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Equilibrium, convergence, and capital mobility in neoclassical models of growth

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Abstract

We study convergence in economies integrated by capital trade. Equilibrium generates transitional dynamics even in the absence of internal adjustment costs or borrowing constraints. Trade lowers the speed of convergence of capital-importing economies but increases the convergence of capital-exporting economies.

Keywords: Economic growth; International trade; Convergence

JEL classification: F21; F43; O40

1. Introduction

The question of why income fails to converge instantaneously in open economies, first posed by Barro et al. (1995), was resolved by these authors by assuming that capital cannot be fully financed by borrowing in world markets. In a comment on their paper, Duczynski (2000) questioned convincingly the plausibility of this explanation because while some countries can be constrained, a borrowing constraint should not bind countries that export capital. Therefore, capital-exporting economies should converge instantaneously.

Since instantaneous convergence has a counterpart in the theory of investment, a common answer to avoid the undesirable feature of instantaneous convergence is that of *internal* adjustment costs in physical or human capital.[1] As in investment theory, the possibility of *external* adjustment costs also has important consequences for convergence. For example, Bianconi and Turnovsky (1997) and Turnovsky and Bianconi (1992) have shown that equilibrium considerations generate transitional dynamics in neoclassical open economies because, in equilibrium, a country can only add to its capital stock if another country exports more capital. As economies have to divert resources into the world market to supply capital, countries will face an upward supply curve for capital or an *external* adjustment cost, as in the theory of investment studied by Foley and Sidrausky (1970) and Mussa (1978).

In this paper, we study the determinants of the rate convergence in the integrated economies of Bianconi and Turnovsky (1997) and Turnovsky and Bianconi (1992). We show that insights gained under exogenous prices or internal adjustment costs do not carry over to an equilibrium setting. For instance, Barro et al. (1995 114) concluded that "economists have long known that capital mobility tends to raise the rate at which poor and rich economies converge." As an integrated world is indeed a closed economy, we show that the rate of convergence cannot increase for all the economies that participate in capital trade. Since capital trade acts as a reduction in the curvature of the production function, trade slows down the convergence for capital-importing economies but accelerates the convergence for capital-exporting economies. Only capital-exporting economies converge faster because they find in the world market a way to partially overcome the diminishing returns to capital.

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[1] As Lucas (1967 323) notes, in the absence of adjustment costs, "following a once-and-for-all change in prices, the firm proceeds immediately to its new long-run position". Some recent examples of internal adjustment costs in growth theory include Barro et al. (1995), Duczynski (2000), Kremer and Thomson (1998), Stokey (1996), and Turnovsky (1997 chap. 7).

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We consider a growth problem with trade in Section 2. Section 3 analyzes a simplified version of the integrated economies of Bianconi and Turnovsky (1997) and compares the convergence rate between open and closed economies.[2] Section 4 generalizes the setting and the results to N-countries. Section 5 concludes the paper.

2. Capital accumulation and trade: one country

Consider a domestic economy with full access to world capital markets. The allocation of consumption and imported capital is carried out to solve

$$V(k(0)) = \max_{c,x} \int_0^\infty u(c(t))e^{-\theta t}dt, \tag{1}$$

subject to:

$$\dot{k}(t) = f(k(t)+x(t)) - c(t) - r(t)x(t), \tag{2}$$

for all $t \geq 0$. $\{c(t), k(t), x(t)\colon t \geq 0\} \in \mathbb{R}_+^2 \times \mathbb{R}$ represents consumption and the domestic and (net) imported capital while θ and $r(t) \in \mathbb{R}_{++}^2$ are the rate of time preference and the world rate of return on capital taken as given by the domestic economy.

Assumption 1. $u\colon\mathbb{R}_+ \to \mathbb{R}_+$ is bounded, strictly increasing in $c(t)$, strictly concave and twice continuously differentiable satisfying: $u(0)=0$, $u_c(0)=\infty$, $u_c(\infty)=0$.

Assumption 2. $f\colon\mathbb{R}_+ \to \mathbb{R}_+$ is a strictly concave and twice continuously differentiable production function satisfying $f(0)=0$, $f_k(0)=\infty$, $f_k(\infty)=0$.

