Umpleby (1987). These studies did not examine the sensitivity of their estimates to changes in the end of the sample period.

<sup>2</sup>For studies focused on the postwar period see Headey and Hodge (2009), Kelley (1988), and Pritchett (1996). The literature that studies the effects of population growth on economic growth has documented positive, negative, and zero effects of population growth.

and the modern epoch. Unified growth models, e.g., Galor and Weil (2000), considered a “smooth” transition into modern economic growth. The pre-modern sub-samples studied here align more with Malthusian tendencies. Boserupian effects may be harder to detect in the shorter span of data considered here or maybe settled agriculture introduced a Malthusian mechanism into an otherwise Boserupian process.<sup>3</sup>

The paper, however, is not an attempt to test for Malthusian dynamics. The analysis, for example, does not focus exclusively on the pre-modern period. The consistency of Malthusian theory with pre-modern observations has been recently documented by Ashraf and Galor (2011). This paper, in fact, provides a clear illustration of the global breakdown of Malthusian theory. A limitation of this paper is the lack of direct data on technological differences across space and time. Comin et al. (2010) assembled a dataset on technology adoption during pre-modern times and argued that population does not play a dominant role in pre-modern technological changes. This paper is consistent with their findings.

The fragility of Kremer’s (1993) findings was also discussed by Ravallion (2010), but in a different context. Ravallion (2010) noted a *spacing implication* of Kremer’s (1993) model: longer time periods between observations imply higher growth rates. Ravallion (2010) showed that global data contradict the spacing implication. The analysis presented here is complementary to Ravallion (2010). For example, I examine different predictions and consider additional data sources; data with evenly spaced observations. I also focus on the effect of influential observations.<sup>4</sup>

## 2 Theoretical background

Kremer (1993) proposed the following growth model. Let  $A(t)$  and  $N(t)$  represent the level of technology and population size at date  $t \geq 0$ .  $A(0) = A_0$  and  $N(0) = N_0$  are

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<sup>3</sup>Archaeologists and prehistorians have long debated the role of population pressure in the origin of agriculture. Views based on population pressures were first proposed in the mid-1970s, but they are less favorable nowadays; see, e.g., Harlan (1992). Alternative views emphasize global climatic change; see, e.g., Fagan (2005). A global factor is consistent with the synchronized adoption of farming across separate regions of the world; see Smith (1995).

<sup>4</sup>Ravallion (2010) updated Kremer’s (1993) global population data for 2000 and 2005, and showed that this also weakens the performance of the model; a point anticipated by Kremer (1993).

positive and given. Production takes place with a Cobb-Douglas production function,  $Y(t) = A(t)N(t)^\eta T^{1-\eta}$  with  $0 < \eta < 1$ , where  $T$  is the fixed amount of land.

Technological change is governed by

$$\dot{A}(t) = \lambda A(t)^\phi N(t)^\gamma, \quad (1)$$

where  $\phi$ ,  $\gamma$ , and  $\lambda$  as positive and fixed parameters. The parameter  $\phi$  measures the returns to scale to knowledge. The constant  $\gamma$  captures the influence of population on the production of knowledge. The influence of exogenous variables, or variables other than  $A(t)$  and  $N(t)$ , is captured by  $\lambda$ .

Income per capita is  $y(t) = A(t)(N(t)/T)^{\eta-1}$ . There is a “subsistence level”  $\bar{y}$  such that changes in technology translate into higher population. That is,  $y(t) = \bar{y}$ , and

$$\frac{\dot{N}(t)}{N(t)} = \frac{1}{1-\eta} \frac{\dot{A}(t)}{A(t)}. \quad (2)$$

Combining (1) and (2) yields

$$\dot{N}(t) = \theta N(t)^\alpha, \quad (3)$$

with  $\theta \equiv \lambda \bar{y}^{\phi-1} T^{(\eta-1)(\phi-1)} / (1-\eta)$  and  $\alpha \equiv (1-\eta)(\phi-1) + \gamma$ .

The parameter  $\theta$  depends on  $\lambda$  and  $T$ . This implies that exogenous determinants of technological change have a positive effect on population growth. If  $\phi < 1$ , arable land also influences population growth positively. The composite parameter  $\alpha$  captures the race between Malthusian and Boserupian effects. If  $\gamma < (1-\eta)(\phi-1) - 1$ , Malthusian effects dominate: population growth and size will be negatively related. If  $\gamma > (1-\eta)(\phi-1) - 1$ , Boserupian effects dominate: population growth and size will be positively related.

Klasen and Nestmann (2006) enriched the previous model. They considered population density as an argument in the production of knowledge, e.g., (1) becomes  $\dot{A}(t) = \lambda A(t)^\phi N(t)^\gamma (N(t)/T)^\sigma$  with  $\sigma > 0$ . In this case,  $\theta \equiv \lambda \bar{y}^{\phi-1} T^{(\eta-1)(\phi-1)-\sigma} / (1-\eta)$  and  $\alpha \equiv (\phi-1)(1-\eta) + \gamma + \sigma$ . Including population density in (1) enhances the relationship

between population size and growth in  $\alpha$  but weakens the positive relationship between arable land and population growth. It is not possible to separately identify the different factors that determine  $\alpha$  and  $\theta$ . I will, however, study empirically the effect of arable land on population growth.

The following result can be easily established for the interesting case of  $\alpha > 1$ . *The solution to the (Bernoulli) differential equation (3) satisfies*

$$\ln [N(t)] = \frac{\ln[\theta(1 - \alpha)t + N_0^{1-\alpha}]}{1 - \alpha}, \quad (4)$$

*defined for  $t < t^* \equiv N_0^{1-\alpha}/\theta(\alpha - 1) > 0$ . In the limit, as  $\alpha \rightarrow 1$ , the model yields exponential growth at a rate  $\theta$ .*

Expressions (3) and (4) distinguish between population growth due to exogenous factors, captured by  $\theta$ , and endogenous influences, captured by  $\alpha$ . Since  $\theta$  may differ across regions, it is important to control for potentially confounding influences to properly identify the role of population, e.g.,  $\alpha$ . By assumption, technological change does not augment output per capita; technological change augments population size. The assumption of a constant “subsistence level”  $\bar{y}$  implies that population is a proxy for technology; see (2). This (Malthusian) assumption is problematic for recent observations, but less so for the pre-modern epoch when differences in subsistence across regions were less apparent; see, e.g., Ashraf and Galor (2011).

The main testable prediction of (3) is that *(i) population growth is an increasing function of current population size,  $N(t)$* . In terms of the solution (4), population growth increases with: *(ii) time,  $t$ , (iii) exogenous factors,  $\theta$ , and (iv) initial population size,  $N_0$* . Kremer (1993) focused on (i) and (iv). Predictions (ii) and (iii) have not been previously examined.<sup>5</sup>

Predictions (i) to (iv) are reduced-form. Ideally, one would like to examine the structural equations (1) and (2). This is not feasible due to the lack of data for  $A(t)$ . For

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<sup>5</sup>Ravallion (2010) interpreted the time index  $t$  as the length between observations. Accordingly, a longer time period between observations imply higher growth rates. Differences in the spacing between observations are specially relevant in the early part of the sample in Kremer (1993). I do not focus on spacing because the data sources used here are roughly evenly spaced.

example, it is not possible to separately test for “scale effects” (e.g.,  $\gamma = 1$  and  $\phi = 0$ ) or “market size effects” (e.g.,  $\gamma < 1$  and  $\phi < 1$ ).<sup>6</sup> As usual, the validity of the reduced-form predictions does not necessarily imply the validity of the structural equations.

### 3 Tests with modern population data

This section examines predictions (i) to (iii), which require modern data. The next section examines (iv) using pre-modern data. The first part of this section focuses on data issues. The next sub-sections present the empirical estimates and sensitivity analyses.

