General Equilibrium (without Production)
or
Exchange
(Chapter 31)

February 16, 2010

General Equilibrium

- Events in one market have effects on other markets (spillovers)
- Demand for x depends upon prices of complements, substitutes; income
- Supply of x depends upon factor prices
- Previously, we've taken these as given– doing *partial equilibrium* analysis
- But its important to understand interdependence of markets– *general equilibrium* analysis

Partial equilibrium analysis says that competitive markets yield efficient outcomes—is this still true in general equilibrium?

General Equilibrium

Our approach:

- Simple environment—the *entire* economy
  - 2 kinds of goods
  - 2 people
- Focus on exchange
  - Abstract away from production of new goods
  - Give people endowments
  - Specify preferences
  - Allow them to trade
- Make predictions about behavior of utility-maximizers
- Evaluate welfare

Endowment Economy

- Consumers A and B; goods 1 and 2
- Endowments: $\omega^A = (\omega_1^A, \omega_2^A)$ and $\omega^B = (\omega_1^B, \omega_2^B)$
- Example: $\omega^A = (6, 4)$ and $\omega^B = (2, 2)$
- This means total endowment of good 1 is $\omega_1^A + \omega_1^B = 6 + 2 = 8$ and of good 2 is $\omega_2^A + \omega_2^B = 4 + 2 = 6$

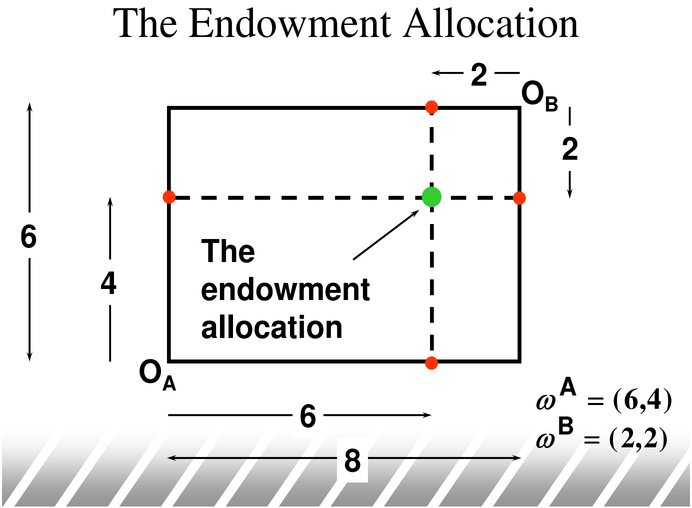
Allocations

- Endowment represents where people start, but through trade, their allocations may change
- General allocation or consumption: $x^A = (x_1^A, x_2^A)$ and $x^B = (x_1^B, x_2^B)$
- (x^A, x^B) is *feasible* if it uses at most the aggregate endowment:

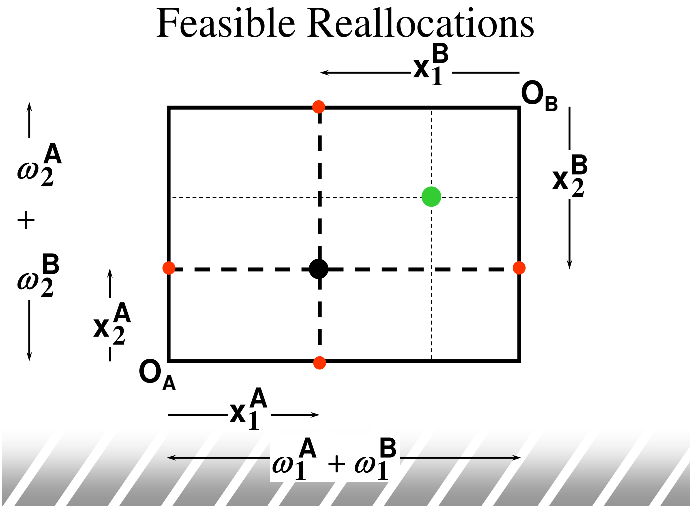
$$x_1^A + x_1^B \leq \omega_1^A + \omega_1^B \text{ and } x_2^A + x_2^B \leq \omega_2^A + \omega_2^B$$

- Helpful graphical tool: Edgeworth Box
- Allows us to simply depict all feasible allocations

Edgeworth Box



Edgeworth Box



How do we think about equilibrium?