Lemma 1. *Let Assumptions 1 and 2 hold. Then, an optimal trajectory solving Eq. (1) satisfies:*

$$f_k(k(t)+x(t)) = r(t), \tag{3}$$

and a system of first order differential equations: Eq. (2) and the Euler equation:

$$\frac{\dot{c}(t)}{c(t)} = \sigma(c(t))[r(t)-\theta], \text{ in which } \sigma(c) \equiv -u_{cc}(c)c/u_c(c), \tag{4}$$

with two boundary conditions: the initial condition $k(0)=k_0$ and the transversality condition $\lim_{t\to\infty} u_c(c(t))k(t)e^{-\theta t}=0$.

Eq. (3) defines the optimal allocation of imported capital. At each instant, the marginal product of capital should be equal to the return in world markets. If the return to capital is set exogenously at $\hat{r}$, for a steady state to exist, $\hat{r}$ must equal θ (Barro et al., 1995). If that is the case, it is possible to show that capital is driven instantaneously to its steady state. Since $x(t)$ is a control variable, the amount of imported capital can be adjusted immediately whenever the return on capital differs from the price on international markets.

[2] Bianconi and Turnovsky (1997) considered labor supply, differential taxation, and government spending. Turnovsky (1997 chap. 6–7) presents an even more general setting with and without internal adjustment costs. We abstain from those considerations in order to provide a simple expression for the convergence rate and a comparison between closed and open economies.

3. Equilibrium in world markets

Consider, as in Bianconi and Turnovsky (1997) and Turnovsky and Bianconi (1992), two economies that take the prices of capital on international markets as given. The solution of the accumulation path for the foreign economy (*) is equal to the trajectory described in Lemma 1, but now $r(t)$ has to be determined as an equilibrium clearing price in the world capital market.

Definition 1. Given $\{k(0), k^*(0)\} \in \mathbb{R}_{++}^2$, a competitive equilibrium is a collection of continuous functions $\{c(t), k(t), x(t), c^*(t), k^*(t), x^*(t), r(t)\colon t \geq 0\}$ that maximize Eq. (1) subject to Eq. (2) and its counterpart in the foreign economy, and feasibility in factor trade given by $x(t)+x^*(t)=0$, for all $t \geq 0$.

As in the integrated models studied in Turnovsky (1997 chap. 6), in a competitive equilibrium it must be true that:

$$f_k(k(t)+x(t)) = r(t) = f_k^*(k^*(t)-x(t)), \tag{5}$$

for all $t \geq 0$. The previous condition implicitly defines $x(t) \equiv x(k(t), k^*(t))$ and $r(t) \equiv r(k(t), k^*(t))$. Both functions are characterized next.

Lemma 2. *Let Assumptions 1–2 and 1*–2* hold. Then, capital traded, $x(k, k^*)$, is a decreasing function of k and an increasing function of k^* and the rate of return of capital, $r(k, k^*)$, is a decreasing function k and k^*. Moreover, $r(k, k^*)$ depends only on the aggregate amount of capital.*

Proof. From Eq. (5): $f_k(k+x(k, k^*)) = f_k^*(k^* - x(k, k^*))$. Thus, $f_{kk}\{1+\partial x(k, k^*)/\partial k\} + f_{kk}^* \partial x(k, k^*)/\partial k = 0$, or

$$\frac{\partial x(k,k^*)}{\partial k} = \frac{-f_{kk}}{f_{kk}+f_{kk}^*} < 0.$$

A similar reasoning produces $\partial x(k, k^*)/\partial k^* = f_{kk}^*/(f_{kk}+f_{kk}^*) > 0$. Since $f_k(k+x(k, k^*)) = r(k, k^*)$, $f_{kk}\{1+\partial x(k, k^*)/\partial k\} = \partial r(k, k^*)/\partial k$ and $f_{kk}^*\{1-\partial x(k, k^*)/\partial k^*\} = \partial r(k, k^*)/\partial k^*$ so after substitutions:

$$\frac{\partial r(k,k^*)}{\partial k} = \frac{\partial r(k,k^*)}{\partial k^*} = \frac{f_{kk}^* f_{kk}}{f_{kk}+f_{kk}^*} < 0, \text{ with } r(k,k^*) = r(k+k^*). \tag{6}$$

□

Lemma 2 indicates that higher domestic capital and lower foreign capital reduce the amount of capital traded. Also, capital accumulation reduces the returns on capital independently of which economy accumulates it because the capital return is determined by aggregate capital (see Ruffin, 1979, for a related analysis in a Solow framework).