**Data.** To examine prediction (i), Kremer (1993) used population estimates from Deevey (1960). These estimates are available “from the inception of the hominid line one million years ago”; see Deevey (1960, p. 195). Deevey (1960) provided highly speculative population data.<sup>7</sup> A particular drawback is that Deevey (1960) assumed that the population density within each of the areas populated by humans increased at an increasing rate from 1 million years ago (MYA henceforth) onwards. As Appendix A shows, this faster-than-exponential growth in population densities implies that population growth and size are positively related by construction.

Figure 1 plots population size and its growth rate from 1MYA to 1990 in the global data from Deevey (1960), Kremer (1993), and Ravallion (2010). As the figure shows, these variables are strongly positively related in Deevey (1960). Figure 1 also suggests that the positive association between population growth and size in Kremer (1993) is driven in part by recent observations. The rest of this section elaborates on this observation.

Several authors have provided longitudinal estimates of regional populations in post-agricultural times. I rely on McEvedy and Jones (1985) and Biraben (1979). None of the available sources are direct measures and it is known that population data are highly uncertain; see, e.g., Caldwell and Schindlmayr (2002). Both estimates, however, are

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<sup>6</sup>Madsen (2008) used patents and R&D data for OECD economies to examine Schumpeterian and semi-endogenous versions of the knowledge production function. These tests require data that is not available in pre-modern times.

<sup>7</sup>The assumptions used by Deevey (1960) are based on limited empirical evidence about hunter-gatherer societies. Deevey himself remarked: “my own treatment of this, published some years ago in *Scientific American*, was not very professional,” Deevey (1968, p. 248).

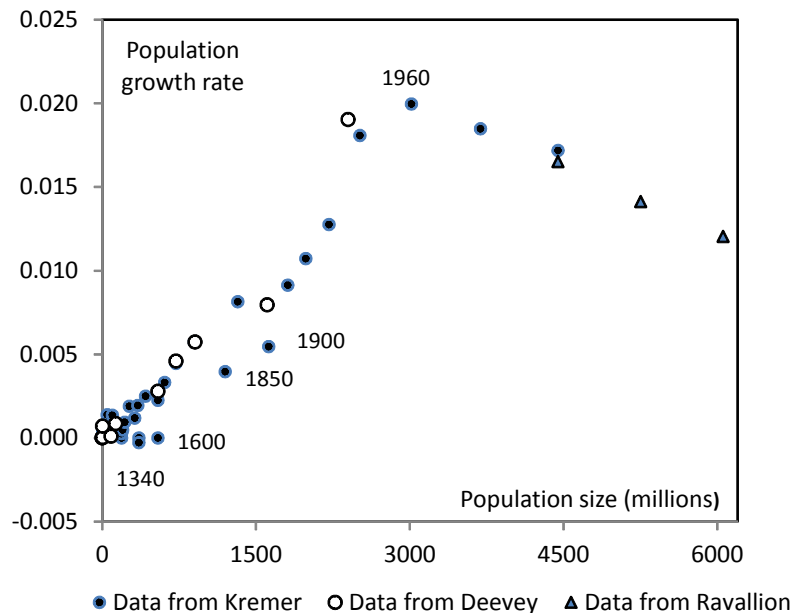


Figure 1: World population size and its growth rate, 1MYA to 2010.

independent and cover a large number of geographic areas: McEvedy and Jones (1985) contains 73 separate regions and data from 200 BC to 1975. Biraben (1979) contains 12 regions and population data from 400 BC to 1970.

**Population growth and population size.** Let  $n_{i,t}$  be the geometric average growth rate of population of region  $i$  between periods  $t-1$  and  $t$ . (The geometric average is more appropriate than the arithmetic average for describing population growth.)

Consider the following simple econometric model

$$n_{i,t} = \beta_0 + \beta N_{i,t-1} + \delta_i + \varepsilon_{i,t}, \quad (5)$$

where  $N_{i,t-1}$  denotes the population size of region  $i$  in period  $t-1$ . The  $\delta_i$  denote unobserved region-specific components that influence the growth rate of population. I consider both random and fixed effects. (The results are invariant to the choice of specification.) The error term,  $\varepsilon_{i,t}$ , captures all other omitted factors.  $\beta_0$  is a constant term.

Prediction (i) implies  $\beta > 0$ . Table 1, Panel A, estimates (5) in the entire sample and

three global population data, using OLS. Column (1) reproduces the estimates in Kremer (1993). Column (2) uses data from McEvedy and Jones (1985) and Column (5) data from Biraben (1979). In all data sources, the point estimates for  $\beta$  are positive and significant. The point estimates in (2) and (5) are actually larger than those that rely on Kremer’s (1993) data, Column (1).

Columns (3) and (4) of Table 1, Panel A, present estimates of (5) using regional data from McEvedy and Jones (1985). Columns (5) and (6) present the same estimates using data from Biraben (1979). Columns (3) and (5) consider random effects, and Columns (4) and (6) fixed effects. The point estimates in these specifications are positive, statistically significant, and virtually identical across data sets and specifications. The point estimates in regional data are larger than the estimates for the world population. These estimates show that global and regional population growth rates are strongly correlated with population size as predicted by the theory.

The previous estimates do not speak directly about (3). To estimate the relevant elasticity  $\alpha$  in (3) Consider a log-log specification:

$$\ln n_{i,t} = \rho_0 + \rho \ln N_{i,t-1} + \delta_i + \varepsilon_{i,t}. \quad (6)$$

Some growth rates  $n_{i,t}$  are negative so I ignore nonpositive growth rates and use  $\rho$  as an *upper bound* for  $\alpha - 1$ .<sup>8</sup>

Table 1, Panel B, presents estimates for (6) under the same specifications as Panel A. All point estimates are positive and significant. A point estimate of  $\hat{\rho} = 1$  implies that population growth is directly proportional to population size. Global data suggest that this might not be a bad approximation. For example, in Kremer’s (1993) data,  $\hat{\alpha} = 1.80$ . Under regional data, the relevant elasticity  $\alpha$  is around 1.5.<sup>9</sup>

*Sensitivity analysis.*— In order to draw generalizations about the performance of the

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<sup>8</sup>I also re-scaled the growth rates so that all values of the normalized population growth rates are positive. The results are sensitive to the normalization used. I do not present these results here but, as expected,  $\rho$  is an upper bound. Notice that dropping nonpositive growth rates is the simplest way to control for events such as the Black Death and the Mongol invasions that resulted in population declines.

<sup>9</sup>Using data from Deevey (1960) yields  $\rho = 0.86$  (s.e. 0.07). This point estimate is indistinguishable from the one obtained in Kremer’s (1993) data.

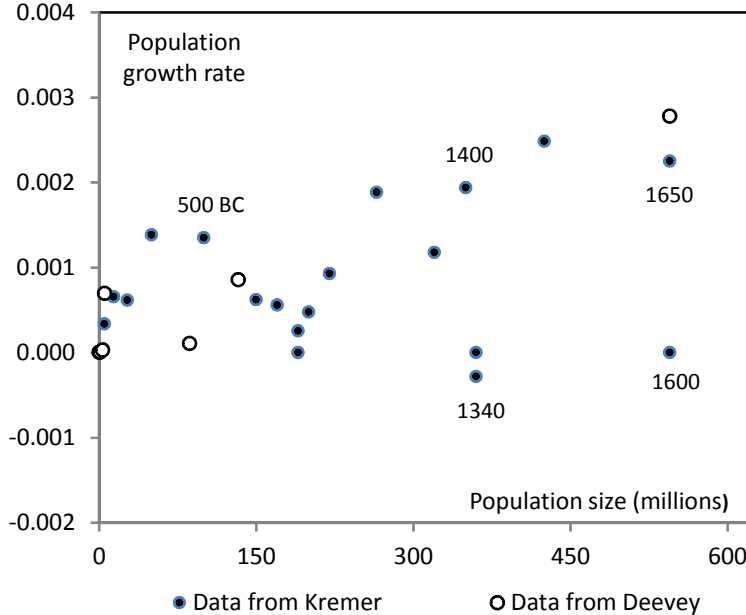


Figure 2: World population size and its growth rate, 1MYA to 1650.

