Choice Under Uncertainty (Chapter 12)

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Express yourself

When should we have the second midterm?

CLICKER VOTE:

- A Tuesday, February 16
- B Thursday, February 18 (current date)

Typical 100B problem

Breaking it down

- You're asked to analyze economic behavior/outcomes/policy
  - Individual choice
  - Market behavior and welfare
  - Effectiveness/consequences of policy
- You need to break it down into smaller pieces
- Apply specific skills/tools to deal with each part
- Put parts together to solve overall problem
- zoom back out, refocus on big picture
  - Not just solving math problem
  - What insight do we gain from this?

Typical 100B problem

Example: uncertainty

- Given setup
- Separately derive budget constraint, indifference curves (find MRS)
- Solve U max problem, optimal bundle
- Learn something about demand for insurance

Typical 100B problem

Example: market demand, equilibrium

- Given individual demands, info about supply
- Derive total demand, supply
- Solve for equilibrium p, q
- Learn something about behavior in the market

Typical 100B problem

Example: Changes to equilibrium (comparative statics)

- Given info about demand, supply
- Find equilibrium p, q
- Introduce demand/supply shift, tax, price floor, ceiling, quota, etc., calculate new p,q
- Observe something about effect on behavior, welfare

Typical 100B problem

Example: Comparison of market structures

- Given market demand, costs/supply
- Find eq. p, q for various market structures
- Compare behavior and welfare

Types of exam questions

One categorization: difficulty

- Easy, just about everyone should get
- Moderate, many, but not all should get
- Challenging, only a handful of the very best students will get

Types of exam questions

Another way to classify:

- Small, deals only with subpart of overall problem
- Large, deals with more parts or entire problem
- Pushes you to focus out on big picture, draw conclusions, push understanding further, deal with new complications– not necessarily more complicated math

What will the quizzes look like?

- Two multiple-choice questions
- Both type 1
- Diagnostic, small grade impact
- Checks for minimum necessary comprehension

Don't think: I did well on the quiz, so I'm prepared for the exam

Do think: I did well on the quiz, so I can focus on the larger parts of the problem, big picture for the exam

Do think: I had trouble on the quiz– I really need to do something about this before the exam

States of Nature and Contingent Plans

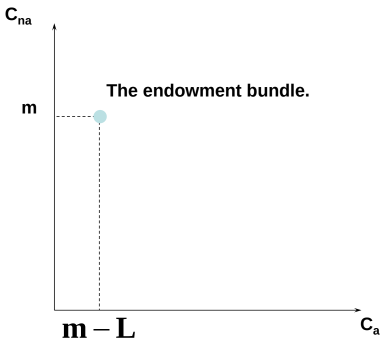
- States of Nature:
  - "accident" (a) vs. "no accident" (na)
  - Probability of: accident $= \pi_a$, no accident $= \pi_{na}$; $\pi_a + \pi_{na} = 1$
  - Accident causes loss of \$L
- "Bundle" = state-contingent consumption plan: Specifies consumption level for each scenario (state)
- Option to buy some insurance: contracts are be state-contingent (e.g. insurer pays only if you have an accident)
- How much should you buy?

Deriving the Budget Constraint

Q: Where to start?

A: The bundle with which you are endowed.

- Without insurance, consumption is:
  - $c_{na} = m$ if no accident
  - $c_a = m - L$ if accident
- The endowment bundle displayed graphically:

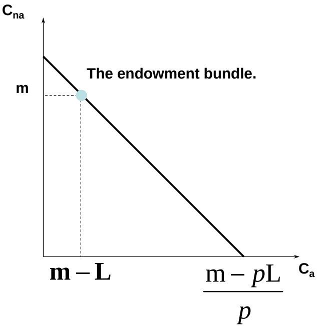


Deriving Budget Constraint

Insurance contract: Buy \$K of fire insurance at price p, claim \$K from company if accident

- If no accident: $c_{na} = m - pK$
- If accident: $c_a = m - pK - L + K = m - L + (1-p)K$
- Given K, it must be true that... (solve for K, substitute):

$$c_{na} = \frac{m - pL}{1-p} - \frac{p}{1-p} c_a$$



Preferences

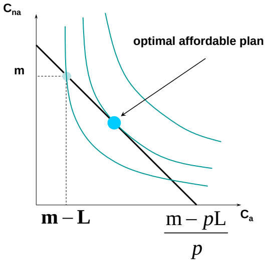
Q: Why do people buy insurance when they face risk?

