Market Demand cont.
Chapter 15

Outline

- Deriving market demand from individual demands
- How responsive is q_d to a change in price? (elasticity)
- What is the relationship between revenue and demand elasticity?

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When deriving market demand from individual demand curves, we add them up

A) Vertically

B) Diagonally

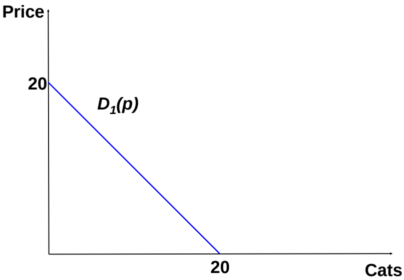
C) Horizontally

D) It depends

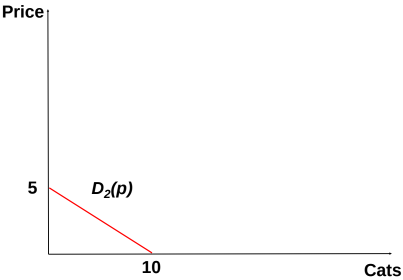
From Individual Demands to Market Demand

Let the (inverse) demand of agent 1 and agent 2 be

$$P(q_1) = 20 - q_1$$
$$P(q_2) = 5 - \frac{q_2}{2}$$



Demand of Agent 1

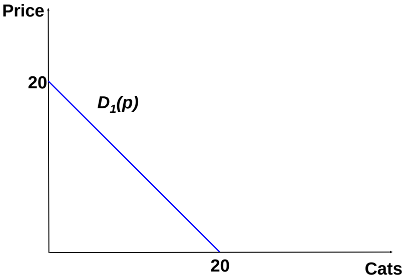


Demand of Agent 2

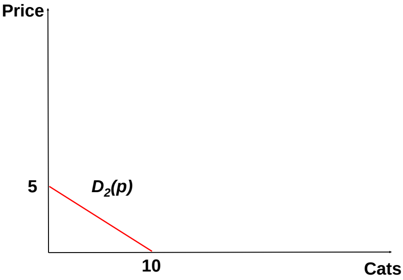
From Individual Demands to Market Demand

To find market (total) demand, we must fix the price and add up the quantities. Easier to do with *demand*, as opposed to *inverse demand*.

$$
\begin{aligned}
D_1(p) &= \max\{20 - p, 0\} \\
D_2(p) &= \max\{10 - 2p, 0\}
\end{aligned}
$$



Demand of Agent 1

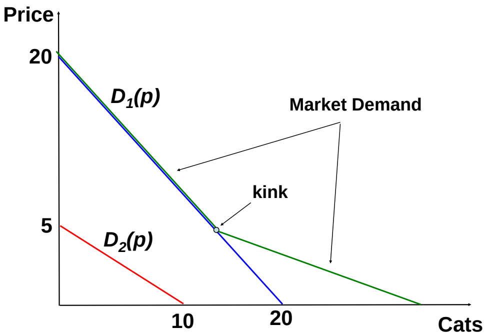


Demand of Agent 2

From Individual Demands to Market Demand

The market demand is the horizonal sum (for a given p) of all individual demand:

$$
\begin{aligned}
D(p) &= \sum_i D_i(p) \\
&= D_1(p) + D_2(p)
\end{aligned}
$$



Price Elasticity of Demand

How sensitive is $D(p)$ to price?

- How much will quantity demanded change in response to a given price change?
- Look at slope of demand curve
- Serious drawback to just using slope
- Heavy dependence on arbitrary units
- Solution?
- Think in terms of *percent* change

Price Elasticity of Demand

How sensitive is $D(p)$ to price?

- Define price elasticity of demand, ϵ, as

$$\epsilon = \frac{\frac{\Delta q}{q}}{\frac{\Delta p}{p}} = \frac{p}{q}\frac{\Delta q}{\Delta p},$$

  or p/q times the slope of the demand curve.