model, it is important to examine if the results in Table 1 are driven by any particular set of observations. A variety of diagnostics can be used to detect influential observations; see Chatterjee and Hadi (1988). For example, *high-leverage* diagnoses atypical observations. Leverage also determines the *Cook's distance* diagnostic, which measures the influence of a given observation on the point estimates. For Table 1, Panel A, post-1900 observations in McEvedy and Jones (1985) and Biraben (1979) and post-1940 observations in Kremer (1993) are high-leverage points. That is, the values of population size in these periods are atypical compared to the majority of the sample. The Cook's Distances also suggest that the point estimates in (5) are sensitive to removing observations from these periods.<sup>10</sup>

How sensitive are the point estimates in Table 1? Recall from Table 1, Panel A, that

<sup>10</sup>An observation is considered high leverage if its leverage exceeds  $4/N$ .obs.; see, Chatterjee and Hadi (1988, p. 100). All Cook's Distances ( $D$ ) should be roughly equal. A relatively large Cook's distance indicates an influential observation. I use the cut-off values based on  $D > F(0.5, 2, N$ .obs.-2); see, Chatterjee and Hadi (1988, p. 119). Influential observations are not necessarily outliers, but their inclusion is likely to bias the estimation of the regression coefficients. For example, because OLS minimizes square deviations, the estimates place a relatively heavy weight on atypical observations. I also computed the previous diagnostics for regional data. Leverage and the Cook's distance have a significant positive time trend. This indicates that recent observations are more influential than pre-modern observations.

the point estimates from Kremer (1993) is 5.08 (s.e. 0.5). This point estimate declines to 4.22 and 4.56 if post-1900 and post-1800 observations are excluded, respectively. These estimates are still significant. If post-1700 observations are excluded, however,  $\beta$  declines to 2.60 (s.e. 1.14) and becomes marginally significant; if post-1600 observations are excluded,  $\beta$  declines to 1.75 (s.e. 1.14) and it is no longer significant. This lack of significance is due to a large decline in the point estimate for  $\beta$ . Figure 2 illustrates the fragility of these estimates; population growth and size are unrelated in Kremer’s (1993) global sample prior to the 1700s.

The sensitivity analysis for Kremer’s (1993) data can be questioned because the degrees of freedom in the global samples are considerably reduced, and because the small global sample makes influential observations more likely to occur. (Robust estimation methods also face a small sample problem.) A decline in the point estimates of  $\beta$ , however, is evident even in regional data. Table 2 presents estimates of (5) for alternative sample periods. Excluding post-1900 observations has a dramatic impact on the point estimates in regional data. All point estimates except the fixed effects in Biraben (1979), which is only marginally significant, become insignificant. Excluding post-1800 observations has a similar effect. Only the fixed effects estimate in McEvedy and Jones (1985) is significantly different from zero in both tables. Finally, no point estimate is significant if post-1700 observations are excluded. In these samples, the estimates of  $\beta$  have the “wrong” (negative) sign. That is, these estimates suggest that population growth and size are negatively rather than positively associated. If one excludes post-1600 observations, some of these negative estimates become significant.

The decline in the significance of the point estimates is *not* due to an increase in the standard errors, but to a decline in the point estimates themselves. This fact is important because the most recent data is likely to be of the highest quality and hence one would expect precision to be lost if these observations are removed. As Table 2 shows, the decline in the significance of the estimates is not due to more “imprecise” estimates.

An alternative way to examine the sensitivity of the results is to focus on modern observations only. To save space, I only discuss post-1600 estimates of (5) for the global

data and the fixed effects regional estimates in McEvedy and Jones (1985), where more data is available. For the global data, and using the same normalization as Table 1,  $\beta = 7.52$  (s.e. 0.70). For the fixed effects estimates,  $\beta = 42.60$  (s.e. 11.70). These estimates are positive, significant, and larger than the estimates in the entire sample. These results and Table 2 imply that the strong positive relationship in the recent samples drives the results in Table 1.

Table 3 considers the sub-samples of Table 2 but uses the log-log specification (6). The general pattern for regional data is consistent with Table 2. In Table 3, the estimates of  $\rho$  also decline, although significance is not always lost. (Recall that  $\rho$  is biased upwards since negative growth rates are omitted.) If post-1600 data is excluded, the upper bound estimate  $\rho$  is reduced by at least 50 percent.

Using alternative sample periods results in substantively different estimates of  $\beta$  and  $\rho$ . Estimates of  $\beta$  lose significance even when a few observations are dropped, as in regional data. These findings suggest that the positive association between population growth and size in Kremer (1993) is driven by the transition from pre-modern high mortality and fertility to current low mortality and fertility. Before this transition occurred, population growth and size were unrelated. In the post-1960 period, population growth and size are negatively related; see Figure 1. As the next sub-sections show, the sensitivity of the results is not unique to prediction (i); empirical results about predictions (ii) and (iii) are also sensitive to changes in the sample.

**Population growth and time.** Recall that (4) implies the existence of a time trend in the growth rate of population. This mechanical relationship provides a complementary test of the model's performance. To estimate a linear time trend  $\tau$  consider:

$$n_{i,t} = \tau_0 + \tau \text{Time} + \delta_i + \varepsilon_{i,t}. \quad (7)$$

Table 4 presents the estimates of (7) for alternative sample periods. The first panel includes the entire sample period. In this panel, and for both data sources, the growth rate of population exhibits a positive time trend. The point estimates for  $\tau$  are fairly similar across specifications and sources, and between the global and the regional samples. The

second and third panel consider samples that end in 1800 and in 1600, respectively.

There are two notable features in Table 4. First, point estimates for  $\tau$  decline uniformly across samples. Second, the decline is shared by the estimates based on global and regional data. The significance of the point estimates also declines as the end point of the sample is reduced. In Biraben (1979), no single estimate is significantly different from zero if post-1600 data are excluded. In McEvedy and Jones (1985), regional estimates remain significant but the results show no trend in world population. Finally, as in Tables 2 and 3, the decline in significance is not due to an increase in the estimated standard errors.

**Exogenous determinants of population growth.** Recall from (4) that exogenous variables also influence population growth. This relationship was not considered by Kremer (1993), but accounting for exogenous factors is important to identify confounding long-term influences on population growth.

As in (5) and (7), consider the following econometric model

$$n_{i,t} = \varphi_0 + \varphi\theta_i + \varphi_x X_{i,t} + \varepsilon_{i,t}, \quad (8)$$

where  $\theta_i$  denotes time-invariant exogenous factors, and  $X_{i,t}$  denotes controls variables. In particular, I use time,  $t$ , and population size,  $N_{i,t-1}$ , as control variables. Since  $\theta_i$  is time-invariant, I cannot rely on fixed-effects. Instead, I will present pooled OLS estimates. (The random effects estimates are virtually identical, and to save space I do not report them.) For these estimates, population size controls for regional-specific factors. Further, since I assume that global arable land and other exogenous factors have remained constant, I can only examine regional data. Finally, I examine McEvedy and Jones (1985) because their data contains a larger number of regions and this permits a better matching with additional sources.

I follow two different approaches. First, I examine the relationship between population growth and arable land, as reported in McEvedy and Jones (1985). Second, I examine biogeographic variables emphasized by Diamond (1997) and coded (at the regional/country level) by Hibbs and Olsson (2004). Country boundaries are endogenous, so arable land is not necessarily exogenous. Biogeography is more likely to be so.

*Arable land.*— Table 5 presents the results for alternative samples. Panel A includes log-arable land as a predictor of population growth. (Total land yields similar results.) Column (1) includes no other controls, Column (2) includes a time trend, and Column (3) controls for population size. Consider the complete sample first. The point estimate in Column (1) is  $\varphi = 9.7$ ; statistically significant. The inclusion of time or population size only changes marginally the value of  $\varphi$  without lowering statistical significance. (Population and time are strongly correlated and hence I do not include both factors simultaneously.) These results show that population has grown at faster rates in regions that support larger population sizes.