- In partial equilibrium analysis:
  - Treat each good separately
  - Find p and q that equate supply and demand
- But this is general equilibrium analysis: where do supply and demand come from?
- A and B can trade with each other
- For everything to be balanced, the amount that A gives up has to equal amount that B receives (for each good, and vice versa)
- In other words $Supply = Demand$ for each good
- This will determine prices for each good
- How do we find supply and demand curves?
- Go back to utility maximization problem
- Need to specify preferences to do this

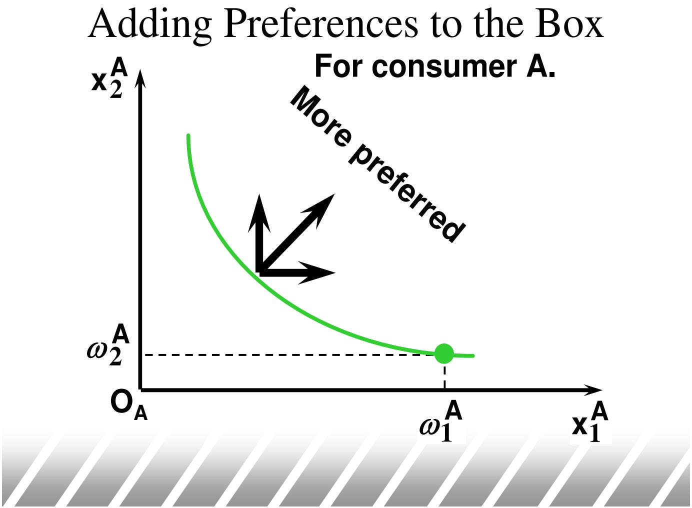
Utility maximization

- Preference are given
- Given prices for each good, endowment bundle serves as income
- Can write down budget constraint

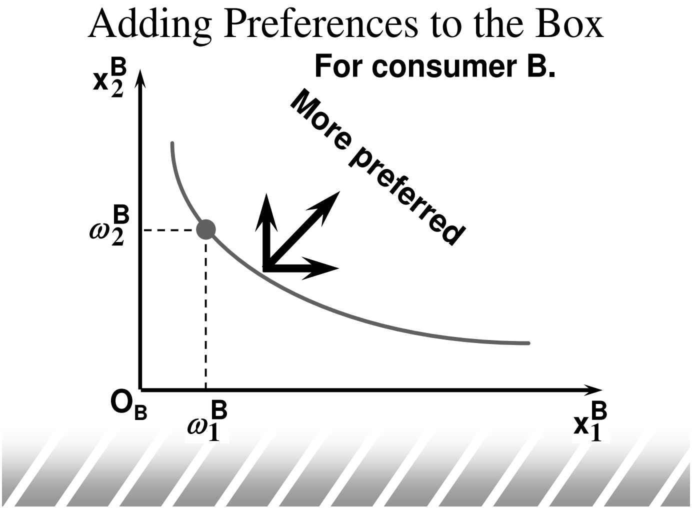
$$p_1 x_1 + p_2 x_2 \leq p_1 \omega_1 + p_2 \omega_2$$

- Solve utility maximization problem
- Gives you optimal allocation, as a function of price ratio
- $x_1^* - \omega_1 > 0$ means person demands more of good 1
- $x_1^* - \omega_1 < 0$ means person is willing to supply good 1
- Key question: what prices will make it so that A demand exactly as much of each good as B supplies?

