

Free-response – 21 out of 50 pts. (2 qns., 10 pts. each + 1)

Answer these questions in your blue-book. Show your work and intermediate steps for partial credit. If you cannot complete part of the problem, e.g. if you get stuck on some algebra, you may still earn partial credit for explaining intuitively how you would solve if you were not stuck. You can expect to see fractions in your answers. Your score will **only** be based on the marks in your blue-book. You will not receive any credit for anything written on your exam paper. You will receive 1 point for correctly writing your name, perm number, version (A,B,C, or D), and TA's name on your blue-book.

1. Consider an exchange economy consisting of two people, A and B , endowed with two goods, 1 and 2. Each person is initially endowed with 5 of each good. Their preferences are given by $U^A(x_1, x_2) = x_1^2 x_2$ and $U^B(x_1, x_2) = x_1 x_2^2$. This question asks you to assume that $p_2 = 1$, then find the competitive equilibrium allocations and p_1 . (Points split equally across parts)

- (a) Find the MRS for each consumer

$$\text{Answer: } MRS^A = \frac{2x_2^A}{x_1^A} \text{ and } MRS^B = \frac{x_2^B}{2x_1^B}$$

- (b) Write the equation of the contract curve (express x_2^A as a function of x_1^A)

$$\text{Answer: } x_2^A = \frac{10x_1^A}{40-3x_1^A}$$

- (c) Write the equation of the budget constraint (in person A 's coordinates) as a function of p_1

$$\text{Answer: } \text{Some form of } x_2^A - 5 = -p_1(x_1^A - 5)$$

- (d) What expression involving x_1^A and x_2^A can we substitute for p_1 in your answer to the previous part?

$$\text{Answer: } \text{The MRS of } A, \text{ which is } \frac{2x_2^A}{x_1^A}$$

- (e) Find the competitive equilibrium price, p_1 and allocations, $x^A = (x_1^A, x_2^A)$ and $x^B = (x_1^B, x_2^B)$

Answer: This amounts to finding the point that is on both the contract curve and budget constraint. The solution is $x^A = (20/3, 10/3)$ and $x^B = (10/3, 20/3)$, from which you can determine $p_1 = 1$.

2. A bakery and coffee shop operate next to each other downtown. The bakery sells bags of donuts (d) for \$30 each and the coffee shop sells bags of coffee (k) for \$15. Making donuts and coffee is costless, but each shop needs to advertise to attract customers. Some of the customers lured by the advertising from one shop will also make a purchase at the other shop. Specifically, the bakery needs to spend $c(d) = \frac{d^2}{15} - \frac{dk}{30}$ on advertising in order to sell d donuts and the coffee shop must spend $c(k) = \frac{k^2}{15} - \frac{dk}{30}$ on advertising in order to sell k cups of coffee.

- (a) (2 points) What is the bakery's marginal private benefit of selling a bag of donuts and the marginal private cost? What is the coffee shop's marginal private benefit of selling a bag of coffee and the marginal private cost?

$$\text{Answer: } MPB_d = p_d = 30, MPC_d = \frac{2d}{15} - \frac{k}{30}, MPB_k = p_k = 15, MPC_k = \frac{2k}{15} - \frac{d}{30}$$

- (b) (2 points) How many bags of donuts and coffee will be sold in the competitive marketplace?

$$\text{Answer: } d = 270, k = 180$$

- (c) (2 points) The city looks out for its business owners and hires a consultant to determine how much the two shops should advertise in order to maximize total profits. What level of d and k does the consultant recommend?

$$\text{Answer: } d = 375, k = 300$$

- (d) (*4 points*) The city decides to subsidize d and k so as to induce the owners to choose the amounts recommended by the consultant. What should be the subsidy per bag of donuts, s_d , and the subsidy per bag of coffee, s_k ?

Answer: $s_d = 10$ and $s_k = 12.5$