The results in alternative sub-samples, however, provide a different view. In the sample that ends in 1800, arable land is no longer significant with or without a time trend or population controls. The diminished significance is not due to larger standard errors or higher imprecision. Moreover, in the sample that ends in 1600, the estimate of  $\varphi$  has the “wrong” (negative) sign, and it is significant; before the modern epoch, population growth was slower in larger geographic areas.

*Biogeography.*— Table 5, Panel B, presents a complementary strategy. I matched the regions in McEvedy and Jones (1985) to the biogeographic information coded by Hibbs and Olsson (2004). Table 5, Panel B, examines the influence of the *number of plants*, which is the average numbers of locally available wild plants suited to domestication 12KYA in various parts of the world, and the *number of animals*, which is the number of species of wild terrestrial mammalian herbivores and omnivores suitable for domestication in various regions of the world. I also consider a specification without a time trend and two other specifications with time trend and a population control.<sup>11</sup>

Column (1) shows that the number of plants has no predictive power for population growth. The number of animals is negative and significant. Instead of yielding higher population growth, a large endowment of domesticable animals lowers population growth. The variation of the point estimates across sub-samples is the same as in Panel A. In the

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<sup>11</sup>The number of domesticable plants and animals are highly correlated in the sample (their correlation coefficient is 0.87). I included the continental axis of orientation from Hibbs and Olsson (2004) as an alternative specification. The results are similar to Panel B, and are available upon request.

sample that ends in 1800, none of the biogeographic variables is significant whereas in the sample that ends in 1600, the signs are reversed; in the pre-1600 sample, animals have a positive influence on population growth. As in Panel A, adding a time trend or population size as a control does not alter the previous conclusions.

Overall, predictions that relate population growth to arable land are supported in samples that include modern observations. Biogeography is unimportant in modern data, but it helped determine population growth in the pre-modern epoch. As in the case of land, the estimated coefficients change signs in pre-modern samples.

These reversals cannot be reconciled with continuity in economic and demographic conditions. For example, the fastest pre-1900 population growth took place in European and Neo-European regions. These areas were not populous at the onset of the demographic transition. After 1900, the demographic transition diffused to large and populous countries. This distinct timing explains the positive effect of arable land on population growth in the complete sample, and the negative effect in pre-modern data. The reversal in the role of biogeography can be explained similarly. Industrialization, for instance, first diffused into regions that lacked endowments for settled agriculture: North America and Australia. These reversals are consistent with the *reversal of fortune* among European colonies documented by Acemoglu et al. (2002). Notice, however, that the results presented here hold on average across *all* world regions.

## 4 A natural experiment

This section provides a causal assessment of (4) using the melting of the ice caps that divided the continents as a *natural experiment*. These tests are aimed at examining prediction (iv) from Kremer's (1993) model.

**A case study.** The land bridges that connected the New World with Asia and New Guinea with Australia (also Tasmania and Flinders Island) disappeared some 12KYA, leaving some populations geographically isolated. Geographic isolation created differences in population sizes across various regions in the world. As in Kremer (1993), assume

that populations were randomly allocated among the Old and the New World, and that population densities were the same in all regions of the world prior to the melting of the ice caps. This assumption can be justified because hunter-gathering was the common mode of production at that time.

To evaluate the effect of population using this *natural experiment*, consider the New World as the treatment group. One needs a control group with similar pre-treatment characteristics. The selection of this control group is important because it should be able to eliminate exogenous influences in population size. One possible control area is the Old World. A comparison between the Old and the New World suggests important advantages associated with the Old World's geographic area. Since densities are assumed equal, the larger area of the Old World translates into a larger population size prior to the melting of the ice caps. The Old World was also more sophisticated and more densely populated near 1500. This comparison suggests that differences in initial population size played a positive role in the Old World's comparative development.

There are, however, important differences unaccounted for in the previous comparison. In particular, larger areas do not only have larger human populations but also a larger number of domesticable animals and plants suitable for domestication; see Diamond (1997). If these differences are not properly controlled for, the inferences attributed to population alone are likely to be biased.

In order to eliminate confounding factors, one needs to compare suitable isolated societies in the New World to a suitable control group in the Old World. It is possible to argue that sub-Saharan Africa may be as close as possible to a control group for the isolation of societies in tropical America. Among the reasons consider: (i) Sub-Saharan Africa and tropical America have similar geography. In both, axes run mostly from North to South; see Diamond (1997, p. 177), and the total area of these continents is relatively equal. (ii) Population sizes in 400 BC were similar in these two regions; see Appendix B. (iii) Agriculture originated in both regions at a much later date than in the Near East or China but roughly at the same time, about 4 KYA; see Smith (1995). Moreover, in neither of them there were many domesticable animals; see Diamond (1997, Table 9.2),

and (iv) the number of large-seeded grass species needed for agriculture was similar; see Diamond (1997, Table 8.1). Finally, (v) climate is similar since Africa and South and Central America are the only continents that cross the Equator.

What insights can be gained by comparing tropical America with tropical Africa? While both regions are similar in size, orientation, climate, and biogeography, sub-Saharan Africa was not isolated from Eurasia.<sup>12</sup> This fact implies that one should expect higher economic conditions in Africa near 1500. I study this comparison in detail in Appendix B. There I show that existing evidence fails to suggest that sub-Saharan Africa had more advanced economic conditions than tropical America at that time. For example, the urbanization rate, a common proxy for pre-modern sophistication, was lower in sub-Saharan Africa than in tropical America in 1500. Estimates of income per capita in 1500 are also lower in sub-Saharan Africa than in tropical America.

In conclusion, once exogenous influences are controlled through a careful selection of the control group, evidence based on the melting of the ice caps fails to support the hypothesis that differences in initial population sizes were central to explain differences in pre-modern economic conditions.

**Some econometric estimates.** This sub-section complements the previous discussion. In this sub-section I examine samples that end in 1500, in accordance with the previous *natural experiment*. Consider the following econometric model:

$$\ln[N_{i,t}] = \pi_0 + \pi_1 \ln[N_{i,0}] + \pi_2 \text{Time} + \pi_3 \{\ln[N_{i,0}] \times \text{Time}\} + \pi_x X_{i,t} + \delta_i + \varepsilon_{i,t}, \quad (9)$$

where  $\ln[N_{i,0}]$  is the log of population in area  $i$  at time  $t = 0$ ; during the first period in the sample. I use this variable as a proxy for the population size prior to the melting of the ice caps. (As a robustness check, I will replace this variable by arable land later on.) The term  $\{\ln[N_{i,0}] \times \text{Time}\}$  is the cross-product of initial population and time. As before,  $\delta_i$  is a region-specific unobserved component that captures invariant characteristics in the

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<sup>12</sup>Repeated contact existed through the Nile River because of ancient Egypt, through the Sahara by the Arab trade that started during the seventh century AD, and by the East African trade through the Indian ocean in medieval times; see, e.g., Connah (2001).

various regions, and  $X_{i,t}$  are additional control variables.

In (9),  $\pi_1$  captures how initial differences in population size yield differences in final population size,  $\pi_2$  captures the time trend in population, and  $\pi_3$  captures the double difference between time and initial population size. The double-difference estimates if areas with larger initial populations grew at faster rates than areas with smaller initial populations. To be consistent with (4),  $\pi_3$  should be positive.

Table 6 presents the results using data from McEvedy and Jones (1985) and Biraben (1979), and three specifications: pooled OLS, random effects, and fixed effects, all with alternative sets of controls. I also present two strategies to account explicitly for possible correlation across the error term. I consider robust standard errors and clustering at the continental level. Clustering implies that unobserved shocks in the initial period will have a common spillover within each continent during all the remaining periods.

Column (1) includes the interaction term  $\ln[N_{i,0}] \times \text{Time}$  with no other covariates. In Column (1),  $\pi_3$  is positive, significant, and virtually identical across data sets. Columns (2) and (3) include a time trend and the initial population size of the region as controls. Time trends have little impact on the point estimates. (The reason being that there was little sustained population growth at that time.) As Panel I Column (3) shows, however,  $\pi_3$  is no longer positive once initial population size is included as a control;  $\pi_3$  is negative and significant. In Panel II,  $\pi_3$  also switches signs in Column (3).