Preferences of A

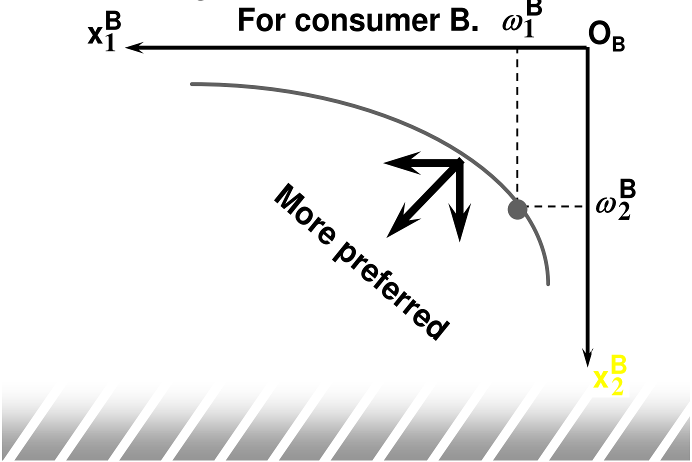


Preferences of B

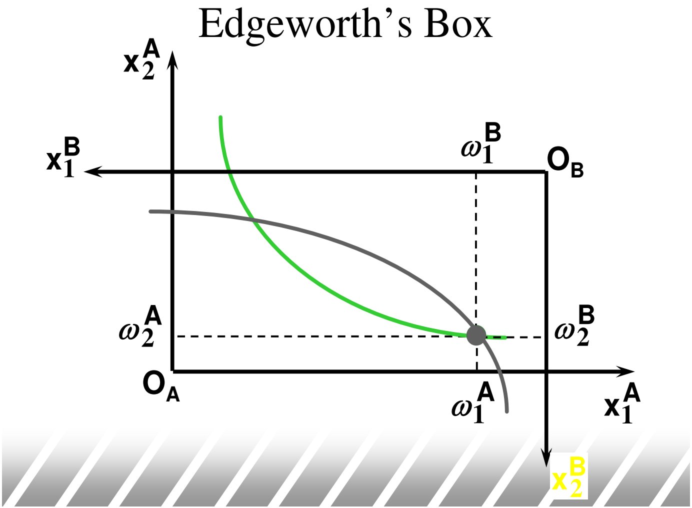


Preferences of B

Adding Preferences to the Box



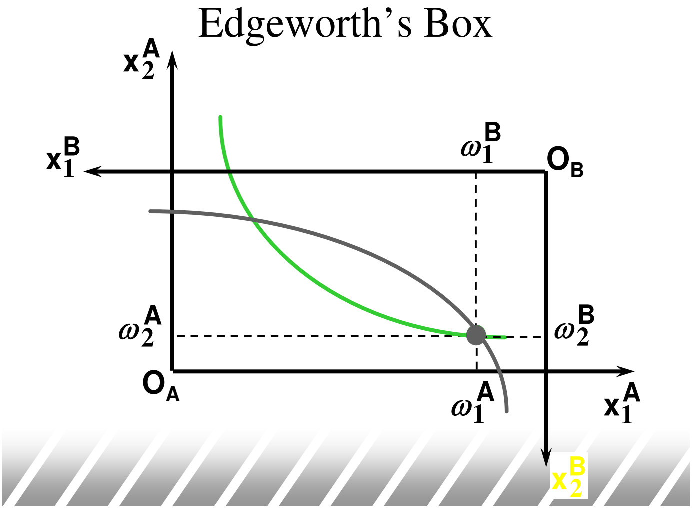
Putting both of them together



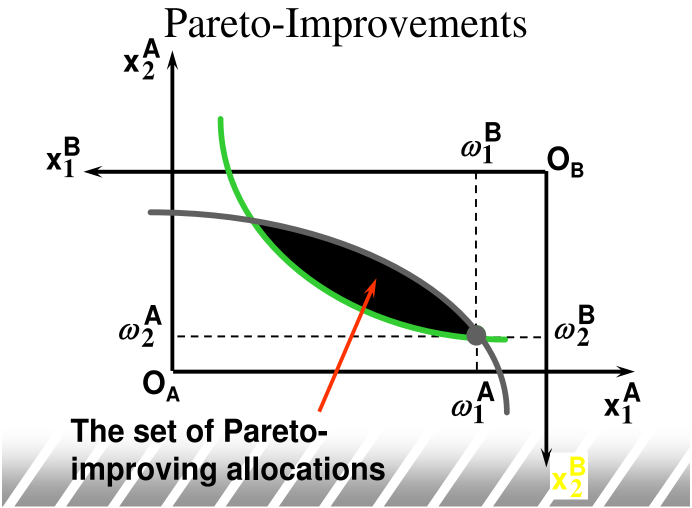
Pareto-improving allocations

- Given a particular allocation, a *Pareto-improving* allocation improves the welfare of at least one consumer *without reducing the welfare of another*.
- How do we depict Pareto-improving allocations in the Edgeworth box?