Since the accumulation of capital in the foreign economy affects world prices, both countries must be considered simultaneously as in the interdependent economies of Bianconi and Turnovsky (1997), Turnovsky and Bianconi (1992), and Turnovsky (1997 chap. 6). However, since only aggregate capital matters for capital returns, the dynamical system only determines $C=c+c^*$ and $K=k+k^*$. The aggregate dynamical system is: $\dot{K}=f(k+x)+f^*(k^*-x)-C$, and $\dot{C}=(\sigma^*(c^*)\ c^*+\sigma(c)c)[r(K)-\theta]$.

Theorem 1. *Let Assumptions 1–2, and 1*–2* hold. Then, the steady state equilibrium under capital trade is locally asymptotically stable.*

Proof. Steady state values are denoted by (ˆ). Linearization of the aggregate dynamical system generates the following eigenvalues:

$$\frac{\theta}{2}\pm\frac{1}{2}\sqrt{\theta^2-4(\sigma(\hat{c})\,\hat{c}+\sigma^*(\hat{c}^*)\,\hat{c}^*)\left(\frac{f_{kk}f_{kk}^*}{f_{kk}+f_{kk}^*}\right)}. \tag{7}$$

These eigenvalues are identical to the solution presented in Turnovsky (1997 eq. 6.26) for integrated neoclassical models once all taxes are eliminated. We let μ represent the negative or stable eigenvalue. □

The world equilibrium mimics the evolution of a closed economy in which the speed of convergence is not instantaneous because prices change as accumulation occurs (see also Bianconi and Turnovsky, 1997). The relationship with a closed economy is perhaps clearer if we note that an open economy with two identical countries collapses into a single neoclassical closed economy. That is, if $f_{kk}=f_{kk}^*$, and $\sigma(\hat{c})\hat{c}=\sigma^*(\hat{c}^*)\hat{c}^*$, convergence is as in closed economies. Moreover, if $f^*(k^*)=\theta k^*$ or $f(k)=\theta k$, convergence in open economies is instantaneous as in Barro et al. (1995).

While the possibility of transitional dynamics in integrated economies is well-known, see for example Turnovsky (1997 chap. 6) and Turnovsky and Bianconi (1992), a comparison between the speed of convergence in closed and open economies has not been previously attempted. As we next show, this comparison indicates that capital trade cannot increase convergence of the world economy beyond the convergence in autarky.

Corollary 1. *Let Assumptions 1–2 and 1*–2* hold, and assume in addition that trade is "small" (so that convergence rates can be compared). Then, the speed of convergence in open economies is bounded by the speed of convergence in closed economies.*

Proof. Recall that:

$$\mu=\frac{\theta}{2}-\frac{1}{2}\sqrt{\theta^2-4\sigma(\hat{c}^*)\,\hat{c}^*f_{kk}^*\left(1+\frac{\sigma(\hat{c})\,\hat{c}}{\sigma^*(\hat{c}^*)\,\hat{c}^*}\right)\left(\frac{f_{kk}}{f_{kk}+f_{kk}^*}\right)}, \tag{8}$$

if $(1+\sigma(\hat{c})\ \hat{c}/\sigma^*(\hat{c}^*)\hat{c}^*)(f_{kk}/f_{kk}+f_{kk}^*)>1$, the world economy converges faster than the capital-exporting economy. The same condition implies $(1+\sigma^*(\hat{c}^*)\hat{c}^*/\sigma(\hat{c})\hat{c})(f_{kk}^*/f_{kk}+f_{kk}^*)>1$ for the capital-importing economy. Since both conditions cannot be satisfied simultaneously, the speed of convergence in autarky bounds the convergence of the integrated economy. □

Corollary 1 has important implications because it suggests that trade reduces the speed of convergence for at least some economies. In autarky, convergence depends on how fast decreasing returns set in. Since productivity in capital-importing economies is high in autarky (so the direction of capital trade is as considered in the text), it is natural to assume that convergence is faster in capital-importing economies in autarky.[3] For capital-importing economies, trade reduces the return to capital and the response of the return to further changes in accumulation; hence, it reduces convergence.