Column (4) includes continental controls (for Africa, America, Asia, Europe, and Australia). The point estimates for  $\pi_3$  are virtually unchanged, although the clustered standard errors yield a reduced significance compared to the robust standard errors. Columns (5) and (6) present the random effects estimates, and Column (7) the fixed effects estimates. In these specifications,  $\pi_3$  is also negative and statistically significant.<sup>13</sup>

Table 7 reproduces Table 6 for alternative specifications and the McEvedy and Jones (1985) data. First, I use 1 AD (or year 1) as the initial date rather than 200 BC because there are fewer observations in this early sample. The pattern across specifications is the same as in Table 6. Second, I use arable land instead of initial population. Population in

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<sup>13</sup>I also added dummy controls for the Black Death and the Mongol invasions to specification (7) in Table 6. The point estimates for these events are negative, but they do not alter the estimate of  $\pi_3$ .

200 BC may be an inadequate proxy for the population size prior to the melting of the ice caps. In 200 BC, the large centers of agricultural production in Asia were consolidated and may have faced relatively stagnant conditions. Arable land may be a better proxy for the size of pre-treatment populations. In specifications that use arable land,  $\pi_3$  is also positive in the absence of controls and negative (though insignificant) once the basic controls are added.

Table 8 reproduces Table 6 for alternative specifications and the Biraben (1979) data. I consider two additional initial dates, 200 BC and 1 AD. In both cases, the pattern seen in Tables 6 and 7 is also present. Finally, it may be useful to discriminate across regions given that their pre-1500 developments were independent. I have estimated specification (7) from Table 6 separately for the Old and the New World. (Fixed effects estimates are representative of all other specifications.) In order to maximize the number of observations, I use  $t_0 = 1$  AD. These estimates are  $\pi_3^{\text{Old World}} = -2.1$  (s.e. 0.3) and  $\pi_3^{\text{New World}} = 0.4$  (s.e. 0.4). Among the regions of the Old World,  $\pi_3^{\text{Eurasia}} = -2.0$  (s.e. 0.4) and  $\pi_3^{\text{Africa}} = -2.7$  (s.e. 1.0). The only positive but non-significant estimate of the interaction term is in the New World, which also contradicts the idea of faster population growth in areas with larger initial populations.

A further difficulty with Kremer’s (1993) model is that the positive association between population growth and initial population size (or arable land) in pre-modern samples is reversed once controls for systematic differences across the various world regions are included. As in the case study previously discussed, omitting systematic differences across regions leads to confounding results by ascribing to Boserupian mechanisms influences that are otherwise Malthusian. For example, a negative and significant estimate of  $\pi_3$  suggests *mean-reverting dynamics* during the pre-modern epoch. That is, these estimates are consistent with the tendency of population to return to some stationary Malthusian equilibrium value after positive or negative deviations. In samples that include modern population data, population does not exhibit mean reversion.<sup>14</sup>

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<sup>14</sup>For example, regressing  $\ln[N_{i,t}]$  on  $\ln[N_{i,t-1}]$  in the full sample in McEvedy and Jones (1985) yields a fixed effects point estimate of 1.030 (s.e. 0.016). Since this point estimate exceeds one, the estimates suggest “divergence” across regions. For the reasons discussed in the previous section, this divergence appears to be a transitory event associated with the demographic transition.

## 5 Concluding remarks

This paper examined several testable predictions from Kremer’s (1993) well-known model of population growth. The results differ from the interpretation presented by Kremer (1993). Overall, and especially outside of the modern epoch, the results find weaker support for the hypothesis that population growth spurs technological change. In particular, I found that the conclusions one can draw about the model’s performance are very sensitive to the sample period: the sign and statistical significance of the relationships between population growth and size, population growth and time, and population growth and exogenous factors are reversed when recent observations are excluded. Further, conclusions based on the melting of the ice caps are sensitive to the controls used, and are typically reversed when confounding influences are taken into account.

These results have important implications for the relationship between population growth and technological change. The findings suggest a sharp contrast between pre-modern and modern economic and demographic regimes. Pre-modern population data, for example, exhibit mean reversion consistent with the notion of a stationary Malthusian equilibrium. The reversal in the influence of biogeography and arable land on population growth further contradicts the idea of continuity in demographic conditions. The central issue is that in Kremer (1993), as well as in ideas-based growth models, past technology and population size reinforce technological change. Integrating (1) gives

$$A(t) = A_0 \left[ \lambda(1 - \phi)^{-1} \int_0^t N(s)^\gamma ds \right]^{1/(1-\phi)}, \quad (10)$$

which is increasing in “accumulated experience”  $\int_0^t N(s)^\gamma ds$ .

Continuity contradicts the evidence presented in this paper. How can the results be rationalized? Consider an alternative view from *leapfrogging* models, e.g., Brezis et al. (1993). In these models, experience is a disadvantage for “radical” technological change. Under a leapfrogging view, population may be beneficial during “normal” times but detrimental for “radical” technological changes. Suppose, for example, that the pre-modern epoch is characterized by technological change  $\hat{A}(t)$  determined exclusively by

exogenous variables such as biogeography, e.g., let  $\hat{A}(t) = \hat{A}_0 \exp\{\lambda t\}$ , which is (10) with  $\phi = 1$  and  $\gamma = 0$ . If a new technology that satisfies (10) becomes available, economies without much experience with  $\hat{A}(t)$  will first adopt the new technology  $A(t)$ . As time goes by, these backward economies will overtake advanced regions. This simple alternative view has the potential to reconcile the presence of positive population externalities in the modern epoch with the multiple reversals documented in the paper.<sup>15</sup>

There are many caveats that make a definite interpretation difficult. The data sources are not ideal and dropping recent observations likely exacerbates measurement errors.<sup>16</sup> Confirmation bias is also important. As discussed by Kremer (1993, pp. 699-700), the basic assumption behind the estimates of population in the past appears to be a Malthusian assumption that associates increases in population with exogenous technological changes. Moreover, earlier historical periods are characterized by small and infrequent discrete technological changes, and the current tests only rely on post-400 BC data. Thus, they do not necessarily discriminate against very gradual technological change, which was the norm prior to the Industrial Revolution. The findings show the need for further theoretical and empirical investigations of the relationship between population growth and technological change.

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<sup>15</sup>In the context of Europe’s industrialization, Diamond (1997, pp. 409-410) argued that “a historian who had lived at anytime between 8500 BC and 1450 AD, and who had tried then to predict future historical trajectories, would surely have labeled Europe’s eventual dominance as the least likely outcome, because Europe was the most backward of those three Old World regions for most of those 10,000 years.[...] Until the proliferation of water mills after about AD 900, Europe west or north of the Alps contributed nothing of significance to Old World technology or civilization.”

<sup>16</sup>There are few alternative data sources to examine pre-modern conditions. Anthropological analyses of genetic diversity in current populations have been able to shed light on the demography of past populations; see, e.g., Relethford (2001, 2003). In economics, a growing literature has started using genetic information to examine current and past differences in economic development; see, e.g., Spolaore and Wacziarg (2009) and Ashraf and Galor (2008).

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Table 1. Population growth and population size.