Edgeworth Box



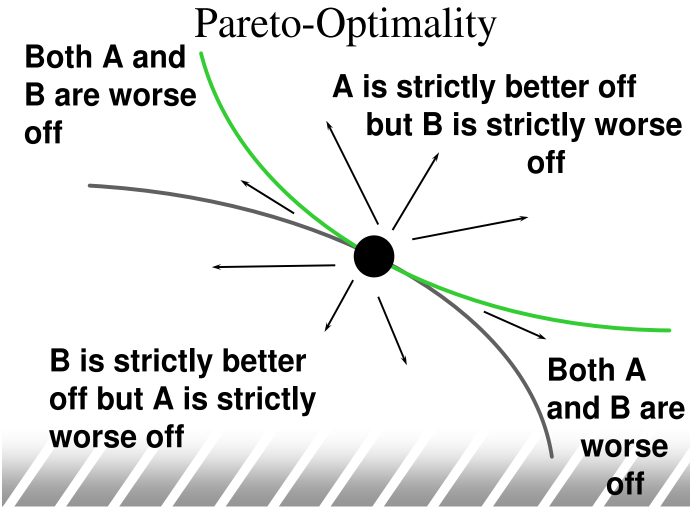
Pareto-improving allocations



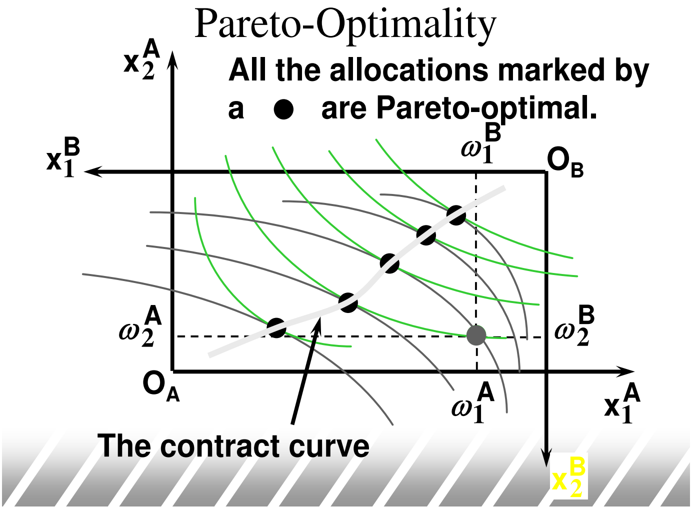
Pareto-optimal allocations

- An allocation is *Pareto-optimal* if it is *feasible* and there is other feasible allocation that is a Pareto-improvement over it.
- In other words, there is no was to make anyone better off without making someone worse off.
- The set of all Pareto-optimal allocations is called the contract curve.

A Pareto-Optimal Allocation



Pareto-Optimal Allocations



Pareto-optimal Allocations

- From the figures, we can see that an allocation at which the indifference curves of the two consumers are tangent must be Pareto-optimal
- Tangency implies they have the same slope
- What is the slope of an indifference curve? The Marginal rate of substitution (MRS)!
- Condition for Pareto-optimality:

$$MRS^A = \frac{\frac{\partial u^A(x_1^A, x_2^A)}{\partial x_1^A}}{\frac{\partial u^A(x_1^A, x_2^A)}{\partial x_2^A}} = \frac{\frac{\partial u^B(x_1^B, x_2^B)}{\partial x_1^B}}{\frac{\partial u^B(x_1^B, x_2^B)}{\partial x_2^B}} = MRS^B$$

- We also require feasibility:

$$x_1^A + x_1^B = \omega_1^A + \omega_1^B \text{ and } x_2^A + x_2^B = \omega_2^A + \omega_2^B$$

