# Public Goods 2 (Chapter 36)

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# What is the optimal level of public goods provision?

- Recall: a *non-rival, non-excludable* good is a **public good**
- Last time: individuals have incentive to free-ride so market does not provide the efficient level of public goods
- What is the optimal level of provision?
- How can we get people to reveal demand, so we (govt.) can provide efficiently? (**mechanism design**)

# A public good with variable quantity

- Suppose there are two goods, $x$ (private, i.e. money) and $G$ (public)
- Cost of producing the public good: $c(G)$
- Two individuals $A$ and $B$, with private consumption $x_A$ and $x_B$
- Budget: $x_A + x_B + c(G) = \omega_A + \omega_B$

# A public good with variable quantity

- For each person, what is marginal benefit to switching some consumption from private to public good?
- $MRS_A$, $MRS_B$
- Each person separate equates marginal benefit with marginal cost.
- But because $G$ is non-rival, 1 extra unit is fully consumed by both $A$ and $B$
- So condition for socially-optimal (Pareto efficient) provision is

$$MRS_A + MRS_B = MC(G)$$

- More generally:

$$\sum_i MRS_i = MC(G)$$

# How can we determine efficient provision level?

- We use a *revelation mechanism*: a scheme that makes it rational for individuals to truthfully reveal their private valuations of a public good
- Example: Vickrey-Groves-Clarke (VCG) mechanism (or Groves-Clarke tax, Clarke tax)
- How does it work?
- Intuition:
  - Your preferences impose externality or others...
  - Assign tax based upon your *stated* valuation and others'
  - Proper tax internalizes externality

# VCG example

- 3 people
- Build a bridge? Yes/no decision
- Private valuations for bridge: \$40 for $A$; \$50 for $B$; \$110 for $C$
- Costs \$180 to build bridge, split equally (\$60 each)
- Given \$60 cost, each person's *net* valuation is $v_A = -20$, $v_B = -10$ and $v_C = 50$.
- Is bridge efficient to build?
- Yes: $40 + 50 + 110 = 200 > 180$ or $v_A + v_B + v_C = -20 - 10 + 50 = 20 > 0$

# Key concept: pivotal

- A person is pivotal if the inclusion of his/her voice (i.e. statement of preferences) would change the group's decision
- Example
  - 5 people make a yes/no decision based on majority voting
  - If vote is 3-2 in favor, each 'yes' voter is pivotal– switching vote changes outcome, but each 'no' voter is not
  - If vote is 4-1 in favor, no one is pivotal– switching from 'yes' to 'no' or 'no' to 'yes' does not affect outcome
- Bridge example: For each person we ask: would bridge be worth building if we only listened to the *other* people?

# Key concept: pivotal

Is each person pivotal?

- *A is not* pivotal: other two have total net value $-10 + 50 = 40 < 0$. Adding $A$'s preferences makes the bridge seem less worthwhile, but not enough so to change the group decision
- *B is not* pivotal: other two have total net value $-20 + 50 = 30 > 0$. Adding $B$'s preferences makes the bridge seem a little less worthwhile, but values $40 + 100 = 150 < 180$, bridge not efficient without $B$
- *C is* pivotal: other two have total net value $-10 - 20 = -30 < 0$. Adding $C$'s voice is what makes the bridge seem worth building.

# Key concept: pivotal

- A person who is pivotal is one who influences the collective outcome (yes/no)
- Since everyone cares about the outcome, a pivotal person imposes an external cost or benefit on others
- Idea behind VCG: use tax to make people pay (or get rewarded) for this externality

# VCG mechanism: how it works

1. Everyone announces own valuation (true value is private so people *could* lie)
2. If sum of announced values exceeds cost (net value $> 0$, project (bridge) gets completed
3. For each person, calculate whether or not pivotal (based on announced values)
4. Collect tax or distribute subsidy
   - If not pivotal: no tax/subsidy
   - If pivotal: tax/subsidy equals net externality imposed on others

# VCG mechanism: applying it

- Incentives created by VCG make lying unprofitable (so assume everyone reveals true valuation)
- $40 + 50 + 110 = 200 > 180$, so bridge is built
- Calculating taxes
  - $A$ is not pivotal so no tax.
  - $B$ is not pivotal so no tax.
  - $C$ is pivotal. Other two don't like bridge, but she insists. She imposes $40 - 60 = -20$ on $A$ and $50 - 60 = -10$ on $B$. Combined net ext. is $-30$, so $C$ pays *tax* of 30
- We get efficient decision on bridge thanks to truthful revelation, but $A$ and $B$ are not happy about this and would not willingly participate in the mechanism.

# VCG mechanism: does it really work?

What are $A$'s incentives, given $B$ and $C$ tell the truth?

- If truthful ($v = 40$), bridge is built, $A$ pays no tax, and experiences $-20$ net value
- If $A$ states $v > 40$, bridge is still built, still not pivotal, so payoff unchanged
- If $A$ states $20 < v < 40$, bridge is still built, still not pivotal, so payoff unchanged
- If $A$ states $v < 20$, then bridge isn't built, but $A$ is pivotal in making that happen. $A$ gets 0 net value because there is no bridge (an improvement over $-20$), but now she imposes a net externality on the others (helps $B$ by 10, but deprives $C$ of 50 net value), so she would have to pay a tax of \$40, yielding a payoff of $0 - 40 = -40$, which is worse what she gets from telling the truth.

So $A$ cannot increase her payoff by lying.

# VCG mechanism: does it really work?

Let's do the same for $B$. Given $A$ and $C$ tell the truth:

- If truthful ($v = 50$), bridge is built, $B$ pays no tax, and experiences $-10$ net value
- If $b$ states $v > 50$, bridge is still built, still not pivotal, so payoff unchanged
- If $B$ states $30 < v < 50$, bridge is still built, still not pivotal, so payoff unchanged
- If $B$ states $v < 30$, then bridge isn't built, but $B$ is pivotal in making that happen. $B$ gets 0 net value because there is no bridge (an improvement over $-10$), but now she imposes a net externality on the others (helps $A$ by 20, but deprives $C$ of 50 net value), so she would have to pay a tax of \$30, yielding a payoff of $0 - 30 = -30$, which is worse what she gets from telling the truth.

So $B$ cannot increase her payoff by lying.

# VCG mechanism: does it really work?

Finally, we repeat for $C$. Given $B$ and $C$ tell the truth:

- If truthful ($v = 110$), bridge is built, $C$ pays tax of \$30, which is subtracted from 50 net value from bridge, so payoff is 20.
- If $C$ states $v > 110$, bridge is still built, still pivotal, same tax, so payoff unchanged
- If $A$ states $90 < v < 110$, bridge is still built, still pivotal, same tax, so payoff unchanged
- If $A$ states $v < 90$, then bridge isn't built, so $A$ doesn't get net benefit of 50, but saves \$30 tax because no longer pivotal. Ends up with zero, which is worse than 20 from telling the truth.

So $C$ cannot increase her payoff by lying.