A. Dependent variable is population growth							
	I. Kremer (1993) 1MYA to 1990	II. McEvedy and Jones (1985) 200 BC to 1975		III. Biraben (1979) 400 BC to 1970			
	World population OLS	World population OLS	Regional data		World population OLS	Regional data	
	(1)	(2)	Random effects (3)	Fixed effects (4)	(5)	Random effects (6)	Fixed effects (7)
$N_{i,t-1}$	5.08*** (0.55)	7.06*** (0.57)	26.5*** (7.92)	36.3*** (7.99)	6.99*** (0.68)	26.3*** (5.79)	39.3*** (6.98)
N. Obs.	37	20	679	679	25	300	300
N. Regions	-	-	73	73	-	12	12
R <sup>2</sup>	0.90	0.93	0.05	0.05	0.87	0.11	0.11
B. Dependent variable is log-population growth							
$\ln N_{i,t-1}$	0.80*** (0.06)	1.30*** (0.10)	0.33*** (0.03)	0.46*** (0.04)	1.48*** (0.31)	0.43*** (0.06)	0.57*** (0.06)
N. Obs.	33	16	566	566	21	176	176
N. Regions	-	-	73	73	-	12	12
R <sup>2</sup>	0.86	0.90	0.16	0.16	0.52	0.45	0.45

Notes: Robust standard errors are in parentheses.  $N_{i,t-1}$  denotes population for region  $i$  in period  $t - 1$ . Panel B dropped the negative growth rates. Point estimates and standard errors in Panel A are multiplied by  $10^5$  to aid visually with the presentation of the results. \*\*\* and \*\* denote significance at the 1 and 5 percent levels.

Table 2. Population growth and population size. Alternative sample periods.

I. McEvedy and Jones (1985)												
	200 BC to 1900			200 BC to 1800			200 BC to 1700			200 BC to 1600		
	World	Regional data		World	Regional data		World	Regional data		World	Regional data	
	pop.	Rand.	Fixed	pop	Rand.	Fixed	pop.	Rand.	Fixed	pop.	Rand.	Fixed
	OLS	effects	effects	OLS	effects	effects	OLS	effects	effects	OLS	effects	effects
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
$N_{i,t-1}$	6.06***	9.79	12.90	5.38***	5.08	20.10**	2.24	-3.63	-2.90	3.93	-3.58	-7.18
	(0.61)	(8.59)	(9.24)	(1.59)	(7.95)	(9.75)	(2.13)	(4.70)	(10.50)	(3.01)	(5.55)	(16.00)
N. Obs.	18	533	533	16	404	404	14	346	346	12	302	302
N. Reg.	-	68	68	-	63	63	-	63	63	-	63	63
II. Biraben (1979)												
	400 BC to 1900			400 BC to 1800			400 BC to 1700			400 BC to 1600		
$N_{i,t-1}$	5.13***	5.52	13.50*	4.06**	5.85	4.29	0.82	-5.82	-19.40	-0.34	-8.99	-29.80*
	(0.64)	(5.11)	(7.42)	(1.66)	(7.25)	(10.30)	(2.68)	(8.16)	(11.80)	(3.91)	(9.61)	(15.40)
N. Obs.	23	276	276	21	252	252	19	228	228	18	216	216
N. Reg.	-	12	12	-	12	12	-	12	12	-	12	12

Notes: The dependent variable is population growth.  $N_{i,t-1}$  denotes population for region  $i$  in period  $t - 1$ . The specifications are the same as in Table 1. Robust standard errors in parentheses. Point estimates and standard errors multiplied by  $10^5$  to aid visually with the presentation of the results. \*\*\*, \*\*, and \* denote significance at the 1, 5, and 10 percent level.

Table 3. Log-population growth and log-population size. Alternative sample periods.

I. McEvedy and Jones (1985)												
	200 BC to 1900			200 BC to 1800			200 BC to 1700			200 BC to 1600		
	World	Regional data		World	Regional data		World	Regional data		World	Regional data	
	pop.	Rand.	Fixed	pop	Rand.	Fixed	pop.	Rand.	Fixed	pop.	Rand.	Fixed
	OLS	effects	effects	OLS	effects	effects	OLS	effects	effects	OLS	effects	effects
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
$\ln N_{i,t-1}$	1.38*** (0.15)	0.21*** (0.04)	0.44*** (0.06)	1.49*** (0.19)	0.05 (0.04)	0.30*** (0.07)	1.52*** (0.30)	-0.06 (0.03)	0.21*** (0.07)	1.77*** (0.36)	-0.12 (0.03)	0.16* (0.09)
N. Obs.	14	421	421	12	295	295	10	241	241	9	211	211
N. Reg.	-	68	68	-	55	55	-	48	48	-	47	47
II. Biraben (1979)												
	400 BC to 1900			400 BC to 1800			400 BC to 1700			400 BC to 1600		
	World	Regional data		World	Regional data		World	Regional data		World	Regional data	
	pop.	Rand.	Fixed	pop	Rand.	Fixed	pop.	Rand.	Fixed	pop.	Rand.	Fixed
	OLS	effects	effects	OLS	effects	effects	OLS	effects	effects	OLS	effects	effects
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
$\ln N_{i,t-1}$	1.48*** (0.46)	0.21*** (0.07)	0.45*** (0.08)	1.44** (0.65)	0.07 (0.07)	0.28*** (0.09)	1.24 (0.89)	0.04 (0.07)	0.25*** (0.09)	1.24 (1.07)	0.04 (0.07)	0.26*** (0.09)
N. Obs.	19	152	152	17	131	131	15	117	117	14	110	110
N. Reg.	-	12	12	-	12	12	-	12	12	-	12	12

Notes: The dependent variable is log-population growth for positive growth rates only.  $\ln N_{i,t-1}$  denotes log-population for region  $i$  in period  $t - 1$ . The specifications are the same as in Table 1. Robust standard errors in parentheses. \*\*\*, \*\*, and \* denote significance at the 1, 5, and 10 percent level.

Table 4. Population growth and time. Alternative sample periods.

I. McEvedy and Jones (1985)									
200 BC to 1970			200 BC to 1800			200 BC to 1600			
	World	Regional data		World	Regional data		World	Regional data	
	population	Random	Fixed	population	Random	Fixed	population	Random	Fixed
	OLS	effects	effects	OLS	effects	effects	OLS	effects	effects
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
Time	4.09**	7.25***	5.01***	1.34**	1.55***	1.42***	0.74	0.54***	0.80***
	(1.51)	(0.43)	(0.49)	(0.51)	(0.22)	(0.22)	(0.47)	(0.20)	(0.24)
N. Obs.	20	679	679	18	404	404	12	302	302
N. Reg.	-	73	73	-	63	63	-	63	63
II. Biraben (1979)									
400 BC to 1975			400 BC to 1800			400 BC to 1600			
Time	3.52***	4.07***	4.07***	1.00	0.60**	0.60**	0.37	0.11	0.11
	(1.43)	(0.57)	(0.58)	(6.14)	(0.30)	(0.30)	(0.69)	(0.31)	(0.31)
N. Obs.	25	300	300	21	252	252	18	216	216
N. Reg.	-	12	12	-	12	12	-	12	12

Notes: The dependent variable is population growth.  $t$  denotes calendar time. The specifications are the same as in Table 1. Robust standard errors are in parentheses. Point estimates and standard errors are multiplied by  $10^5$  to aid visually with the presentation of the results. \*\*\*, \*\*, and \* denote significance at the 1, 5, and 10 percent levels.

Table 5. Population growth, land, and biogeography. Alternative sample periods.

A. Pooled OLS: Arable land (in km <sup>2</sup> )									
200 BC to 1970									
200 BC to 1800									
200 BC to 1600									
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
Log arable land	9.70***	6.31***	10.66***	10.07	5.30	13.30	-1.53**	-1.68**	-1.90**
	(2.21)	(1.95)	(2.45)	(11.20)	(10.40)	(14.80)	(0.69)	(0.69)	(0.83)
Time trend?	No	Yes	No	No	Yes	No	No	Yes	No
Population?	No	No	Yes	No	No	Yes	No	No	Yes
N. Obs.	679	679	679	404	404	404	302	302	679
B. Pooled OLS: Biogeography									
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
Number of plants	-0.33	0.17	0.60	1.57	0.80	2.00	-3.32**	-3.85***	-0.61***
	(0.36)	(0.33)	(0.42)	(1.89)	(0.19)	(2.10)	(1.44)	(1.49)	(0.17)
Number animals	-12.88***	-9.19***	-13.81***	-2.76	-1.65	-2.91***	2.18**	2.99***	3.16***
	(1.91)	(1.86)	(2.04)	(1.70)	(1.68)	(1.77)	(0.88)	(0.94)	(0.96)
Time trend?	No	Yes	No	No	Yes	No	No	Yes	No
Population?	No	No	Yes	No	No	Yes	No	No	Yes
N. Obs.	573	573	573	362	362	362	270	270	270

Notes: The dependent variable is population growth. All specifications are pooled OLS. Robust standard errors are in parentheses. Point estimates and standard errors for arable land are multiplied by 10<sup>3</sup> to aid visually with the presentation of the results. \*\*\*, \*\*, and \* denote significance at the 1, 5, and 10 percent levels.