Example

Identifying Pareto-optimal allocations

- Recall total endoments: $\omega_1^A + \omega_1^B = 6 + 2 = 8$ and $\omega_2^A + \omega_2^B = 4 + 2 = 6$
- Let $u^A(x_1^A, x_2^A) = \ln(x_1^A) + 2\ln(x_2^A)$ and $u^B(x_1^B, x_2^B) = \ln(x_1^B) + 2\ln(x_2^B)$.
- MRS of consumer A:

$$MRS^A = \frac{\frac{1}{x_1^A}}{\frac{2}{x_2^A}} = \frac{x_2^A}{2x_1^A}$$

- Similarly,

$$MRS^B = \frac{\frac{1}{x_1^B}}{\frac{2}{x_2^B}} = \frac{x_2^B}{2x_1^B}$$

- So a Pareto-optimum is a feasible allocation for which

$$\frac{x_2^A}{2x_1^A} = \frac{x_2^B}{2x_1^B}$$

Clicker Vote

Which of these allocations is Pareto Optimal?

A) $(x_1^A, x_2^A, x_1^B, x_2^B) = (4, 2, 6, 3)$
B) $(x_1^A, x_2^A, x_1^B, x_2^B) = (4, 1, 4, 5)$
C) $(x_1^A, x_2^A, x_1^B, x_2^B) = (4, 4, 5, 1)$
D) $(x_1^A, x_2^A, x_1^B, x_2^B) = (4, 3, 4, 3)$
E) $(x_1^A, x_2^A, x_1^B, x_2^B) = (6, 4.5, 2, 1.5)$

Clicker Vote

Which of these allocations is Pareto Optimal?

A) $(x_1^A, x_2^A, x_1^B, x_2^B) = (4, 2, 6, 3)$

B) $(x_1^A, x_2^A, x_1^B, x_2^B) = (4, 1, 4, 5)$

C $(x_1^A, x_2^A, x_1^B, x_2^B) = (4, 4, 5, 1)$

$\Longrightarrow$ **D)** $(x_1^A, x_2^A, x_1^B, x_2^B) = (4, 3, 4, 3)$

$\Longrightarrow$ **E)** $(x_1^A, x_2^A, x_1^B, x_2^B) = (6, 4.5, 2, 1.5)$

Let's look at why. . .

Example

Identifying Pareto-optimal allocations

- Recall that a Pareto-optimum is a feasible allocation for which

$$\frac{x_2^A}{2x_1^A} = \frac{x_2^B}{2x_1^B}$$

- In other words, A and B need to have the same $x_1 : x_2$ ratio, which makes sense, because they have identical preferences
- Clicker option **A** meets the tangency condition, but is not feasible.
- **B** is feasible, but does not meet the tangency condition
- **C** satisfies neither condition.
- However, both **D** and **E** satisfy both conditions, so they both are Pareto-optimal.

Example

Finding *all* Pareto-optimal allocations (deriving the contract curve)

- We can simplify tangency condition to:

$$\frac{x_2^A}{x_1^A} = \frac{x_2^B}{x_1^B}$$

- Recall endowment/feasibility requirement:

$$x_1^A + x_1^B = 8 \text{ and } x_2^A + x_2^B = 6$$

- Re-write tangency condition, substituting $x_1^B = 8 - x_1^A$ and $x_2^B = 6 - x_2^A$:

$$\frac{x_2^A}{x_1^A} = \frac{6 - x_2^A}{8 - x_1^A}$$

  or

$$x_2^A = \frac{3}{4} x_1^A$$

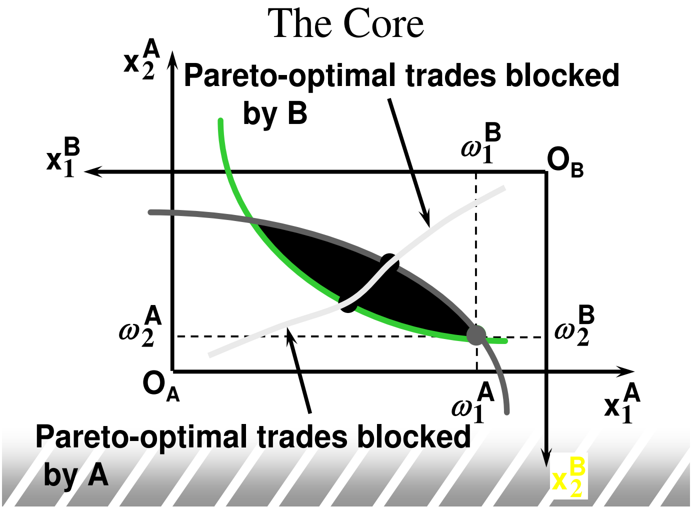
- This is the equation of the contract curve.
- In this case, it's just the diagonal of the rectangle