- At a particular point on the demand curve:

$$\epsilon = \frac{p}{q}\frac{\partial q}{\partial p}$$

Price elasticity: Example

Workout 15.4

- Demand for kitty litter: $\ln D(p, m) = 1000 - p + \ln m$, where p is price and m is income
- Rewrite demand: $D(p, m) = e^{1000}e^{-p}e^{\ln m} = me^{1000}e^{-p}$
- What is the price elasticity of demand for kitty litter when
  1. $p = 2$ and $m = 500$?
     Differentiate to find: $\frac{\partial q}{\partial p} = -me^{1000}e^{-p} = -D(p, m)$ So

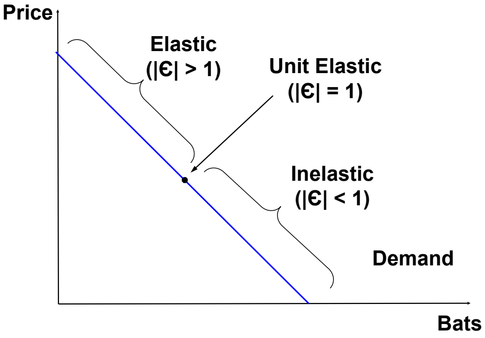
     $$\epsilon = \frac{p}{D(p, m)}(-D(p, m)) = -p$$

     So $\epsilon = -2$
  2. $p = 3$ and $m = 500$?
     $\epsilon = -3$
  3. $p = 4$ and $m = 1500$?
     $\epsilon = -4$

Price Elasticity of Demand

Demand curve slopes downward ($\frac{\mathrm{d}q}{\mathrm{d}p} < 0$) so $\epsilon \leq 0$.

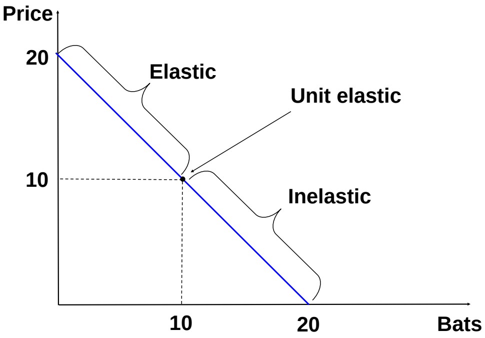
- $|\epsilon| > 1 \implies$ demand is elastic
- $|\epsilon| < 1 \implies$ demand is inelastic
- $|\epsilon| = 1 \implies$ demand is unit elastic



Price Elasticity of Demand

With linear demand: $q = 20 - p$ (inverse: $p(q) = 20 - q$)

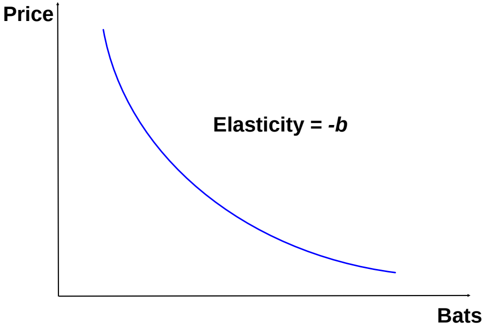
- Above midpoint $\Longrightarrow$ demand is elastic
- Below midpoint $\Longrightarrow$ demand is inelastic
- At midpoint $\Longrightarrow$ demand is unit elastic



Price Elasticity of Demand

Iso-elastic demand: $q = ap^{-b}$

- $\frac{\mathrm{d}q}{\mathrm{d}p} = a \cdot (-b)p^{-b-1}$
- $\epsilon = \frac{p}{q}\frac{\mathrm{d}q}{\mathrm{d}p} = \frac{p}{ap^{-b}} a \cdot (-b)p^{-b-1} = -b$
- $|\epsilon| = b$



Other Elasticities

Suppose $F = F(x, y)$

- How sensitive is F to a change in x?
- Elasticity of F w.r.t. x (or x elasticity of F) is given by

$$\frac{x}{F(x,y)}\frac{\partial F}{\partial x}$$

- Example: income elasticity of demand
  - How sensitive is D to a change in income?
  - $\epsilon_m = \frac{m}{D(p,m)}\frac{\partial D}{\partial m}$
  - $\epsilon_m \geq 0 \implies$ normal good
  - $\epsilon_m < 0 \implies$ inferior good