Table 6. Population size and initial population before 1500.

I. McEvedy and Jones (1985), $t_0=200$ BC							
	Pooled OLS				Random Effects		Fixed Effects
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$\ln[N_{i,0}] \times \text{Time}$	5.25*** (0.51) [0.56]	5.12*** (0.46) [0.60]	-2.61*** (0.64) [0.14]	-2.58*** (0.46) [0.17]	-2.57*** (0.41) [0.09]	-2.57*** (0.40) [0.10]	-2.56*** (0.46) [0.09]
Time	-	4.53*** (1.25)	8.22*** (0.81)	8.51*** (0.78)	7.98*** (0.56)	8.01*** (0.58)	7.97*** (0.63)
$\ln[N_{i,0}]$	-	-	0.98*** (0.04)	0.95*** (0.06)	0.98*** (0.11)	0.96*** (0.09)	-
Continental controls?	No	No	No	Yes	No	Yes	-
N.Obs./N.Reg.	252	252	252	252	252/22	252/22	252/22
II. Biraben (1979), $t_0=400$ BC							
$\ln[N_{i,0}] \times \text{Time}$	4.54*** (0.32) [1.05]	5.24*** (0.33) [1.07]	-2.05*** (0.50) [1.17]	-2.05*** (0.50) [1.18]	-2.05*** (0.39) [1.17]	-2.05*** (0.40) [1.18]	-2.05*** (0.42) [1.16]
Time	-	-4.56*** (1.41)	9.36*** (0.11)	9.36*** (0.10)	9.36*** (0.90)	9.36*** (0.97)	9.36*** (0.99)
$\ln[N_{i,0}]$	-	-	1.04*** (0.04)	0.91*** (0.05)	1.04*** (0.13)	0.91*** (0.16)	-
Continental controls?	No	No	No	Yes	No	Yes	-
N.Obs./N.Reg.	216	216	216	216	216/12	216/12	216/12

Notes: The dependent variable is log population size. Robust standard errors are in parentheses. In brackets are standard errors clustered at the continental level. The point estimates and s.e. for interaction between initial population and time,  $\ln[N_{i,0}] \times \text{Time}$ , have been multiplied by  $10^4$  to aid visually with the presentation of the results. The point estimates and s.e. for Time have been multiplied by  $10^3$ . \*\*\*, \*\*, and \* denote significance at the 1, 5, and 10 percent levels based on the robust standard errors.

Table 7. Population size and initial population before 1500.

	Pooled				Random		Fixed
	OLS				Effects		Effects
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Data: McEvedy and Jones (1985)							
A. Different initial date, $t_0=1$ AD							
$\ln[N_{i,0}] \times \text{Time}$	6.08*** (0.27) [0.17]	6.14*** (0.25) [0.22]	-1.59*** (0.35) [0.40]	-1.57*** (0.32) [0.42]	-1.59*** (0.25) [0.43]	-1.59*** (0.25) [0.44]	-1.59*** (0.30) [0.43]
Time?	No	Yes	Yes	Yes	Yes	Yes	Yes
Initial population?	No	No	Yes	Yes	Yes	Yes	-
Continental controls?	No	No	No	Yes	No	Yes	-
N. Obs./N. Reg.	378	378	378	378	378/64	378/64	378/64
B. Using arable land instead of initial population size							
$\ln[\text{Land}_i] \times \text{Time}$	1.80*** (0.43) [1.15]	2.36*** (0.48) [1.34]	-3.13 (9.52) [5.27]	-3.42 (7.92) [0.42]	-0.71 (2.87) [5.96]	-0.79 (2.89) [6.00]	-0.79 (2.78) [5.97]
Time?	No	Yes	Yes	Yes	Yes	Yes	Yes
Initial population?	No	No	Yes	Yes	Yes	Yes	-
Continental controls?	No	No	No	Yes	No	Yes	-
N. Obs./N. Reg.	416	416	416	416	416/71	416/71	416/71

Notes: The dependent variable is log population size. Robust standard errors are in parentheses. In brackets are standard errors clustered at the continental level. The specifications are the same as in Table 6. \*\*\*, \*\*, and \* denote significance at the 1, 5, and 10 percent levels.

Table 8. Population size and initial population before 1500.

	Pooled				Random		Fixed
	OLS				Effects		Effects
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Data: Biraben (1979)							
A. Different initial date, $t_0=200$ BC							
$\ln[N_{i,0}] \times \text{Time}$	4.29*** (0.28) [1.06]	4.98*** (0.31) [1.09]	-2.04*** (0.54) [0.88]	-2.04*** (0.54) [0.88]	-2.04*** (0.39) [0.88]	-2.04*** (0.41) [0.88]	-2.04*** (0.42) [0.87]
Time?	No	Yes	Yes	Yes	Yes	Yes	Yes
Initial population?	No	No	Yes	Yes	Yes	Yes	-
Continental controls?	No	No	No	Yes	No	Yes	-
N. Obs./N. Reg.	204	204	204	204	204/12	204/12	204/12
B. Different initial date, $t_0=1$ AD							
$\ln[N_{i,0}] \times \text{Time}$	4.72*** (0.26) [0.94]	5.74*** (0.25) [0.87]	-1.60*** (0.52) [0.88]	-1.60*** (0.50) [0.89]	-1.60*** (0.43) [0.88]	-1.60*** (0.45) [0.89]	-1.60*** (0.46) [0.87]
Time?	No	Yes	Yes	Yes	Yes	Yes	Yes
Initial population?	No	No	Yes	Yes	Yes	Yes	-
Continental controls?	No	No	No	Yes	No	Yes	-
N. Obs./N. Reg.	192	192	192	192	192/12	192/12	192/12

Notes: The dependent variable is log population size. Robust standard errors are in parentheses. In brackets are standard errors clustered at the continental level. The specifications are the same as in Table 6. \*\*\*, \*\*, and \* denote significance at the 1, 5, and 10 percent levels.

## 6 Appendix A: Population in the very long run

Table A1 reproduces Deevey's (1960) population and density estimates. These estimates and Hassan (1981) follow a similar principle; they assume that the area inhabited by humans has increased over time and that the population densities within each of these areas has also increased since 1MYA. Deevey (1960), however, assumed that population density increased at increasing rates. That is, the rate of change of population density is also increasing. As a consequence, population size and population growth will be mechanically positively related.

Table A1. Population size and population density in Deevey (1960).

Year	Population density (per square km.) (1)	Growth rate (percent) (2)	Population size (millions) (3)	Growth rate (percent) (4)
1MYA	0.00425	0.000148	0.125	0.000297
0.3MYA	0.012	0.000438	1	0.000439
25KYA	0.04	0.000000	3.34	0.002738
10KYA	0.04	0.022091	5.32	0.069741
6KYA	0.1	0.000000	86.5	0.010753
1	0.1	0.219216	133	0.085569
1650	3.7	0.281297	545	0.289935
1750	4.9	0.471737	728	0.438435
1800	6.2	0.574993	906	0.576606
1900	11	0.801971	1610	0.801665
1950	16.4	2.084141	2400	1.939178
2000	46	-	6270	-

For example, the correlation between the growth rate of population density, Column (2), and the growth rate of population, Column (4), is 0.99. This correlation will be zero if population densities increase at a constant rather than at an increasing rate.<sup>17</sup> The

<sup>17</sup>Let the inhabited area at date  $t$  be  $T(t)$  and let population density be  $d(t) \equiv N(t)/T(t)$ . Assume that  $T(t) = T_0 \exp\{g_T t\}$ , and  $d(t) = d_0 \exp\{g_d(t)t\}$ , with  $\partial g_d(t)/\partial t > 0$ . Then,  $N(t) \equiv d(t)T(t) = N_0 \exp\{[g_T + g_d(t)]t\}$ , which yields an increasing population growth rate. If  $g_d(t) = g_d$ , population size, population density, and occupied area will grow at different but constant rates.

increasing population growth rate in Figure 1 is a consequence of this assumption. The use of Deevey's (1960) will also bias a time series analysis of population growth rates. As a contrast, the correlation between the growth rate in occupied area and the growth rate of population in Table A1 is negative,  $-0.57$ .<sup>18</sup>

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<sup>18</sup>There are many other potential biases in past population estimates but they do not need special emphasis here; see, e.g., Petersen (1975) and Whitmore et al. (1990).