Core Allocations

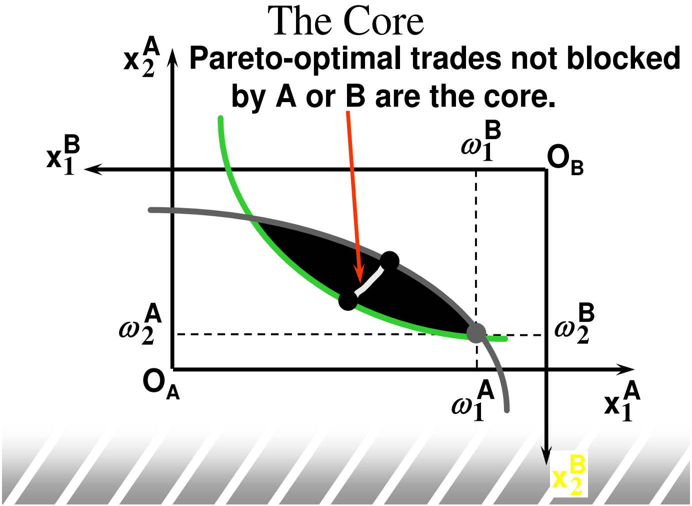
How can we narrow down our prediction about the outcome resulting from trade?

- We've derived the contract curve: $x_2^A = \frac{3}{4}x_1^A$
- Note that clicker options **D** and **E** are both on this curve.
- However, Pareto Optimal, but given her endowment, $(\omega^B = (2, 2)$, Person B would never agree to trade to allocation **E** $=$ (6,4.5,2,1.5).
- We need to restrict attention to Pareto-optimal allocations that are Pareto-improvements over the initial endowment.
- These allocations are called *core allocations*, or *the core*—the set of all PO allocations that are welfare improving for both consumers relative to their own endowments
- An allocation will be in the core if it is feasible and it is not *blocked* by any consumer

Core Allocations



Core Allocations



Competitive General Equilibrium

- A competitive equilibrium are prices(p_1, p_2) and allocations (x^A, x^B) such that
  1. Given the prices (p_1, p_2), the allocation (x^A, x^B) solves each consumer's utility maximization problem. That is, x^A is the solution of the consumer A's utility maximization problem, taking (p_1, p_2) as given, and similarly for x^B.

     $$\max_{(x_1^A, x_2^A)} u^A(x_1^A, x_2^A)$$

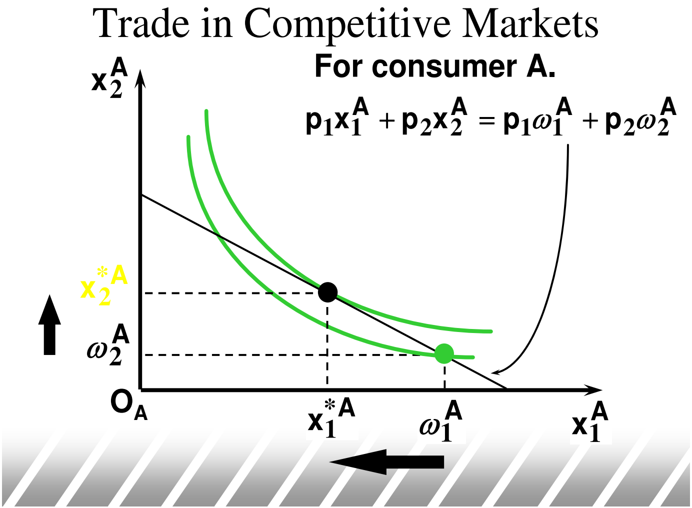
     s.t.