In capital-exporting economies, the return to capital is relatively low in autarky, and, hence, trade with capital-scarce countries increases the return to accumulation without having decreasing returns set in (i.e., the change in prices is much smaller than predicted by changes in autarky). For example, accumulation reduces the return to capital by f_{kk}^* in autarky but only by a fraction $f_{kk}^*f_{kk}/(f_{kk}+f_{kk}^*)$ when trade is available. Since the return to capital increases compared to autarky, capital-exporting economies increase accumulation and converge at faster rates in an integrated world.

4. An *N*-country integrated world

We next extend the Bianconi and Turnovsky (1997) and Turnovsky and Bianconi (1992) setting to an *N*-country world under equal time preferences. Since there are no distortions, we consider a central planner allocation. The Pareto allocation can be defined as the solution of: $W(K(0))=\max_{c^i}\sum_{i=1}^{N}\int_0^\infty u^i(c^i(t))e^{-\theta t}dt$, subject to the dynamics of capital accumulation with $\{c^i(t), k^i(t): t\geq 0\}\in\mathbb{R}_+^2$ as consumption and capital in economy $i=1, \ldots, N$.

The allocation of capital and its accumulation are independent problems. The allocation of capital is given by: $F(K(t))=\max\ \{\sum_{i=1}^{N} f^i(k^i(t)):\sum_{i=1}^{N} k^i(t)=K(t)\}$. Let $C(t)$ be world consumption and define $U(C(t))=\max\ \{\sum_{i=1}^{N} u^i(c^i(t)):\sum_{i=1}^{N} c^i(t)=C(t)\}$. Thus, the planner problem is given by:

$$W(K(0))=\max_{C}\left\{\int_0^\infty U(C(t))e^{-\theta t}dt : \dot{K}(t)=F(K(t))-C(t)\right\},$$

with a solution that is identical to the solution of a neoclassical model. The speed of convergence is a generalization of previous results:

Theorem 2. *Let Assumptions 1–2 hold for all i=1, …, N countries. Then, the speed of convergence of the integrated economy is:*

$$\mu=\frac{\theta}{2}-\frac{1}{2}\sqrt{\theta^2-4\left(\frac{1}{N}\sum_{i=1}^{N}\sigma^i(\hat{c}^i)\,\hat{c}^i\right)\left(\frac{1}{N}\sum_{i=1}^{N}\frac{1}{f_{kk}^i}\right)^{-1}}. \tag{9}$$

As before, convergence is instantaneous if and only if $f_{kk}^i=0$ for at least one country because the measure of the external

[3] Assume that $\sigma(\hat{c})\hat{c}=\sigma(\hat{c}^*)\hat{c}^*$. Then, the world economy converges faster than the capital-exporting economy when $2f_{kk}/(f_{kk}+f_{kk}^*)>1$, or when $f_{kk}<f_{kk}^*$, which implies that decreasing returns are less important in capital-importing or poor economies.

adjustment cost is inversely related to the degree of decreasing returns to capital (i.e., a linear technology in any economy fixes the price of capital and eliminates the adjustment costs). Moreover, if all economies are identical, convergence is as in a closed economy, and in a two-country world, we obtain Eq. (8).

Finally, since world convergence depends on the average values of the elasticity of intertemporal substitution and the average degree of decreasing returns, world convergence is bounded by the convergence in autarky; while an integrated world will converge faster than some countries, others will still enjoy faster convergence in autarky. In other words, trade cannot increase convergence beyond the limits set in autarky.

5. Conclusions

This paper examined a neoclassical growth model of integrated economies following Bianconi and Turnovsky (1997) and Turnovsky and Bianconi (1992). As these authors have shown, in an integrated world there is gradual convergence toward the steady state by the dependence of capital returns on aggregate capital (as in an external adjustment cost).

More importantly, we compared convergence in open and closed economies. Since an integrated world behaves as a closed economy, we showed that convergence in interdependent economies is bounded by the convergence in autarky as the degree in which decreasing returns affect convergence in an integrated world is a weighted average of the response in autarky. Under capital trade, the degree of decreasing returns is 'pooled' so economies in which decreasing returns exert a strong influence (i.e., rich economies) find in world markets a way to partially overcome those diminishing returns. This is done at the expense of economies in which the decreasing returns are less important.

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