## 7 Appendix B: Sub-Saharan Africa as a “control”

In this Appendix, I use sub-Saharan Africa as a control group and examine the isolation of tropical America. I treat North Africa (including ancient Egypt) as part of Eurasia because biogeographically it is closer to Eurasia than to sub-Saharan Africa; see Diamond (1997, p. 161). I focus on South and Central America because in terms of biogeography these regions and the West Indies conform a single Neotropical region. (North America is part of the Nearctic zone.) Communication between South and Central America was far more common than between Central and North America.

Table B1. Estimated pre-modern population in Africa and the Americas.

Region	Area	Biraben (1979)				McEvedy and Jones (1985)		
		400 BC	AD	1000	1500	AD	1000	1500
Africa								
North	2	10	14	9	9	8	11	8
Sub-Saharan	25	7	12	30	78	8	22	38
The Americas								
North	20	1	2	2	3	0.4	0.7	1.3
South and Central	20	7	10	16	39	4	8	13
Indian subcontinent	5	30	46	40	95	34	77	100
World population		153	252	253	461	170	265	425

Notes: Population in millions. Area (mill. km<sup>2</sup>) from McEvedy and Jones [40]. North Africa includes the Maghreb, Libya and Egypt. The area in North Africa does not include the Sahara. North America includes the US, Canada, and the Caribbean.

Table B1 presents estimates of regional populations from the sources in the text. For both sources, population in sub-Saharan Africa in 1500 was larger than in South and Central America. Since sub-Saharan Africa had contact with Eurasia during the post-agricultural period, one should expect higher technological sophistication than in tropical America.

Table B2. Cities in Africa and the Americas.

Year	North Africa	Sub-Saharan Africa			Total	North America	South and Central America
		Muslims	Middle Nile and Ethiopia	Rest (indigenous)			
A. Number of cities with populations over 20,000 inhabitants							
800	10	0	2	3	5	0	10
1000	13	0	1	4	5	0	9
1200	18	6	2	4	12	0	10
1300	18	8	2	5	15	0	11
1400	18	8	2	9	19	0	18
1500	19	13	3	8	24	1	16
B. Number of cities with populations over 40,000 inhabitants							
800	4	0	0	1	1	0	2
1500	7	4	0	2	6	0	6

Source: Chandler (1987, 39-57). The indigenous cities in sub-Saharan Africa cover mostly Ghana, Zimbabwe and the Bantus. The middle Nile corresponds to Dongola (modern Sudan) and Kaffa. North Africa includes cities in the Mediterranean (i.e., Arabian, Egypt, Spanish Africa, and Alos) and the Maghreb.

There is no standard measure of development or technological sophistication in pre-modern societies. However, using urbanization as a proxy for differences in economic prosperity during pre-industrial periods contradicts the hypothesis that population spurs innovation.<sup>19</sup> Table B2 lists the number of cities in Africa and the Americas. The inventory of cities with sizes over 20 and 40 thousand inhabitants is from Chandler (1987). Table B2 reports different time periods as the influence of Eurasia differed over time and divides sub-Saharan Africa in three sub-regions. The cities in regions with high Arab influence are coded as Muslims while the Middle Nile and Ethiopia are regions with influence from trade through the Indian ocean and North Africa. The rest of sub-Saharan Africa can be considered as indigenous formation.

Before the 1500, the Islamic world was the main Eurasian influence in sub-Saharan Africa. Around the time Islam spread into Africa, after the seventh and eighth centuries,

<sup>19</sup>Cities are a complex form of organization that often result from advances in agricultural productivity or incentives given by external or internal trade. Physical evidence on the existence of cities also tends to be well preserved; see Acemoglu et al. (2001, Section 2) for a related discussion in support of this view.

there was a total of 5 cities with more than 20 thousand inhabitants in Africa. In the Americas, in 800 AD, there were twice as many cities, 10. In 800 AD, the number of large cities, cities with more than 40 thousand inhabitants, was also twice as large in the Americas.

In 1500 there were 13 cities in regions with Arab influence. The number of non-Arab cities in sub-Saharan Africa increased from 5 to 11 between 800 and 1500, but the number of cities with more than 20 thousand inhabitants in South and Central America was still larger in 1500. The total number of cities with more than 40 thousand inhabitants was the same in sub-Saharan Africa as in the Americas. Of the 6 large cities in sub-Saharan Africa, 2 have an African origin. The 6 large cities in the Americas were indigenous.

Based on Tables B1 and B2, construct the urbanization rate in sub-Saharan Africa in the following way. Multiply the number of cities at each size by the cut-off size to obtain a total estimate of urban populations. For the Americas this number in 1500 is  $10 \times 20,000 + 6 \times 40,000 = 440,000$ . If the Muslim cities are counted as part of Africa, the same estimate for sub-Saharan Africa gives 600,000. If Muslim cities are excluded, the size of the urban population is 260,000. Using the population size from Biraben (1979) as the denominator gives urbanization rates for Africa that are 68 percent the rate in the Americas (or 30 percent if Arab cities are excluded). Using the population size from McEvedy and Jones (1985) gives estimates that are 47 and 20 percent the rate in the Americas. In all cases, the urbanization rate in sub-Saharan Africa is lower than in tropical America.

The relationship between urbanization and GDP per capita is positive and robust. In particular, Acemoglu et al. (2002, Table 2) find a coefficient of 0.038 (s.e. 0.006 and  $R^2 = 0.69$ ) between urbanization rates and log income per capita. This value holds for a cross section estimate in 1913 as well as for a cross-country regression in 1995. If the value of 0.038 is assumed stable, the range of urbanization rates suggest that income in tropical America was between 5 to 20 percent higher than in sub-Saharan Africa.

## 8 Appendix C [NOT FOR PUBLICATION]: Omitted comparative statics

Predictions (ii) to (iv) follow from the differentiation of (4). To see these results, notice that the population growth rate is

$$\frac{d \ln [N(t)]}{dt} = \frac{\theta}{\theta(1 - \alpha)t + N_0^{1-\alpha}}.$$

Simple substitutions show that the previous solution of (4) is such that

$$\frac{d \ln [N(t)]}{dt} = \theta \exp\{(\alpha - 1) \ln N(t)\}, \quad (\text{C1})$$

which is increasing in population size. Further, since  $N(t)$  is endogenous, it is possible to relate population growth to its “exogenous” determinants:

$$\frac{d^2 \ln [N(t)]}{dt dt} = \frac{\theta(\alpha - 1)}{[\theta(1 - \alpha)t + N_0^{1-\alpha}]^2} > 0, \quad (\text{C2})$$

$$\frac{d^2 \ln [N(t)]}{dt d\theta} = \frac{N_0^{1-\alpha}}{[\theta(1 - \alpha)t + N_0^{1-\alpha}]^2} > 0, \quad (\text{C3})$$

$$\frac{d^2 \ln [N(t)]}{dt dN_0} = \frac{\theta(\alpha - 1)N_0^{-\alpha}}{[\theta(1 - \alpha)t + N_0^{1-\alpha}]^2} > 0. \quad (\text{C4})$$

The rest of the paper proceeds as follows. Section 2 provides a brief background. Section 3 examines predictions that involve modern data, and Section 4 predictions that rely on pre-modern data. Section 5 concludes this paper.