Other Elasticities

Example: Workout 15.4 continued

- Recall that $\ln D(p, m) = 1000 - p + \ln m$, so $D(p, m) = me^{1000}e^{-p}$
- What is the income elasticity of demand?
- Differentiate w.r.t m:

$$\frac{\partial D}{\partial m} = e^{1000}e^{-p} = \frac{D(p, m)}{m}$$

- $\epsilon_m = \frac{m}{D(p,m)}\frac{D(p,m)}{m} = 1$
- Interpretation: $m \uparrow$ by \$1 $\Longrightarrow D \uparrow$ \$1? **NO!** $m \uparrow$ by 1% $\Longrightarrow D \uparrow 1\%$

Elasticity & Revenue

What happens to revenue when you change p?

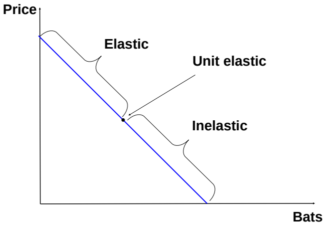
- Revenue: $R = pq$
- Change in revenue w.r.t. p:

$$\frac{\partial R}{\partial p} = p \cdot \frac{\partial q}{\partial p} + q \cdot 1 = q\epsilon + q = q(1+\epsilon)$$

- How does a price increase change revenue?
  - $R \uparrow$ if demand is *inelastic*
  - $R \downarrow$ if demand is *elastic*
  - R is unchanged if demand is *unit elastic*

Elasticity & Revenue

Q: What price maximizes revenue?



$$\frac{\partial R}{\partial p} = q(1+\epsilon^*) = 0 \iff \epsilon^* = -1$$

A: The price at which demand is unit elastic

Example: $D(p) = 40 - 2p$. Unit elasticity occurs at

$$\epsilon^* = \frac{p^*}{40-2p^*} * (-2) = -1 \implies p^* = 10$$

Marginal Revenue

What happens to revenue when quantity q changes?

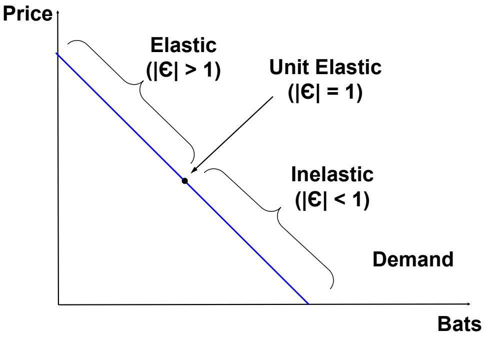
- *Marginal Revenue*:

$$\begin{aligned} MR &= \frac{\partial R}{\partial q} = p \cdot 1 + q\frac{\partial p}{\partial q} \\ &= p + p\frac{q}{p}\frac{\partial p}{\partial q} = p(1 + \frac{1}{\epsilon}) \end{aligned}$$

- Example: if $\epsilon = -1/2$ then $MR = -p < 0$, so reducing the quantity will increase revenue.

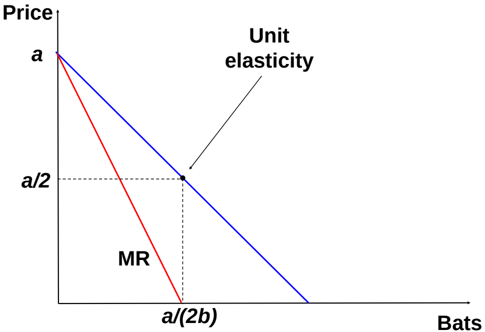
Marginal Revenue

Linear demand: $p(q) = a - bq$ (inverse demand)



Marginal Revenue

Linear demand: $p(q) = a - bq$ (inverse demand)



$MR = a - 2bq$, so revenue maximizing $(p, q) = (a/2, a/2b)$.