     $$p_1 x_1^A + p_2 x_2^A = p_1 \omega_1^A + p_2 \omega_2^A$$

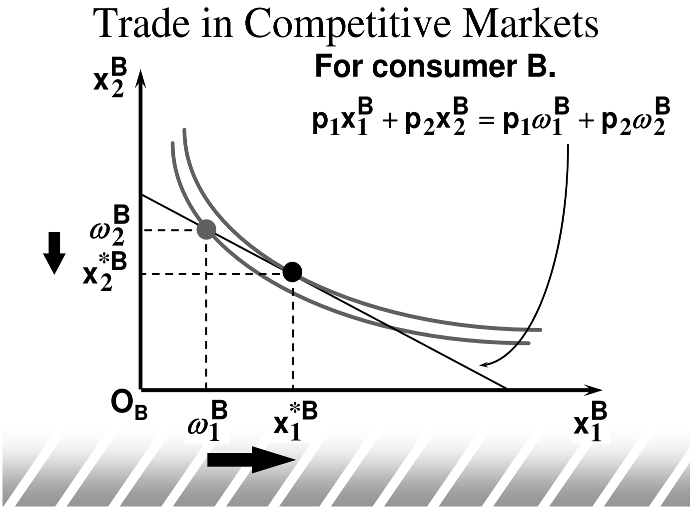
  2. Market clearing conditions: the total demand of good 1 is equal to the total endowment (supply) of good 1 and similarly for good 2.

     $$x_1^A + x_1^B \leq \omega_1^A + \omega_1^B \text{ and } x_2^A + x_2^B \leq \omega_2^A + \omega_2^B$$