To answer this, we have to consider preferences

- $U(c_a, c_{na})$ captures attitude towards uncertainty/risk
- Person might be risk averse or risk neutral (or risk loving)
- Consider our three favorite examples:
  - A Perfect Substitutes
  - B Cobb-Douglas
  - C Perfect Complements
  - D Not sure
  - E Don't have clicker yet
- CLICKER VOTE: which of these reflect some degree of risk aversion?

Optimal Choice (Graph)

Insurance is a way of mitigating risk. If you are risk averse, you are happiest buying some positive amount of insurance.



- Comparative statics:
  - risk aversion $\uparrow \Longrightarrow K$?
  - $p \uparrow \Longrightarrow K \downarrow$
  - $L \uparrow \Longrightarrow K$?
- What about an algebraic solution? First, a detour. . .

Example: a lottery

- Win \$90 or \$0 equally likely
- $U(90) = 12$ and $U(0) = 2$
- *Expected Utility* is

$$EU = .5 * U(90) + .5 * U(0) = .5 * 12 + .5 * 2 = 7.$$

- *Expected Money* is

$$EM = .5 * 90 + .5 * 0 = \$45.$$

Risk Attitudes

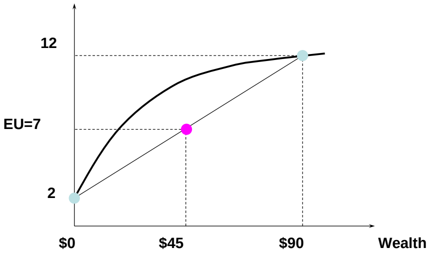
How do we characterize attitude towards risk?

Recall: $EU = 7$ and $EM = \$45$

- $U(45) > 7 \implies$ risk-averse
- $U(45) < 7 \implies$ risk-loving
- $U(45) = 7 \implies$ risk-neutral

Risk Attitudes

We typically assume diminishing marginal utility (DMU) of wealth.

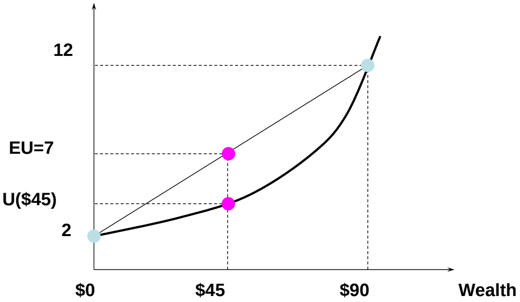


So $EU < U(EM)\dots$

this implies risk aversion!

Risk Attitudes

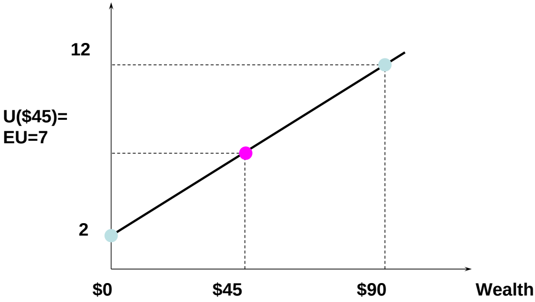
Example: Risk-loving preferences



$EU > U(EM)$

Risk Attitudes

Example: Risk-neutral preferences



$EU = U(EM)$

Optimal Choice (Algebra)

Calculating the MRS

- $EU = \pi_f U(c_a) + \pi_{na} U(c_{na})$
- Indifference curve $\implies$ constant EU
- Differentiate:
  - $dEU = 0 = \pi_a MU(c_a) dc_a + \pi_{na} MU(c_{na}) dc_{na}$
  - $MRS = \frac{dc_{na}}{dc_a} = -\frac{\pi_a MU(c_a)}{\pi_{na} MU(c_{na})}$
- Solution satisfies

$$\frac{p}{1-p} = \frac{\pi_a MU(c_a)}{\pi_{na} MU(c_{na})}.$$

Competitive Insurance

How optimal insurance purchase (K) and consumption levels c_a, c_{na} depend upon probabilities (given) and price.

Q: What determines the price of insurance?

A: Market conditions

Consider a competitive insurance market:

- Free entry $\implies$ zero expected economic profit
- So $pK - \pi_a K - (1 - \pi_a)0 = (p - \pi_a)K = 0$.
- $\implies p = \pi_a$
- Insurance is fair

Competitive Insurance

- With fair insurance, rational choice satisfies

$$\frac{\pi_a}{\pi_{na}} = \frac{\pi_a}{1-\pi_a} = \frac{p}{1-p} = \frac{\pi_a MU(c_a)}{\pi_{na} MU(c_{na})}.$$

- In other words, $MU(c_a) = MU(c_{na})$
- Risk-aversion $\Longrightarrow c_a = c_{na}$
- Full insurance!

Not-Fair Insurance

Suppose the insurance market is not competitive

- Insurers can expect positive profits
- $pK - \pi_a K - (1 - \pi_a)0 = (p - \pi_a)K > 0$
- Then $p > \pi_a$ and $\frac{p}{1-p} > \frac{\pi_a}{1-\pi_a}$
- $\implies MU(c_a) > MU(c_{na})$
- Risk-averse $\implies c_f < c_{na}$: less than full (not-fair) insurance

Proposed Gamble

I flip a fair coin. Heads: I pay you \$120; tails: you pay me \$100. *Any takers?*

CLICKER VOTE:

A Accept

B No thank you!

Proposed Gamble: II

What if I offered this same gamble at the beginning of every lecture (and you had to tell me today what you would choose each time)?

CLICKER VOTE:

- A Accept every time
- B Reject every time
- C Some combination

Analysis

Why is the same gamble more attractive when it is repeated?

- Each gamble has positive expected value
- Each coin toss is independent
- Law of Large Numbers: expected money from compound gamble $=$ N times the EM $=$ a big positive number
- Portfolio of gambles is diverse, so very little chance of net loss

Diversification

Example:

- Two firms, A and B. Shares cost \$10
- With prob $= .5$, $\Pi_A = 100$ and $\Pi_B = 20$
- With prob $= .5$, $\Pi_A = 20$ and $\Pi_B = 100$
- You have \$100 to invest. How?

Diversification

Example:

- Buy only firm A's stock?
- $\$100/10 = 10$ shares
- Earn \$1000 w/ prob .5 and \$200 w/ prob .5
- Expected earning: $\$500 + \$100 = \$600$
- Same for buying only B

Diversification

Example:

- Buy 5 shares of each firm?
- Earn $600 for sure
- Diversification has maintained expected earnings while lowering risk
- Typically there's a tradeoff between earnings and risk

Recap

What are rational responses to risk?

- Buying insurance
- A diverse portfolio of contingent consumption goods (assets)

How do insurance companies operate?

- You buy insurance in response to risk
- Insurance company gets your premium, but now faces risk of having to pay claim
- To the extent that claims are independent, this is ok for them because they have a diverse portfolio of risks
- Same w/ home lenders: they get your mortgage payments, but lose if you default
- To diversify risk, lenders wad contracts/mortgages together into bundles, then sell them (in pieces) as relatively safe (diversified) securities
- Thus, our risk and insurance courses through the veins of the financial system

How do insurance companies operate?

So what can and does go wrong?

- Diversification works if risks are independent, but not if correlated
- My proposed gamble: imagine if I decided outcome w/ one coin-toss at the end of the quarter. Takers?
- Risk of house burning down: Seattle vs. SoCal
- Insurance companies are exposed to systemic risks
- Wildfires, earthquakes, hurricanes can wipe out entire cities/regions at once
- Natural disasters are disasters for insurers
- Insurers know this: there is an enormous re-insurance industry