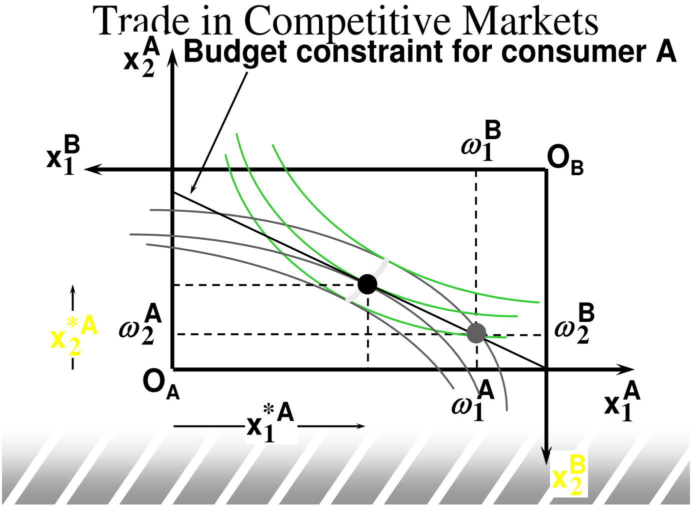
A's Utility Maximization



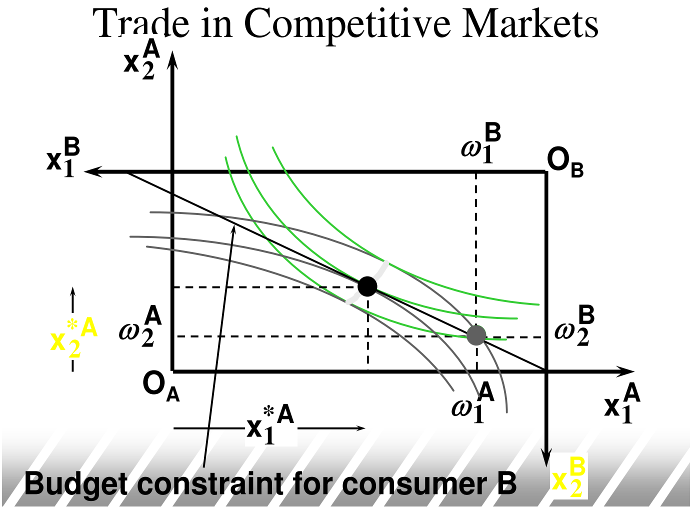
B's Utility Maximization



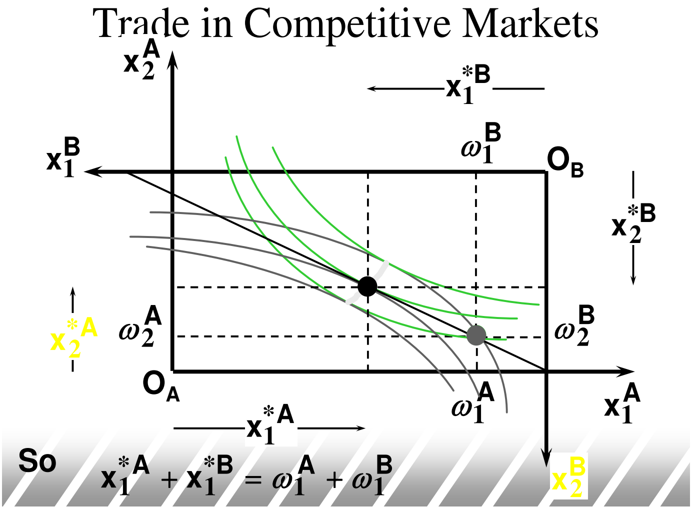
A's Utility Maximization



B's Utility Maximization



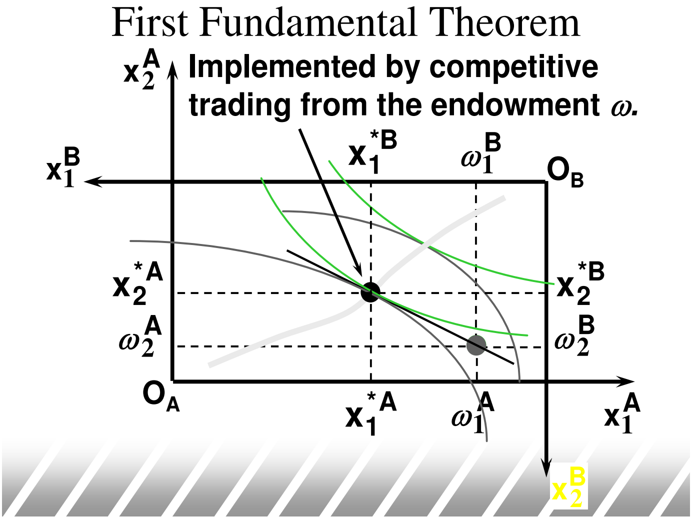
Market Clearing



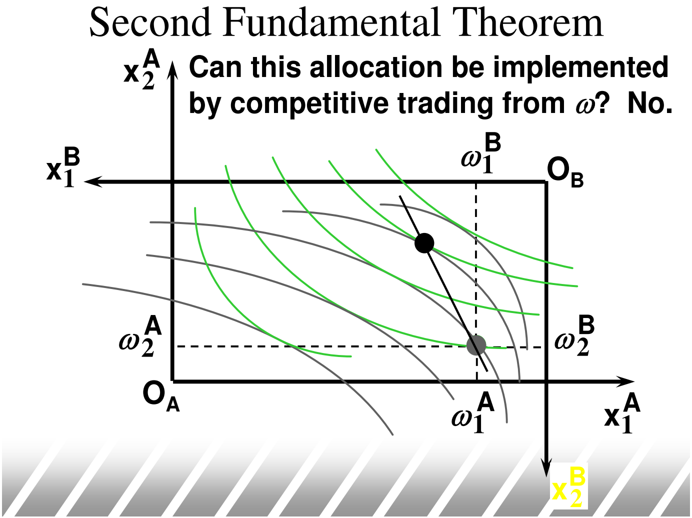
Fundamental Welfare Theorems

- First Welfare Theorem: Given that consumers preferences are well-behaved, trading in perfectly competitive markets implements a Pareto-optimal allocation of the economys endowment. That is, $CE \Rightarrow PO$.
- Second Welfare Theorem: any Pareto-optimal allocation (i.e. any point on the contract curve) can be achieved by trading in competitive markets provided that endowments are first appropriately rearranged amongst the consumers. That is, $PO \Rightarrow CE$.

First Welfare Theorem



Second Welfare Theorem



Second Welfare Theorem